SOME PROPERTIES OF LIQUID HELIUM BELOW 1° K

H. C. KRAMERS

STELLINGEN

I

Een onderzoek van de buiging van de fononenbundel door een nauwe opening zou een onafhankelijke methode zijn om geheel uit te sluiten, dat z.g. acoustische fononen een belangrijke rol spelen bij de in dit proefschrift beschreven experimenten over de voortplanting van warmtepulsen in het gebied beneden 0.4°K.

Dit proefschrift, § 4.5.

II

De oplossing, die Ziman geeft van de differentiaalvergelijking, die de voortplanting van warmtepulsen beschrijft in het gebied, waar de vrije weglengte van de fononen groot is vergeleken met de diameter van de meetbuis, is niet in overeenstemming met de fysische situatie in de gebruikelijke experimenten over de voortplanting van warmtepulsen in vloeibaar helium.

J. M. Ziman, Phil. Mag. (7) **45**, 360 (1954).

III

Bij de omrekening van oude waarnemingsresultaten op een nieuwe verbeterde temperatuurschaal, dient uitgegaan te worden van de oorspronkelijke calibratie van de bij de meting gebruikte thermometers.

W. H. Keesom, Helium, Elsevier 1942, blz. 217.

IV

Voor een onderzoek van de geldigheid van de formules van H. London in het temperatuurgebied beneden 1°K, verdienen metingen van het mechanocalorisch effect de voorkeur boven die van het fonteineffect.

V

Daniels en Robertson besluiten uit hun experimenten over de entropie van ceriummagnesiumnitraat, dat voor molaire entropieën tussen 0.506 R en 0.1 R de temperatuur constant is. Dit berust op een onjuiste interpretatie van de waarnemingen.

J. M. Daniels, F. N. H. Robertson, Phil. Mag. (7) **44**, 630 (1953).

VI

Bij het onderzoek van de paramagnetische relaxatie in het temperatuurgebied van vloeibaar helium zijn de verschijnselen veelal niet met één relaxatietijd te beschrijven. Met behulp van een kleine uitbreiding van een methode van Benzie en Cooke is het mogelijk de gemiddelde relaxatietijd te bepalen; bovendien kan men een schatting maken van de grootte van het interval van relaxatietijden.

R. J. Benzie, A. H. Cooke, Proc. Phys. Soc. **63 A**, 201 (1950).

VII

Het is mogelijk om voor lage tot zeer lage frequenties, met behulp van het electrisch-mechanisch trillingssysteem van een galvanometer een trillingskring met grote kringkwaliteit te realiseren. Geheel analoog aan het gebruik van een kwartskristal voor hoge frequenties zou men op grond hiervan een generator voor lage frequenties kunnen construeren.

VIII

Een van de beste methoden voor het op visuele wijze bepalen van de helderheid van kometen is de methode aangegeven door BEYER. Voor uitgebreide, maar zwakke kometen kan deze echter tot belangrijke systematische fouten leiden.

M. BEYER, Astr. Nachr. **250**, 233 (1933).

IX

Bij de beschrijving van de temperatuurafhankelijkheid van de electrische weerstand, wordt veelal als enige parameter een karakteristieke temperatuur θ ingevoerd, welke zijn fundering vindt in de theorie der roostertrillingen. Hierop kan ernstige critiek worden uitgeoefend.

X

Het verdient aanbeveling bij de beschrijving van kristallen met anorganische structuren, behalve tabellarische gegevens, ook een Debije-Scherreropname te publiceren, teneinde het opsporen van isomorfieën te vergemakkelijken.

XI

Bij het natuurkundig practicum voor prae-candidaten in het eerste jaar zou een systeem, dat meer het karakter heeft van een een-jaarlijkse cursus, de doelmatigheid van dit practicum zeer ten goede komen. Dit geldt in het bizonder voor hen, die natuurkunde als bijvak beoefenen.

XII

Bij het natuurkundig practicum voor prae-candidaten dient meer de nadruk gelegd te worden op de methoden der experimentele natuurkunde dan op het waarnemen van fysische verschijnselen.

XIII

De status van de z.g. wetenschappelijke staf aan de universiteit dient duidelijker vastgelegd te worden.

XIV

Het verdient aanbeveling bij onderzoekingen, die zich daartoe lenen, medewerkers van verschillende wetenschappen in te schakelen.

SOME PROPERTIES OF LIQUID HELIUM BELOW 1° K

SOME PROPERTIES OF LIQUID HELIUM BELOW 1° K

PROEFSCHRIFT

TER VERKRIJGING VAN DE GRAAD VAN
DOCTOR IN DE WIS- EN NATUURKUNDE
AAN DE RIJKSUNIVERSITEIT TE LEIDEN,
OP GEZAG VAN DE RECTOR MAGNIFICUS
DR. A. E. VAN ARKEL, HOOGLERAAR IN DE
FACULTEIT DER WIS- EN NATUURKUNDE,
PUBLIEK TE VERDEDIGEN OP WOENSDAG
1 FEBRUARI 1956 TE 16 UUR

DOOR

HENDRIK CORNELIUS KRAMERS

GEBOREN TE HEERDE IN 1922

1955
DRUKKERIJ HOLLAND N.V. – AMSTERDAM

Promotor: PROF. DR. C. J. GORTER

Teneinde te voldoen aan de wens van de Faculteit der Wis- en Natuurkunde volgt hier een kort overzicht van mijn universitaire studie.

Ik ving mijn studie aan in 1939 te Leiden en zette deze in 1941 voort te Groningen. Na een onderbreking ten gevolge van de bekende oorlogsomstandigheden legde ik daar in 1946 het candidaatsexamen letter A af.

Vanaf Januari 1947 werkte ik op het Kamerlingh Onnes Laboratorium te Leiden onder leiding van Prof. Dr C. J. GORTER.

In Juni 1951 legde ik het doctoraalexamen experimentele natuurkunde af. De tentamens in de theoretische natuurkunde en de mechanica werden mij afgenomen door Prof. Dr. H. A. KRAMERS en Dr. J. KORRINGA.

In 1947 werkte ik onder leiding van Dr. D. BIJL aan een onderzoek betreffende de paramagnetische relaxatie van enkele aluinen, welk werk door mij werd voortgezet in het academiejaar 1947–1948 tijdens het verblijf van deze te Oxford.

Vanaf October 1948 hield ik mij bezig met de opbouw van een opstelling voor metingen aan vloeibaar helium beneden 1° K; van de eerste resultaten (de soortelijke warmte) kon mededeling worden gedaan op het congres voor lage temperaturen te Oxford in 1951. Na de afsluiting van deze experimenten werkte ik aan het onderzoek van de voortplanting van warmtepulsen in vloeibaar helium in hetzelfde temperatuurgebied.

Na mijn doctoraal examen werd ik belast met een deel van de leiding van het practicum, aanvankelijk nog als assistent, vanaf 1 Januari 1953 echter als hoofdassistent, terwijl ik vanaf 1 Januari 1955 de rang van wetenschappelijk ambtenaar kreeg. In deze functie werkte ik nauw samen met Dr. M. J. STEENLAND.

De soortelijke-warmte-experimenten werden verricht tezamen met J. D. WASSCHER, de metingen aan warmtepulsen met achtereenvolgens mevr. T. VAN PESKI–TINBERGEN en J. WIEBES, terwijl de electronische apparatuur voor dit laatste onderzoek tot stand kwam in samenwerking met Ir. F. A. W. VAN DEN BURG.

De leden van de technische staf van het Kamerlingh Onnes Laboratorium, die in het bizonder betrokken zijn geweest bij deze experimenten, waren de cryogeen-technici D. DE JONG en T. NIEBOER en vooral de chef-glasblazer A. R. B. GERRITSE, die ook bij het ontwerpen van de glazen apparatuur van advies diende.

CONTENTS

INTRODUCTION

Liquid ^{4}He is probably the most remarkable liquid existing. It is in equilibrium with its vapour under atmospheric pressure at 4.2° K and is, therefore, with the exception of its rare isotope ^{3}He, the substance with the lowest boiling temperature. Moreover, helium is the only fluid which does not solidify under its saturated vapour pressure. Between 4.2° and 2.19° K, the so-called helium I region, the liquid behaves in many ways as an ordinary liquid, but in the realm of helium II – below 2.19° K, the so-called λ-point – it shows an abnormal behaviour. Many experiments have been performed on the properties of liquid helium II in the region above 1° K. An exceptionally large conduction of heat, leading to the idea of superfluidity, was observed. Moreover, the fountain effect and the mechanocaloric effect were observed. The phenomenon of the fountain effect is the existence of a pressure difference between two vessels connected by means of a narrow slit and maintained at different temperatures. The mechanocaloric effect is the production of heat (cold) in a vessel when helium is flowing out (in) through a narrow aperture. Later on the so-called second sound, showing the possibility of practical undamped propagation of heat waves, was discovered.

The temperature region below 1° K, which cannot be attained by the usual method of reducing the pressure above the liquid, has up till now produced only few experimental results. The normal helium region could be extended down to 0.8° K by using a diffusion pump with a very large capacity, but this is a comparatively small achievement [1, 2]. The obvious method for making experiments on the liquid below 1° K is, of course, to bring it in contact with a paramagnetic salt and to use the well-known demagnetization method for cooling.

In this way experiments on the specific heat, only above 0.6° K [3, 4] and on second sound [5, 6] were tried. The latter were performed by observing the propagation of heat pulses. Some preliminary results of experiments on the fountain effect [7] and the heat conductivity [8] were published. Recently some data on the attenuation of normal sound have also appeared [9].

The experiments below 1° K have one annoying obstacle in common: the temperature cannot be kept constant, but increases continuously towards the temperature of the surrounding bath (1° K). Therefore, a good insulation is required to preserve the temperature-equilibrium. Moreover, the amount of heating used in making the observations should be kept small as there exists no bath with a large heat capacity at these low temperatures.

Experiments on the specific heat and on the propagation of heat pulses will be described in the following pages. The measurements on the specific heat could be extended down to about 0.25° K by the use of a paramagnetic salt with small heat capacity. With the apparatus employed it was also possible to get new data on the specific heat up to 1.8° K.

The propagation of heat pulses was investigated in much greater detail than in the earlier experiments mentioned above. On the basis of the data obtained it could be suggested that second sound in the proper sense exists only in the temperature region above 0.65° K.

This thesis consists of four chapters. In the first a review is given of theoretical considerations required for an explanation of the experimental results. Chapter II contains the experimental arrangement and in chapter III and IV the results of the specific heat measurements [10] and of the observations on the heat pulses [11, 12] are presented and discussed.

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CHAPTER I

THEORETICAL CONSIDERATIONS

1.1. *Introduction.* It is the purpose of this chapter to give a picture of the theoretical background of the experiments under consideration. It is not the object to give a full account of the large amount of theoretical work done on the helium II problem. Review papers [1, 2, 3, 4, 5] may be referred to for a complete narration.

The present chapter contains mainly the part of the theory dealing with the specific heat and the propagation of second sound. Moreover, the considerations with respect to the latter are restricted to small amplitudes. The phenomena occurring in the vicinity of the λ-temperature only are also excluded.

A very successful approach to the problem of liquid helium has so far been made by means of the so-called two-fluid model. Curious phenomena as the fountain effect and the mechanocaloric effect could be explained. Moreover, on the basis of this model it was predicted that apart from normal sound a second kind of wave propagation was possible in the liquid. This effect, the so-called "second sound", was actually found experimentally by PESHKOV [6] in 1944.

In section 1.2 some consideration is given to the two-fluid model and on the basis of this model wave propagation in the liquid is considered in some detail in section 1.3. No attention will be paid to effects of large velocities; the hydrodynamical equations used will be restricted to the so-called acoustical approximation, i.e. to first order terms in the velocities. Included are, however, irreversible processes of this order as viscosity and heat conductivity.

The microscopic background of the two-fluid model is discussed in section 1.4. A microscopic theory is in any case required for an explanation of the results of the specific heat experiments. It seems that in the temperature region under consideration, i.e. not very close to the λ-point and in particular below 1° K, the theory of excitations originally put forward by LANDAU [7, 8] and improved especially by KRONIG and collaborators [9, 10, 11] and by FEYNMAN [5, 12, 13] gives the most promising approach. The connection between this theory and the two-fluid model is discussed in section 1.5.

The irreversible processes occurring in liquid helium II can be analysed by considering the interactions between the excitations. A summary of the work of **Landau** and **Khalatnikov** [14, 15, 16, 17, 18] is given in section 1.6.

In section 1.7 attention is paid to the influence of the mean free path which becomes presumably very large on second sound propagation at low temperatures. It is argued in the same section that below 0.6° K no second sound in the proper sense can occur.

1.2. *The two-fluid model.* The two-fluid model describes the properties of liquid helium below the λ-point by considering the liquid as a kind of mixture of two inseparable fluids. The first one behaves as a normal liquid and is, therefore, called the normal fluid. There exists, for instance, a normal viscosity. The second part behaves as an ideal non-viscous liquid and is, therefore, called the superfluid.

At any point in the liquid the density is now built up of two parts:

$$\varrho = \varrho_n + \varrho_s. \tag{1.01}$$

At the temperature of the transition to the normally behaving helium I liquid, the so-called λ-point, ϱ_n becomes equal to ϱ and ϱ_s equal to zero: the liquid loses its peculiar properties. At absolute zero it is supposed that no normal fluid is left and, therefore, ϱ_s becomes equal to ϱ.

Historically the two-fluid model was first formulated by **Tisza** [19] in 1940. He based the model on the ideas of F. **London** [20] who gave as a possible explanation of the liquid helium problem the analogy with the phenomenon of the condensation in momentum space occurring in a perfect gas, obeying **Bose–Einstein** statistics. It proved, however, very difficult to extend the theory of a perfect gas to the liquid, particularly because of the large interactions between the molecules in the latter. Certainly these interactions cannot be included by means of simple perturbation methods. Independently **Landau** [7] developed his ideas on a quantum hydrodynamics and arrived at a two-fluid model. At that time, however, **Landau**'s ideas were considered to be rather obscure and intuitive and did not contribute much to a clear picture of the microscopic background of the two-fluid model. In its present clarified shape this theory will be discussed in a later section of this chapter.

In the meantime the two-fluid model was disconnected from any microscopic explanation [21]. As a pure phenomenological theory it contributed much to the explanation and description of the properties of liquid helium II. The fountain effect and the mechanocaloric effect could be expressed in terms of thermodynamical quantities. The conclusion of the possibility of a second kind of wave propagation followed immediately from the presence of two hydrodynamical equations, one for the normal and one for the super-fluid.

The thermodynamic foundations of the two-fluid model were laid by H. **London** [22] and by **Gorter** [23], who arrived at somewhat different conclusions. H. **London** supposed the whole entropy to be carried by the normal fluid. The equations of GORTER included the more general case of a non-zero entropy of the superfluid and a possible entropy of mixing. Finally DE GROOT, JANSEN and MAZUR [24] have considered the problem on the basis of the thermodynamics of the irreversible processes. Their conclusions are in agreement with GORTER's. It is not the purpose to discuss here the thermodynamic foundations in detail. In the following the equations of the two-fluid model will be written in the general form used by GORTER. The more special equations used by LONDON must also be mentioned, because they are in accordance with the formulation of the two-fluid model on the basis of the theory of excitations described in section 1.4. As will be seen in that case the entropy of the superfluid is zero per definitionem.

Effects of the first degree in the velocities will be considered only. The mutual friction term of GORTER and MELLINK [25] for instance is omitted.

The equations of motion for the normal and the super-fluid respectively are [23]:

$$(1.02) \quad \varrho_n \frac{d\mathbf{v}_n}{dt} = -\frac{\varrho_n}{\varrho} \operatorname{grad} P - \frac{\varrho_n \varrho_s}{\varrho} S^* \operatorname{grad} T + \eta_n (\nabla^2 \mathbf{v}_n + \tfrac{1}{3} \operatorname{grad} \operatorname{div} \mathbf{v}_n)$$

$$(1.03) \quad \varrho_s \frac{d\mathbf{v}_s}{dt} = -\frac{\varrho_s}{\varrho} \operatorname{grad} P + \frac{\varrho_n \varrho_s}{\varrho} S^* \operatorname{grad} T$$

$\mathbf{v}_n$ and $\mathbf{v}_s$ are the velocities of the normal fluid and the superfluid, P is the pressure, η_n the coefficient of the normal viscosity and ∇^2 is the Laplace operator. As has been stated, the superfluid is supposed to have no viscosity. The second term on the right in both equations is the diffusion force which has, of course, the same magnitude but opposite sign in equation 1.02 and equation 1.03.

The quantity S^* has the dimension of the entropy, in the original description by GORTER

$$S^* = \partial S / \partial x$$

x being the relative concentration of the normal fluid ϱ_n / ϱ.

According to LONDON and LANDAU,

$$S^* = \varrho S / \varrho_n$$

S being the entropy per gram of the liquid.

From these equations the dependence of the pressure gradient due to the fountain effect on the temperature gradient can be shown directly by considering the case of zero velocities and accelerations (the steady state). This yields

$$(1.04) \quad \operatorname{grad} P = \varrho_n S^* \operatorname{grad} T$$

which specifies to

$$(1.05) \quad \operatorname{grad} P = \varrho S \operatorname{grad} T$$

according to H. LONDON.

When a liquid is flowing into or from a vessel through a hole which only allows the superfluid to pass, heat is absorbed or evolved in a reversible way. This is the so-called mechanocaloric effect. This effect and the fountain effect are in the general case connected by the relation

$$\operatorname{grad} P/\operatorname{grad} T = \varrho Q/T. \quad [24]$$

The amount of heat withdrawn or developed per gram of the liquid flowing in or out is

$$Q = \varrho_n T S^* / \varrho \tag{1.06}$$

according to the general theory of GORTER.

It reduces to

$$Q = ST \tag{1.07}$$

in H. LONDON's derivation.

Finally it should be remarked that TISZA [21] made the special assumption

$$x = S/S_\lambda. \tag{1.08}$$

S_λ being the entropy at the λ-point. This formula appears to be approximately realised in the temperature region above 1° K.

1.3. *Wave propagation through the liquid.* The equations of motion 1.02 and 1.03 can be used for a consideration of propagation of a wave through the liquid. To give the derivation extra equations determining the conservation of the quantities involved have to be added.

The first of these equations is, of course, the equation of continuity of the total density,

$$\frac{\partial \varrho}{\partial t} + \operatorname{div} (\varrho_n \mathbf{v}_n + \varrho_s \mathbf{v}_s) = 0. \tag{1.09}$$

As to the second equation different choices can be made [4]. The equation used here is that of the conservation of the entropy. It is supposed that at any time and at any point in the liquid the quantity x depends only on the temperature. In that case

$$\frac{\partial (\varrho S)}{\partial t} - \frac{\varrho_n \varrho_s}{\varrho} S^* \operatorname{div} (\mathbf{v}_s - \mathbf{v}_n) - \frac{\lambda}{T} \nabla^2 T = 0. \tag{1.10}$$

The last term in this equation is due to the effect of irreversible heat conductivity of common type. λ is the appropiate coefficient. As will be seen, this effect has certainly to be included. Probably it gives the main contribution to the absorption of second sound. As usual in the derivation of the velocity of sound, only small linear deviations from the overall equilibrium values of the velocities, the pressure and the temperature are considered. Moreover, thermal expansion can be neglected in this case. Thus, for instance:

$$(\partial P/\partial \varrho)_S = (\partial P/\partial \varrho)_T.$$

Taking these simplifications into consideration, using the second law of thermodynamics on equation 1.10, introducing the relative velocity $\mathbf{w}=\mathbf{v}_n-\mathbf{v}_s$ and the total current flow $\mathbf{j}=\varrho_n\mathbf{v}_n+\varrho_s\mathbf{v}_s$, one gets after a simple transformation of 1.02 and 1.03 the set of four equations:

$$\frac{\partial \mathbf{w}}{\partial t}-S^* \operatorname{grad} T'+\frac{\eta_n}{\varrho\varrho_n}(\nabla^2+\tfrac{1}{3}\operatorname{grad}\operatorname{div})(\mathbf{j}+\varrho_s\mathbf{w})=0 \tag{1.11}$$

$$\frac{\partial \mathbf{j}}{\partial t}+\frac{\partial P}{\partial \varrho}\operatorname{grad}\varrho'-\frac{\eta_n}{\varrho}(\nabla^2+\tfrac{1}{3}\operatorname{grad}\operatorname{div})(\mathbf{j}+\varrho_s\mathbf{w})=0 \tag{1.12}$$

$$\frac{\partial \varrho'}{\partial t}+\operatorname{div}\mathbf{j}=0 \tag{1.13}$$

$$\frac{c_v}{T}\frac{\partial T'}{\partial t}+\frac{\varrho_n\varrho_s}{\varrho^2}S^*\operatorname{div}\mathbf{w}-\frac{\lambda}{\varrho T}\nabla^2 T'=0. \tag{1.14}$$

The fluctuations of T and ϱ from their equilibrium values are indicated by a dash.

As a solution of these equations a plane wave in the z direction is considered. The dependence of $\mathbf{w}$, $\mathbf{j}$, ϱ' and T' on t and z is now of the form $\exp\{i\omega(t-z/v)\}$. The result is

$$v^2 w_0+vS^* T_0'+\tfrac{4}{3}i\frac{\eta_n\omega}{\varrho\varrho_n}(j_0+\varrho_s w_0)=0$$

$$v^2 j_0+v\frac{\partial P}{\partial \varrho}\varrho_0'-\tfrac{4}{3}i\frac{\eta_n\omega}{\varrho}(j_0+\varrho_s w_0)=0$$

$$v\varrho_0'-j_0=0$$

$$v^2\frac{c_v}{T}T_0'-v\frac{\varrho_n\varrho_s}{\varrho^2}S^*w_0-\frac{i\lambda\omega}{\varrho T}T_0'=0$$

the suffix 0 indicating the amplitudes.

By the elimination of T_0' and w_0 one gets two homogeneous equations in j_0 and ϱ_0' compatible for

$$v_{\mathrm{I}}^2=\frac{\partial P}{\partial \varrho}+\frac{i\omega}{\varrho}\tfrac{4}{3}\eta. \tag{1.15}$$

By first eliminating j_0 and ϱ_0' two equations in w_0 and T_0' result, yielding in a similar way:

$$v_{\mathrm{II}}^2=\frac{\varrho_n\varrho_s}{\varrho^2}\frac{TS^{*2}}{c_v}+\frac{i\omega}{\varrho}\left(\frac{4}{3}\frac{\varrho_s}{\varrho_n}\eta_n+\frac{\lambda}{c_v}\right) \tag{1.16}$$

v_{I} is the velocity of normal sound, v_{II} that of second sound.

Quadratic terms in ω are neglected in this derivation. Therefore, it is only valid for not too high frequencies. More explicitly $16\eta_n^2\varrho_s\omega^2/9\varrho^2\varrho_n$ must be small compared to the "static" value of v_{I}^2 and $4\eta_n\varrho_s\lambda\omega^2/3\varrho^2\varrho_n c_v$ small compared to the "static" value of v_{II}^2.

The nature of the two "sounds" is clear from this derivation. In the first sound the energy is oscillating between the kinetic energy connected to the total current $\mathbf{j}$ and the elastic energy connected to the local deviations of ϱ from its equilibrium value. This process is reversible apart from the damping caused by viscosity. Damping from the heat conductivity does not occur, thermal expansion being neglected. In practice this is fully justified.

On the other hand in the second sound the energy is oscillating between the kinetic energy connected to the relative motion of the two fluids and the energy connected to the local deviation of T from its equilibrium value. Because of the supposition that the local value of x is always in equilibrium with the local value of T, this latter energy behaves as an elastic energy of the normal fluid (or the superfluid). Again this process is reversible apart from the damping, to which this time both viscosity and heat conductivity contribute.

In H. LONDON's description the static value of v_{II}^2 becomes

$$v_{\mathrm{II}}^2=\frac{\varrho_s}{\varrho_n}\frac{S^2T}{c_v}. \tag{1.17}$$

The general form of the complex dispersion equation of the velocities can be written

$$v^2=v_0^2+i\omega\alpha, \tag{1.18}$$

the suffix 0 indicating the velocity at zero frequency.

Of course, real dispersion only occurs in the second approximation. The absorption coefficient β can be easily found; it is equal to the imaginary part of ω/v:

$$\beta=\frac{1}{2}\frac{\omega^2}{v_0^3}\alpha. \tag{1.19}$$

This gives, however, only a part of the actual attenuation. KHALATNIKOV's calculations which will be discussed later on lead to the introduction of second viscosity. This means adding terms γ_1 grad div $\mathbf{v}_s$ and γ_2 grad div $\mathbf{v}_n$ to equation 1.03 and γ_3 grad div $\mathbf{v}_s$ and γ_4 grad div $\mathbf{v}_n$ to equation 1.02. The values of α are consequently changed and become

$$\alpha_{\mathrm{I}}=\frac{1}{\varrho}\left(\tfrac{4}{3}\eta+\gamma_1+\gamma_2+\gamma_3+\gamma_4\right) \tag{1.20}$$

$$\alpha_{\mathrm{II}}=\frac{1}{\varrho}\left\{\frac{\varrho_s}{\varrho_n}\left(\tfrac{4}{3}\eta+\gamma_4\right)-(\gamma_2+\gamma_3)+\gamma_1\frac{\varrho_n}{\varrho_s}\right\}+\frac{\lambda}{\varrho c_v}. \tag{1.21}$$

The β values, of course, are changed in the same manner.

Absorption effects due to a finite relaxation time for the transformation of the normal into the super-fluid and *vice versa* were not considered. If such effects are to be included, x is no longer a function of T only. These effects were discussed by KRONIG *et al.* [26] and GORTER *et al.* [27] and may be responsible for the high attenuation of first and second sound near the λ-point. Because this is out of the range of the present experiments, they will not be discussed here.

It should be emphasized that only effects due to absorption in the bulk liquid have been considered. For a consideration of the extra attenuation in narrow tubes (surface effects) may be referred to publications by

Dingle [28] and by Peshkov [29]. In those papers attention is also given to the coupling occurring between the two sounds.

Large amplitude effects (shock waves) have also been omitted. These have been considered by several authors (see Dingle [2]).

1.4. *The theory of elementary excitations.* As has already been mentioned, a microscopic explanation of the helium II problem has been tried along two lines, both ways of attack leading to the two-fluid model. At first sight the theory of the Bose–Einstein condensation appears to be the most promising one, especially in view of the apparent absence of a λ-point in ^{3}He which obeys, per force, Fermi–Dirac statistics. Unfortunately, however, only little progress could be made with respect to quantitative conclusions, although the recent approach by Feynman to this problem appears somewhat more promising [30].

Instead of starting with an explanation of the λ-transition the theory of excitations considers the problem beginning at absolute zero. It is not intended to give a complete account of the development of this theory from the original quantum-hydrodynamics of Landau [7], via papers by H. A. Kramers [31] and Kronig *et al.* [9, 10, 11] to the recent work of Feynman [12, 13, 14]. The only purpose is to give a very short summary of this theory as it stands now.

At absolute zero ^{4}He is, of course, in its quantummechanical ground state. Since the zero-point energy is very large, the small attractions between the atoms are not sufficient to form a crystal-lattice as occurs with all other substances (with the exception of ^{3}He). Helium remains therefore a liquid down to absolute zero [32].

If internal friction would occur in this liquid, the kinetic energy of helium flowing through a tube would gradually be diminished, the energy being transferred to the wall. Such a process should have to start by first exciting the internal motion of the liquid in the neighbourhood of the walls. This can easily be seen by considering the situation with respect to the coordinate system of the moving liquid. As has been pointed out by Landau, these excitations cannot be created, unless the velocity of flow is very large and consequently for moderate velocities the liquid is a frictionless superfluid. Its motion is of a potential kind, i.e. the curl of its velocity is zero.

When the liquid is heated, excited states are occupied. The obvious possibilities for those low energy states are standing sound waves. By appropriate linear combination of these, progressive waves can be formed and by quantization the so-called phonons are produced. Certain localization can be ascribed to them by forming wave-packets in the usual way. Thus one can form a picture of "particles" moving in the underlying superfluid liquid. For the phonons applies, of course:

$$\omega = v_1 k. \tag{1.22}$$

The energy attributed to one phonon is

$$\varepsilon = \hbar\omega \tag{1.23}$$

and it carries a momentum

$$\mathbf{p} = \hbar\mathbf{k}. \tag{1.24}$$

Consequently also

$$\varepsilon = v_I p. \tag{1.25}$$

k is the wave vector, i.e. the vector with the direction perpendicular to the planes of equal phase and with a magnitude of 2π over the number of wavelengths per cm. v_I is the velocity of the phonons, i.e. the velocity of sound.

The energy of a phonon at a temperature T is of the order $\varkappa T$, $\varkappa$ being the Boltzmann constant. Thus at 1° K $\omega \approx 10^{11}$ Hz. The average wavelength $\frac{2\pi}{k}$ of a phonon is, therefore, of the order of 10^{-6}, i.e. about 30 times the atomic distance. For this reason in a theory of phonons no attention has to be given to the atomic structure, in other words the liquid can be considered as a continuum.

The procedure of quantization, roughly indicated here, has been placed on a sound quantum mechanical base by KRONIG and THELLUNG [9].

A second kind of excitation may occur with wavelengths of the order of the distance between the atoms (about 3×10^{-8} cm), i.e. k of the order of 2×10^{8} cm^{-1}. The energy of these short wave excitations is much larger than that of the long wave phonons. On the other hand it can be argued that exciting a state with wave vector of the order of the reciprocal of the atomic separation is easier than states with somewhat larger or smaller k. Consequently a minimum may be supposed to occur in the $\varepsilon(k)$ plot of these excitations.

The energy of the short wave excitations may then be written, in any case as a first approximation,

$$\varepsilon = \Delta + \frac{\hbar^2(k^2 - k_0^2)}{2\mu} = \Delta + \frac{(p^2 - p_0^2)}{2\mu} \tag{1.26}$$

where μ is an effective mass, Δ the energy gap with the ground state of the liquid. To these new excitations the same "particle"-like behaviour can be assigned as that of the phonons.

The exact nature of the short wave excitations is as yet not completely understood. LANDAU concluded to similar excitations by quantizing vortex motion in the quantumliquid. He called them "rotons". This name is now in common use, so it will be used here, though it is by no means sure that rotation is characteristic of these excitations.

The nature of the rotons was more closely considered by FEYNMAN. Of course, for a calculation of the parameters of equation 1.26 from first principles a more exact picture is required than that given above.

Feynman's treatment has not been completely successful in this respect. The difficulty is to find the right wave function. He shows that several possibilities lead to an equation of the form 1.26. They involve, however, always a close consideration of the atomic structure of the liquid. Landau's procedure of quantizing vortex motion without taking account of this appears now to be dubious.

Feynman [13] and also H. A. Kramers [31] were the first to emphasize the importance of the atomic structure for an explanation of the rotons, although A. Bijl [33] just in considering the motion of one atom with respect to others, arrived at excitations of a similar character even before the introduction of the name "roton".

Because $\Delta/\varkappa$ is experimentally found to be about 9° K, at temperatures of the order of 1° K only those rotons with wave vector just about $k \approx k_0 = 2.0 \times 10^8$ cm^{-1} are excited. This is perhaps the reason for the good results with the simple $\varepsilon(k)$ dependence of equation 1.26.

The $\varepsilon(k)$ curve of the excitations in helium II is shown in fig. 1.1. Normally only phonons just near the origin are present. (At 1.5° K the average wave number of a phonon is about 10^7 cm^{-1}.)

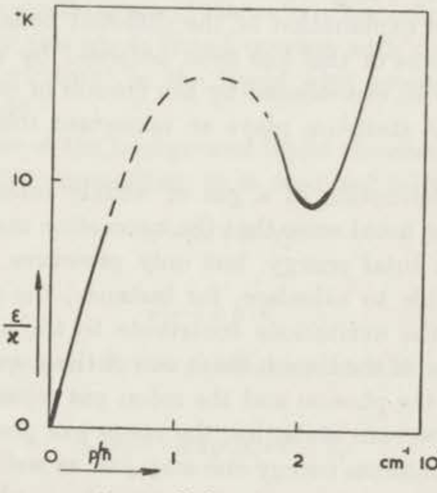

Fig. 1.1. Energy spectrum of the elementary excitations in He II.

The particle-like excitations thus formed can "collide" with each other. If the density of the excitations and their interaction is not too large, they can be considered as a "gas" in thermodynamic equilibrium to which normal statistical methods can be applied. The difference with a real gas is that the number of particles in general is not preserved. The interaction of the phonons is a consequence of the non-linearity of the equation of state of the liquid [9]; the interaction between phonons and rotons and rotons with each other is a more difficult problem, the nature of the rotons not being specified. Some kind of interaction, however, will certainly occur. The excitations can also collide with the walls of the container, thus being responsible for the transport of energy and momentum to the walls. Consequently friction will occur and the liquid will, for instance, show viscosity. A relation of the excitations with the *normal* part of the liquid presented in the two-fluid model becomes evident.

The condition of not too many excitations together with not too large interaction puts a limit to the applicability of this theory. With rising temperature the amount of excitations increases rapidly. What exactly occurs to them at the λ-point is as yet rather unintelligible, but in any case it is evident that near this point the density of the excitations becomes too large to conserve the picture of a gas of weakly interacting single "particles". Probably there is no sense in talking of single excitations in this region. As will be seen, deviations pointing to this appear to occur in the region of a few tenths of a degree below the λ-point only.

FEYNMAN [5] considered the reason for the scarcity of the number of excited states in the whole temperature range not too near the transition. This scarcity, as has been said, is essential for the formation of a gas of single excitations. He concludes that this scarcity is a natural consequence of the BOSE–EINSTEIN statistics obeyed by the ^{4}He liquid. In a FERMI–DIRAC liquid probably a large amount of low energy states is present. This would give a simple explanation of the different behaviour of ^{3}He.

Another explanation of this has been proposed by DE BOER [34]. He considers a roton to be represented by the motion of two atoms in a cell. Then, of course, the statistics plays an important role in the behaviour of a roton.

Accepting now the notion of a gas of weakly interacting excitations (weak meaning in the usual sense that the interaction makes no appreciable contribution to the total energy, but only preserves the internal equilibrium), it is possible to calculate, for instance, the specific heat. It is obvious that only the excitations contribute to the energy, the specific heat, the entropy etc. of the liquid. Each one of these quantities is the sum of contributions of the phonon and the roton gas separately. The phonon gas obeys BOSE–EINSTEIN statistics, the roton gas probably also, but in view of the large minimum energy one may just as well apply BOLTZMANN statistics in the latter case.

The energy of the phonon gas per gram of the liquid is consequently

$$E_{\text{ph}} = \frac{1}{\varrho}\,\frac{1}{(2\pi)^3}\int \frac{\varepsilon}{\exp(\varepsilon/\varkappa T)-1}\,\mathrm{d}^3\mathbf{k}. \tag{1.27}$$

By inserting the expressions 1.24 and 1.25, integrating and differentiating with respect to T the specific heat can be calculated. The outcome is a simple low temperature DEBIJE term, proportional to T^3. It can be written

$$c_{\text{ph}} = \frac{16}{15}\,\pi^5\,\frac{\varkappa^4}{\hbar^3}\,\frac{1}{\varrho}\,\frac{T^3}{v_{\text{I}}^3}. \tag{1.28}$$

The only difference with the usual DEBIJE specific heat occurring in crystals is the absence of transverse waves.

The calculation of the roton contribution can be carried out in a similar way. The total energy of the roton gas per gram of the liquid is

(1.29)
$$E_r = \frac{1}{\varrho} \frac{1}{(2\pi)^3} \int \varepsilon \exp(-\varepsilon/\varkappa T) \, \mathrm{d}^3\mathbf{k}.$$

This yields for the specific heat

(1.30)
$$c_r = \frac{2\mu^{1/} p_0^2 \Delta^2}{\varrho (2\pi)^{3/2} \varkappa^{1/2} \hbar^3} \frac{1}{T^{3/2}} \left[1 + \frac{\varkappa T}{\Delta} + \frac{3}{4}\left(\frac{\varkappa T}{\Delta}\right)^2\right] \mathrm{e}^{-\Delta/\varkappa T}.$$

The free energy and the entropy can, of course, be determined in the same way, using well-known thermodynamic expressions.

At low temperatures expression 1.28 should give the main contribution to the total specific heat, at higher temperatures expression 1.30 predominates and the temperature dependence is much steeper.

1.5. *The derivation of the two-fluid model.* The derivation of the equations of the two-fluid model from the theory of excitations is given in several papers (e.g. Kronig [11]). A short review is given here.

The first problem is to get a notion of ϱ_n. For the time being interactions between the excitations and the walls of the container are neglected. One has now firstly the whole liquid moving with velocity $\mathbf{v}_s$. Secondly the excitations may "drift" in the liquid with average velocity $\mathbf{w}$ with respect to the liquid.

Due to the motion of the background liquid the energy of an excitation of wave vector $\mathbf{k}$ (i.e. momentum $\mathbf{p}$) is modified by the Doppler effect:

$$\omega' = \omega + \mathbf{k} \cdot \mathbf{v}_s.$$

Consequently also

(1.31)
$$\varepsilon' = \varepsilon + \mathbf{p} \cdot \mathbf{v}_s.$$

The distribution function of the excitations at rest with respect to the liquid (i.e. $\mathbf{w} = 0$) is

(1.32)
$$f_0(p) = 1/\exp(\varepsilon/\varkappa T - 1).$$

This is modified to

(1.33)
$$f(\mathbf{p}) = 1/\{\exp(\varepsilon - \mathbf{w} \cdot \mathbf{p})/\varkappa T - 1\},$$

if the excitations have a drift velocity.

In the latter two formulae it is supposed that Bose–Einstein statistics are obeyed, but this has no influence on the resulting general formulae. Also ε and $\mathbf{p}$ are chosen as variable instead of ω and $\mathbf{k}$, because this is somewhat more illustrative.

Developing $f(\mathbf{p})$ in a series of ascending powers of $\mathbf{w}$ gives

(1.34)
$$f(\mathbf{p}) = f_0(p) - \frac{\partial f_0}{\partial \varepsilon} \mathbf{w} \cdot \mathbf{p} + \ldots.$$

It is easy now to calculate the total momentum, i.e. the total mass flow of a unit of volume of the liquid. The total momentum of the excitations in the z direction is

$$P_z = \frac{1}{h^3} \int f(\mathbf{p})\, p_z \,\mathrm{d}^3\mathbf{p}.$$

Using the relation 1.34 and neglecting all but the first power in w_z yields

$$P_z = w_z \times \frac{4\pi}{3h^3} \int_0^\infty \frac{\partial f_0}{\partial \varepsilon} p^4 \,\mathrm{d}p = \varrho_n w_z. \tag{1.35}$$

This can be considered as the *definition* of the normal density ϱ_n. By adding $\varrho \mathbf{v}_s$ one finds the total current of the flowing liquid

$$\mathbf{j} = \varrho \mathbf{v}_s + \varrho_n \mathbf{w} = \varrho_n \mathbf{v}_n + \varrho_s \mathbf{v}_s \tag{1.36}$$

with $\mathbf{v}_n = \mathbf{w} + \mathbf{v}_s$ i.e. the mean velocity of the excitations in the laboratory system and $\varrho_s = \varrho - \varrho_n$. This is in complete agreement with the two-fluid model.

In the same way taking into account also the squares of the velocities one finds for the total kinetic energy

$$E_{\mathrm{kin}} = \tfrac{1}{2}\varrho_n v_n^2 + \tfrac{1}{2}\varrho_s v_s^2.$$

It is now easy to find the equations of the two-fluid model from the laws of conservation of energy and momentum. They can be expressed in ϱ_n and thermodynamic quantities. The latter can be found from statistical formulae analogous to 1.27 and 1.29. An additional assumption is made by putting curl $\mathbf{v}_s = 0$. Only potential movement is supposed to be possible in the superfluid [12].

The law of conservation of energy, only involving the excitations, of course, yields

$$\frac{\partial(\varrho S)}{\partial t} + \mathrm{div}\,(\varrho S \mathbf{v}_n) = 0. \tag{1.37}$$

This equation shows clearly that the entropy is only connected with the normal fluid and is identical with equation 1.10 in H. London's interpretation of the two-fluid model, except for the heat conductivity term which is neglected in the present derivation.

Secondly the law of conservation of momentum of the excitations gives

$$\partial(\varrho_n \mathbf{w})/\partial t = -\varrho S \;\mathrm{grad}\; T. \tag{1.38}$$

This equation is analogous to 1.11.

Equations 1.37 and 1.38 give together the wave equation for second sound, irreversible effects being neglected. The interpretation of second sound in the excitation theory is now also clear. On the one hand one has the movement of the excitation gas with respect to the underlying liquid. On the other hand one has the local density fluctuations of the energy of the excitation gas. These density fluctuations are analogous to density fluctuations of the gas itself. Thus there is a close resemblance between a second sound wave in the excitation gas and an ordinary sound wave in a gas of molecules. This analogy is not quite exact, because the motion of the background liquid plays an essential role.

Finally the law of conservation of momentum applied to the entire liquid yields the expected equation

$$\partial(\varrho_{\mathrm{n}}\mathbf{v}_{\mathrm{n}}+\varrho_{\mathrm{s}}\mathbf{v}_{\mathrm{s}})/\partial t=-\frac{1}{\varrho}\operatorname{grad}P. \tag{1.39}$$

Together with the law of conservation of mass 1.13 the relations 1.37, 1.38 and 1.39 form the four equations of the two-fluid model. 1.13 and 1.39 together give the wave equation of first sound. It should be stressed again that these derivations are only correct, if one can neglect other than linear terms in the velocities. Otherwise quantities as ϱ_{n} and S are dependent on the velocities and consequently lose their simple meaning. This is, however, not the only complication, not even the most important one occurring at large velocities (see section 1.8).

A few words must be added about the velocity of second sound according to this theory. As the equation of state of the phonons is known (formula 1.25), the ϱ_{n} due to the phonons can be calculated by means of formula 1.35. Moreover, as has been seen, S_{ph}, $c_{v\mathrm{ph}}$ can also be calculated. All quantities of equation 1.17 are now known and the velocity of second sound in the temperature region below 0.5° K, where only phonons are important, can be computed. At absolute zero it is found that $v_{\mathrm{II}}^2=\frac{1}{3}v_{\mathrm{I}}^2$. This means $v_{\mathrm{II}}=137$ m/sec. Since Tisza [21] had predicted a value of v_{II} decreasing with temperature below 1° K and finally becoming zero at $T=0$, it could be considered as a great triumph for Landau's theory, when experiments revealed that indeed a rapid increase of velocity below 1° K occurred. As will be shown, however, complications arise at these low temperatures.

At higher temperatures, where rotons play an important role, equation 1.17 can be employed to calculate ϱ_{n} from measurements of v_{II}. Also by using equation 1.35 information can be gained about the constants occurring in the equation of state of the rotons. Formula 1.35 yields for the roton contribution of ϱ_{n}:

$$\varrho_{\mathrm{nr}}=\tfrac{2}{3}(2\pi)^{-3/2}\frac{\mu^{1/2}\hbar}{\varkappa^{1/2}}\left(\frac{p_0}{\hbar}\right)^4\times\frac{1}{T^{1/2}}\left[1+\frac{6\mu\varkappa}{\hbar^2}\left(\frac{\hbar}{p_0}\right)^2T\right]\mathrm{e}^{-\Delta/\varkappa T}. \tag{1.40}$$

This equation and a similar equation of c_v (formula 1.30) make it possible to calculate Δ, p_0 and μ from experiments on the second sound velocity and the specific heat together.

In the derivation of the four equations of the two-fluid model irreversible effects were neglected. This is equivalent to the assumption that the quantities are in thermal equilibrium, though they may still be functions of coordinates and time. This is only true, if the times required for establishment of equilibrium in the gas of excitations is negligibly small compared to the times characterizing the change of the macroscopic conditions (e.g. the reciprocal of the frequency of a wave of first or second sound), or, what is essentially the same, if the mean free path of the excitations is negligibly small compared to the distances over which the macroscopic conditions change appreciably (e.g. the wavelength). If this condition is no longer fulfilled, but the ratio of times or distances is still small, effects of deviations from equilibrium occur, but can be treated as linear deviations. These have been considered in the theories of Landau and Khalatnikov. The result of this theory is the introduction of extra terms in the equations of the two-fluid model. These are the terms of first and second viscosity and heat conductivity already mentioned in section 1.2 and section 1.3. In section 1.6 some attention is given to Khalatnikov's calculations.

If the mean free paths are of the order of the wave lengths, a quantitative description of the phenomena is practically impossible. The extreme case of *very large* mean free paths gives again the possibility of a quantitative theory, if the dimensions of the experimental vessel are small compared to them. The latter two cases are discussed in section 1.7.

1.6. *The theory of Khalatnikov.* Essentially Khalatnikov tries to solve the problem of the coefficients of the irreversible effects in helium II by considering the interactions or collisions between the excitations in a way very analogous to that of the kinetic theory of gases [35].

In a formal way [17] one can start with the general Boltzmann equation for the gas of excitations,

$$\frac{\partial f}{\partial t}+\frac{\partial}{\partial \mathbf{r}}(f\dot{\mathbf{r}})+\frac{\partial}{\partial \mathbf{p}}(f\dot{\mathbf{p}})=J(f). \tag{1.41}$$

f is the appropiate non-equilibrium distribution function and $J(f)$ is the so-called collision integral, dependent on the interactions of the particles in question. For small effects in the sense of the last part of section 1.5, i.e. for small velocities and small deviations from equilibrium, the first order of approximation is sufficient. This means that in the left hand side of equation 1.41 the equilibrium function f_0 of equation 1.33 can be inserted. In a formal way the Boltzmann equation can now be solved for $f-f_0$ which proves to contain terms proportional to div $\mathbf{v}_n$ and div $\mathbf{v}_s$, grad T and a term with a tensorial character. This new value of f can now be inserted in the conservation laws for energy and momentum in a way analogous to the procedure followed in section 1.5 with respect to f_0. The first two terms of $f-f_0$ give rise to second viscosity terms, the third term to the heat conductivity term and the last term to first viscosity only connected with $\mathbf{v}_n$. The result is the set of equations 1.11 to 1.14 of section 1.3 in which the second viscosity terms mentioned in section 1.5 are inserted.

In this way Khalatnikov indicates the formal introduction of the effects mentioned. For an actual calculation of the coefficients mentioned, however, one has to know the nature and magnitude of different kinds of interactions occurring. This is a very difficult problem especially because, particularly with respect to the rotons, the nature of the excitations is still an open question. This problem has been attacked by Landau and

KHALATNIKOV in a number of papers [14, 15, 16, 18]. A very short summary of the results will be given here (see also WILKS [45]).

Firstly the normal viscosity and the heat conductivity will be considered. These effects are determined by the transport of momentum and energy. This transport can be achieved by rotons as well as by phonons. For this reason the coefficients of the viscosity (η) and heat conductivity (λ) are the sum of a roton (η_r, λ_r) and a phonon (η_{ph}, λ_{ph}) contribution. The suffix n of η_n will be omitted from now onwards.

To calculate the viscosity KHALATNIKOV starts by writing down two separate BOLTZMANN equations, one for rotons and one for phonons, only taking into account a velocity gradient. The procedure for the calculation of the heat conductivity is similar, now using the appropriate BOLTZMANN equations with a temperature gradient. The rotons behave as heavy particles compared to the phonons. (The value of p_0 at $T = 1°$ K is about 50 times the average momentum of a phonon.) Therefore, the influence of the phonons on η_r and λ_r is negligible. The interaction of the rotons is unknown. The best approach which can be made to the solution of the problem is by taking the interaction energy to be proportional to a δ-function of the distance between two rotons. This is very similar to the case of a gas of solid spheres. The result is, of course, a value of η_r independent of temperature and a value of λ_r proportional to $1/T$. A common constant has to be found from the experiment. KHALATNIKOV uses for this the rather constant value of η measured between 1.4° K and 1.8° K [36]. The phonon viscosity (η_{ph}) is supposed to be relatively small in this region.

The calculation of η_{ph} and λ_{ph} is much more complicated. Three different interaction processes have to be taken into account. Firstly the elastic scattering of phonons, secondly collision processes of phonons accompanied by the creation or annihilation of a phonon (inelastic scattering) and lastly the scattering of phonons by rotons. The first problem is to find the distribution function f in the case of the inequilibrium conditions due to the gradient in question.

The mentioned processes are each characterised by a characteristic time dependent on temperature. This time indicates the rate of restoration of equilibrium by each process and is closely connected with the collision time.

In the calculation of the phonon elastic scattering an important role is played by the nonlinear terms in the energy momentum relation of a phonon. KHALATNIKOV has to make a very rough guess as to its magnitude and, therefore, the results in this case are in general rather uncertain. It is, however, not unreasonable to accept with KHALATNIKOV that this process has by far its largest probability (i.e. its largest cross section) for small angle scattering. This results in a very rapid exchange of energy in one direction of motion of the phonons.

The inelastic scattering processes cannot be calculated. The most important one is a 5-phonon process, i.e. 3 phonons are reverted into two or *vice versa*. KHALATNIKOV has to adopt the magnitude of the appropriate time constant $\Theta^{\mathrm{i}}_{\mathrm{phph}}$ to experimental data on the absorption of first sound. $\Theta^{\mathrm{i}}_{\mathrm{phph}}$ is proportional to T^9. (The upper index i stands for "inelastic", e will indicate "elastic".)

The process of scattering of phonons by rotons again is calculated by Khalatnikov. He assumes that phonons and rotons interact, because the heavy rotons influence the local value of the density and, inasmuch a phonon is a density wave, collisions can take place.

As the energy spectrum of a roton is known, if data are available on quantities as $\frac{\partial \Delta}{\partial \varrho}, \frac{\partial^2 \Delta}{\partial \varrho^2}, \frac{\partial p_0}{\partial \varrho}$, the cross section of this process can be calculated. Also the value of $\frac{\partial v_{\mathrm{I}}}{\partial \varrho}$ is required.

The time constant $\Theta^{\mathrm{e}}_{\mathrm{rph}}$ proves to be proportional to $1/N_{\mathrm{r}}T^4$, N_{r} being the number of rotons per cm^3. As the above mentioned quantities are not very accurately known (they can be found from experiments on the pressure dependence of v_{II} [46, 47] and v_{I} [48, 49], the non-temperature-dependent factor of $\Theta^{\mathrm{e}}_{\mathrm{rph}}$ is only approximately determined.

The main function of the elastic scattering of phonons, as has been mentioned, is a rapid exchange of energy for phonons moving in a certain direction. For this reason f has the form of a simple equilibrium distribution function, but with a temperature T', in general not equal to the local mean temperature of the phonongas. $T'-T$ is a function of the direction with respect to the direction of the gradient (in the viscosity case also of the direction of the macroscopic velocity).

General equilibrium within the phonon gas is opposed by the process of scattering of phonons by rotons. This latter process proceeds much slower than the small-angle phonon-phonon scattering just mentioned, but faster than their large-angle scattering.

The main influence of the 5-phonon process is a tendency to attain the equilibrium *number* of phonons belonging to the temperature of the phonon gas. Moreover, the rapid non-elastic process in which two phonons are converted into one and *vice versa*, noted by H. A. Kramers [31], contributes probably also to this. Owing to the conservation laws it can only occur without change of direction. This process has been excluded by Khalatnikov. The cross section of the 5-phonon process has a large maximum for small angles.

At temperatures below 0.9° K it is found that $\Theta^{\mathrm{i}}_{\mathrm{phph}} \ll \Theta^{\mathrm{e}}_{\mathrm{rph}}$ and, therefore, the equilibrium with respect to the number of phonons is actually established in the phonon gas. At higher temperatures these time constants appear to be of about the same order of magnitude and the non-equilibrium of the number of phonons has also to be accounted for in the evaluation of f.

At the lower temperatures ($T < 0.7°$ K) $\Theta^{\mathrm{e}}_{\mathrm{rph}}$ becomes very large, because the number of rotons decreases very rapidly with decreasing temperature. In the case of viscosity the phonon-phonon scattering process then replaces the phonon-roton process in the limitation of the momentum transport. According to Khalatnikov this process cannot affect the heat conductivity. A similar situation exists in the theory of the heat conductivity in non-metal solids [37].

In the final part of the calculation the computed values of f are inserted in the equation for transport of momentum and energy respectively. After integration over the directions η_{ph} and λ_{ph} are readily found.

The main temperature-dependent factor of η_{ph} is $T^{-1/2}e^{\Delta/\varkappa T}$, of λ_{ph}: $T^{-3/2}e^{\Delta/\varkappa T}$. Neglecting the contribution from the phonon-phonon elastic collisions to the reciprocal viscosity (only occurring at the lowest temperatures) one finds

$$\lambda_{\mathrm{ph}}/\eta_{\mathrm{ph}} = 5 v_{\mathrm{I}}^2/T. \tag{1.42}$$

Comparison of η_{ph} and η_{r} shows that the latter contribution is the main one above 1.4° K (actually this has been adapted in this way), but that at lower temperatures η_{r} is negligible. λ_{r} and λ_{ph} are of equal magnitude at 2° K, but again at lower temperatures λ_{r} rapidly loses its importance.

For an analysis of the absorption of second sound it is important to compare the viscosity and heat conductivity contributions. The ratio of these two according to equation 1.21 and using 1.42 is

$$\alpha_{\mathrm{II}\,\eta}/\alpha_{\mathrm{II}\,\lambda} = \tfrac{4}{3}\eta\,\frac{\varrho_{\mathrm{s}}}{\varrho_{\mathrm{n}}}\Big/\frac{\lambda}{c_v} \approx 0.05 \tag{1.43}$$

independent of temperature. This is true, if η_{r} and λ_{r} can be neglected (certainly below 1° K).

The second viscosity coefficients are calculated by Khalatnikov along a somewhat different line. He argues that inequilibrium conditions of *numbers* of excitations are responsible for the anomalous high attenuation of first sound. Therefore, processes involving annihilation and creation of excitations play here an important role. Only the already mentioned 5-phonon process and a process in which 2 rotons are transformed into 1 roton and a phonon or *vice versa* contribute appreciably. Other processes have too small a probability. These processes are characterised by the relaxation times $\Theta^{\mathrm{i}}_{\mathrm{phph}}$ and $\Theta^{\mathrm{i}}_{\mathrm{rph}}$.

The approach to equilibrium numbers in the roton and phonon gas can be described by the equations

$$\begin{cases} \dot{N}_{\mathrm{r}} + \operatorname{div}(\mathbf{v}_{\mathrm{n}} N_{\mathrm{r}}) = -A_{\mathrm{rr}}\,\delta\mu_{\mathrm{r}} + A_{\mathrm{rph}}\,\delta\mu_{\mathrm{ph}} \\ \dot{N}_{\mathrm{ph}} + \operatorname{div}(\mathbf{v}_{\mathrm{n}} N_{\mathrm{ph}}) = A_{\mathrm{phr}}\,\delta\mu_{\mathrm{r}} - A_{\mathrm{phph}}\,\delta\mu_{\mathrm{ph}} \end{cases} \tag{1.44}$$

$\delta\mu_{\mathrm{r}}$ and $\delta\mu_{\mathrm{ph}}$ are deviations from equilibrium of the partial Gibbs functions for the roton and the phonon gas respectively. The coefficients A are directly connected with the relaxation times.

From the equation 1.44 together with the four equations of the two-fluid model the laws of propagation of the "sounds" can be deduced, again only for the case of linear deviations. μ_{r} and μ_{ph} are considered as a function of t and z of the form $\exp i\omega\,(t - z/v)$ and one can proceed with the six equations in a way analogous to that of section 1.3.

It is also possible to express μ_r and μ_{ph} in terms of div $\mathbf{v}_n$ and div $\mathbf{v}_s$. By appropriate elimination one gets in this way the four equations of the two-fluid model now including the different second viscosity coefficients mentioned in section 1.3. It should be emphasized that this procedure of introducing the second viscosity terms is a very formal one. The advantage is that only four equations are required for describing the wave propagation in liquid helium II. Moreover, those equations only contain normal thermodynamic quantities. In the way of description with six equations the quantities μ_r and μ_{ph} which are directly connected to the phonon-roton picture still occur.

As has been mentioned, the two fundamental scattering processes cannot be calculated exactly. Both effects contain a numerical factor which has to be deduced from the experimental results on the absorption of first sound. The main contribution of this absorption is due to the "second viscosity" effect, only a small term originates from the normal viscosity. KHALATNIKOV uses experiments of PELLAM and SQUIRE [18] to evaluate the numerical factors. They have been recalculated by CHASE [39] on the basis of his latest experiments.

The contribution of the second viscosity effect to the absorption of *second* sound is only a small one compared to the heat conductivity term. It is of the same order as the contribution from the normal viscosity.

The conclusion to be drawn from all this is that although many uncertain points exist in the theory of KHALATNIKOV, the general tendency of the absorption of first and second sound to increase towards low temperatures is explained rather well. The main effect, however, is the decrease of the phonon and especially the roton density with decreasing temperature. Therefore, details of the theory of interactions may be wrong without very much affecting the results. It is clear that especially measurements below 1° K can give a good check on the theory, because the coefficients of the irreversible processes increase appreciably with decreasing temperature. In any case KHALATNIKOV's theory is the only one available at present. Finally it should be remarked that five independent irreversible coefficients are introduced (of the four second viscosity coefficients only three are independent), but that only three combinations of them are in principle measurable, i.e. η, α_I and α_{II}. The heat conductivity cannot be measured independently, because convection effects always occur in helium II.

Here it should be emphasized that the introduction of these coefficients for an explanation of the absorption effects of first and second sound makes only sense, when the absorption coefficient is proportional to ω^2. This is equivalent to the assumption of small frequencies compared to the reciprocal relaxation times of the fundamental processes mentioned. If this assumption is no longer valid, the only approach to a solution of the absorption problem can be made by considering those processes directly. This should be a very difficult problem to solve exactly. In practice this may occur at low enough temperatures, because all relaxation times increase with decreasing temperature. In the case of first sound Chase and Herlin [40] find indeed two maxima in the absorption curve as a function of temperature. Below those the absorption decreases with temperature, i.e. the frequency of sound is now larger than the reciprocal relaxation times. The two maxima can be contributed to the relaxation times of the two inelastic processes mentioned. The original Khalatnikov picture is only valid for the high temperature slope of the absorption curve. With second sound the situation is somewhat more complicated (see section 1.7).

This theory can, of course, not explain the high rise of the absorption near the λ-point. As this effect is beyond the scope of the present experiments, it will not be considered here.

Finally for a comparison with experiments reference is made to chapter IV.

1.7. *The mean free path and second sound.* As has been pointed out, Khalatnikov's theory can only lead to the simple introduction of coefficients for viscosity etc., if the deviation from equilibrium conditions is small. Restricting ourselves from now onwards to the case of second sound this condition means that the time of establishment of equilibrium at ordinary temperatures (mainly determined by $\Theta^{e}_{\mathrm{rph}}$) should be small compared to the reciprocal of the frequency of the wave; or, what is essentially the same, the mean free path of the phonons has to be small compared to the wave length. Khalatnikov himself estimates the mean free path at 1° K to be of the order of 10^{-4} cm, but at 0.6° K of the order of 0.1 cm, increasing very rapidly (proportional to T^{-9}) below that temperature. The low temperature estimates are, of course very uncertain, because the effect of phonon-phonon interaction, dominating in this region, is not adequately dealt with. The number of rotons is very small below half a degree.

A second sound wave at 0.6° K of frequency say $\nu = 20000$ Hz has a wave length also of a few mm and so one sees that the propagation of such a wave becomes very difficult; the condition mentioned above is no longer fulfilled. At still lower temperatures the damping is so large that there is no trace left of a periodic exchange of energies characteristic of second sound. In other words, second sound can no more exist. In contrast to first sound the absorption of second sound does not decrease again at very low temperature. The reason of this is the fast decrease with temperature of the density of the gas of excitations. The situation is rather analogous to that of the propagation of sound in a very diluted gas, i.e. only some kind of diffusion process of phonons is left. In chapter IV the results of experiments on the propagation of heat pulses will also be described for this region. An explanation of these results is very difficult, because many effects are contributing. Firstly a heat pulse contains a broad band of second sound wavelengths. The long waves will survive over a longer distance, the shorter ones are damped away immediately. Secondly the mean free path of a phonon is a much more complicated conception than that of a molecule in a gas. High frequency phonons will have a much shorter "mean" free path compared to the low frequency phonons. As the phonons are subject to a Planck distribution, again a smearing out of the effects occurs. Thirdly, it is seen, that the mean free paths become easily of the order of centimetres, i.e. of the order of the dimensions of the experimental vessel. This may again seriously affect the picture.

When indeed the mean free path increases rapidly with decreasing temperature, this last effect becomes predominating at low enough temperatures. In that case the path of a phonon is no longer limited by a collision with a roton or another phonon, but only by the walls of the vessel. The general picture is now again a very simple one and is essentially that of a phonon gas in Knudsen conditions. As will be seen in chapter IV the results of measurements on heat pulses below 0.4° K indicate such a behaviour.

The possibilities of the effects noted in this section have been first pointed out by Gorter [41] and also by Atkins [42]. Gorter especially emphasized the non-existence of second sound at a low enough temperature for normal second sound frequencies and normal dimensions of the experimental space. Possibly second sound would still be found in experiments on a very large volume of liquid and with very small frequencies.

There is one last effect which may influence the phenomena at the lowest temperatures. Helium normally used for experiments contains a very small concentration of ^{3}He atoms (for helium from wells about 1 in 10^7). It is not unreasonable to suggest that these atoms may limit the free path of the phonons, when the other effects of roton-phonon or phonon-phonon collisions give no more an effective limitation. One could, for instance, imagine an interaction of phonons with ^{3}He atoms in a similar way as that of phonons with rotons, i.e. by means of the local disturbance of the density by the ^{3}He atom.

Such a limitation of the mean free path is in agreement with experiments on second sound in mixtures of ^{3}He and ^{4}He in which the amount of ^{3}He is much larger [43]. In these experiments second sound indeed exists down to the lowest temperature values without any appreciable attenuation. As will be shown, the present experiments also indicate an influence from the presence of a small amount of ^{3}He.

1.8. *Final remarks.* In the present state of the theory of excitations it is not possible to explain adequately large amplitude effects, the possible existence of a critical velocity of flow or the mutual friction introduced by Gorter and Mellink on the basis of experiments. As has been pointed out by Onsager [44] and by Feynman [5], some kind of turbulence occurring in the superfluid may be responsible. This implies that curl $\mathbf{v}_s$ is no more zero. A complete solution of this question, however, appears to be very difficult.

The second limitation has been noted already. No explanation can be given of the phenomena near the λ-point nor of the λ-point itself.

The conclusion as to the reliability of the theory within the limits set by these remarks has to be postponed till the discussion of the experimental results.

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CHAPTER II

THE EXPERIMENTAL ARRANGEMENT

In this chapter the arrangement for the experiments which are the subject of this exposition will be described. A large part of the cryogenic arrangement to which sections 2.1 to 2.5 will be devoted is common to both the experiments. Section 2.1 contains the general set-up of the cryostat and some details of the construction; in sections 2.2 and 2.3 the magnetic cooling and the thermal insulation are considered; section 2.4 gives a short survey of the well-known method for measuring the temperature magnetically, while section 2.5 contains some details of the performance of an experiment.

In sections 2.6 and 2.7 the special apparatus respectively for the specific heat and the heat pulse experiments is described. Finally section 2.8 reviews the electronics needed in the heat pulse experiments.

1.1 *General set-up and construction.* The central part of the apparatus consisted of a glass vessel part of it being occupied by the paramagnetic salt for the cooling process by demagnetization and part serving as a helium reservoir (see fig. 2.1). It was connected to a pumping tube (P_1) by means of a narrow glass capillary which served as a mechanical support. The capillary and the inner vessel were surrounded by a vacuum jacket which could be evacuated by means of a second pumping tube (P_2). The vacuum jacket was immersed in the helium bath. The helium dewar and the hydrogen dewar which surrounded it were of the normal "tail" type used for experiments with a magnet, i.e. its lower part had a small diameter to allow not too large a distance between the poles of the magnet. The temperature of the helium bath could be regulated in the usual way by adjusting the pressure. This pressure was read on a mercury manometer for values above 3 mm Hg and on a carefully calibrated Mac Leod gauge for lower values.

The pumping tubes which connected the inner vessel and the "vacuum"-jacket were fixed to the head of the cryostat. They could be connected separately to the main high-vacuum tube leading to the diffusion pump (see fig. 2.2). Because for filling at the liquifier the cryostat had to be transported, the flat ground joint d was inserted. The high-vacuum part up to the pump was entirely made of glass. An advantage of this construction was the considerable reduction of the possibility of leaks. A three stage mercury diffusion pump of iron construction, with a Leyboldt rotation pump as backing pump, was used as the high vacuum pump.

The small auxiliary tube b with a valve served as a connection to the high pressure vessel from which helium could be condensed into the inner

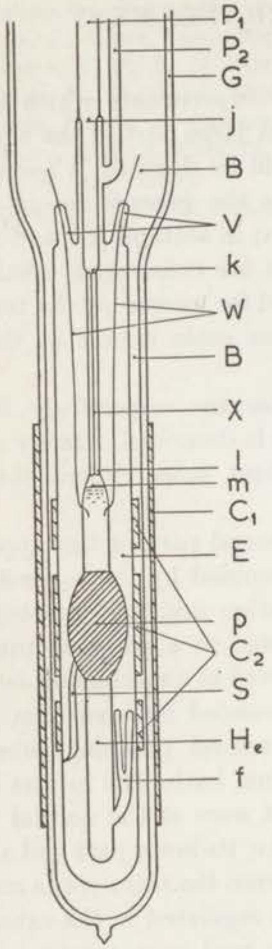

Fig. 2.1

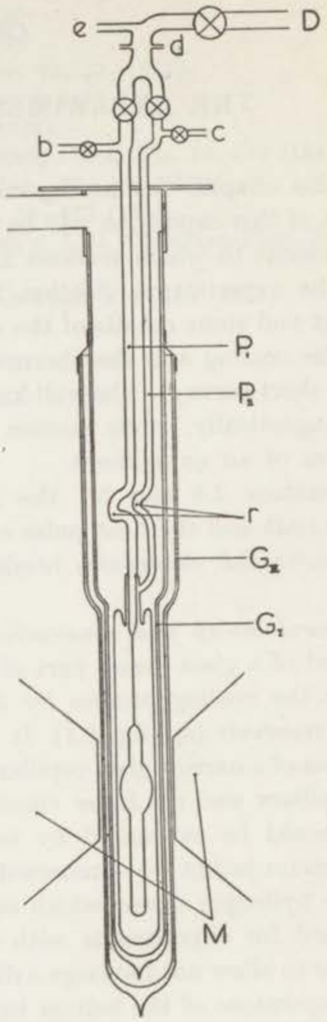

Fig. 2.2

Fig. 2.1. The cryostat (internal).

P_1, P_2 = pumping tubes; G = helium dewar; B = helium bath; V = platinum-glass seals; X = capillary; C_1 = primary coil; E = vacuum jacket; p = paramagnetic salt; C_2 = secondary coils; S = one of the three glass supports; He = helium reservoir; f = safety fuse; m = inner helium level; W = superconducting wires for the electrical connections.

Fig. 2.2. The cryostat (external).

D = main high vacuum tube; e = manometer connection; d = ground joint; b = filling tube; c = auxiliary tube; P_1, P_2 = pumping tubes; r = radiation traps; G = helium dewar; G_{II} = hydrogen dewar; M = pole pieces of the magnet (the upper part of the dewars is much longer than indicated).

vessel after the helium dewar had been filled at the liquifier. The second pumping tube was also provided with an auxiliary connection which could be used for filling the vacuum space with a small amount of helium gas for heat exchange with the bath.

The shape of the inner vessel depended on the special conditions of the experiment in question, while the part of it which served as a container for the salt was always ellipsoidal in shape.

The inner vessel was further provided with three very thin glass tubes (fig. 2.1 : S) drawn out in points which were bent in such a way that the apparatus fitted tightly into the outer wall of the vacuum space. Finally a safety device consisting of a thin-walled glass tube was connected to the inner vessel. Its purpose was to prevent a large scale explosion in case the capillary was blocked after filling the inner vessel.

Electrical leads to the inner vessel had to pass two walls. This problem was solved in the usual way by means of platinum wire melted in lead glass. To reduce heat inflow the wiring in the vacuum space was made of superconducting metal. In the earlier experiments spiralled lead wire was used; later on, because of its suitable mechanical properties, tantalum proved to be more convenient. Since tantalum allows of much thinner wires, spiralisation was not necessary.

2.2 *Cooling*. The temperature region below 1° K is only attainable by means of the magnetocaloric effect of paramagnetic substances. Only above 0.75° K it is still possible to use the common method of pumping on liquid helium, but a large capacity diffusion pump is required and some special precautions have to be taken. In the experiments under consideration measurements had to be made down to about 0.1° K and consequently the magnetic method was employed.

The principle is to have an amount of liquid helium in contact with a paramagnetic salt, keeping both in a vessel which can be insulated thermally from the surrounding helium bath. The bath should be at a temperature as low as can possibly be attained by the normal method of pumping (about 1.05° K in the present case). To start with, the thermal contact is made between the inner vessel and the bath in the usual manner, by means of an exchange gas introduced in the interspace. The salt is now magnetized isothermally and consequently the entropy of the system is decreased. After evacuating the interspace the salt is demagnetized adiabatically as a result of which the temperature goes down.

Some details of the demagnetization process will now be considered more closely. The entropy balance of the process is

$$S_{\mathrm{i}}^{\mathrm{p},o}-S_{\mathrm{i}}^{\mathrm{p},H}=(S_{\mathrm{i}}^{\mathrm{p},o}-S_{\mathrm{f}}^{\mathrm{p},o})+(S_{\mathrm{i}}^{\mathrm{He}}-S_{\mathrm{f}}^{\mathrm{He}})+\delta S_{\mathrm{irr}}, \tag{2.01}$$

S indicating the entropy. The indices p, o and p,H stand for paramagnetic salt in fields 0 and H, He for helium, i and f indicate the initial and final temperatures of the demagnetization process. The last term on the right is introduced to allow of a possible entropy production during the process which may arise when the salt and helium in the vessel are not permanently in thermal equilibrium.

The left-hand side of equation (2.01) represents the gross cooling effect. This can be calculated easily from the initial temperature T_i and the field H. It has its largest value when the salt is completely saturated by the magnetic field. In that case the value per mole is

$$S_i^{p.o} - S_i^{p.H} = R \ln(2s+1) \tag{2.02}$$

s being the total spin quantum number of the magnetic ion. Clearly the cooling effect is larger when a salt with larger s-value is used. Moreover with larger s the saturation value is easier to attain. For instance, in a specific case with $T_i = 1.08°$ K and $H = 12000$ Ø the cooling effect is only 38 % of its saturation value when $s = 1/2$ ($CuK_2(SO_4)_2 \cdot 6H_2O$), but 54 % if $s = 3/2$ ($CrK(SO_4)_2 \cdot 12H_2O$) and 62 % if $s = 5/2$ ($Fe(NH_4)(SO_4)_2 \cdot 12H_2O$). In this evaluation the LANDÉ-factor is assumed to be 2 except for the copper potassium sulphate for which it equals 2.18 [1].

The first bracketed term on the right contains the entropy change of the salt. As is well-known for not too low temperatures

$$S_i^{p.o} = -\tfrac{1}{2}\frac{b}{T^2} + \tfrac{1}{3}aT^3 + \text{constant} \tag{2.03}$$

b being the coefficient of the specific heat originating from electric and magnetic interactions and a that of the crystal lattice. The latter of these two contributions can be neglected; the $1/T^2$ term may have an important influence on the cooling. The $1/T^2$ term is no more a good approximation at the lowest temperatures (e.g. for KCr-alum below 0.2° K).

The second bracketed term in (2.01) contains the helium entropy change. It is clear that because of the large entropy increase of liquid helium with temperature it is very important to have an initial temperature as low as possible. For instance, the amount of liquid which can be cooled with $T_i = 1.0°$ K as the initial temperature is about twice that with $T_i = 1.1°$ K, using the same magnetic field.

If the vessel is not completely filled with the liquid, an amount of vapour present as a consequence at the initial temperature may contribute to the entropy content which has to be removed. A calculation of this contribution gives ($T_i = 1.08°$ K):

$$S_i^{v} = 0.07\,\frac{V_v}{V_l}\,S_i^{l} \tag{2.04}$$

where S_i^v en S_i^l are the entropies of the amounts of vapour and liquid and V_v and V_l their respective volumes. If $V_v/V_l = 0.2$, the largest value used in the experiments, this contribution is less than 2 %. Owing to the fact that the vessel is not closed, but connected by means of a capillary to the upper pumping tube, however, this contribution may be larger although this upper tube is pumped continuously during demagnetization. The coefficient in the expression (2.04) increases with T_i but not rapidly.

The term δS_{irr} is probably not very large. A large contribution would result in a marked influence of the time of demagnetization on T_f. No appreciable effect was found. A bad thermal contact between salt and helium may indeed be expected at temperatures of the order of a few tenths of a degree. In that region, however, this cannot have large influence because of the very small heat capacity of the liquid.

Fig. 2.3 shows for $T_i = 1.08°$ K and $H = 12000$ Ø the calculated number of moles of the salt which are necessary for cooling one mole of liquid helium as a function of T_f. Two curves are drawn the upper one being that for $CuK_2(SO_4)_2 \cdot 6\,H_2O$ ($s = 1/2$) and the lower one that for $CrK(SO_4)_2 \cdot 12\,H_2O$ ($s = 3/2$). In both curves the part down to 0.7° K shows a rapid increase with decreasing temperature. This is the region where the helium entropy has the main influence. Below 0.7° K the curves are rather flat: the entropy of the liquid has become almost negligible. At the lowest temperature one observes again a fast increase, due to the influence of the salt entropy. The cause of the intersection of the two curves just below 0.1° K is the comparatively much smaller value of b in the case of the copper salt.

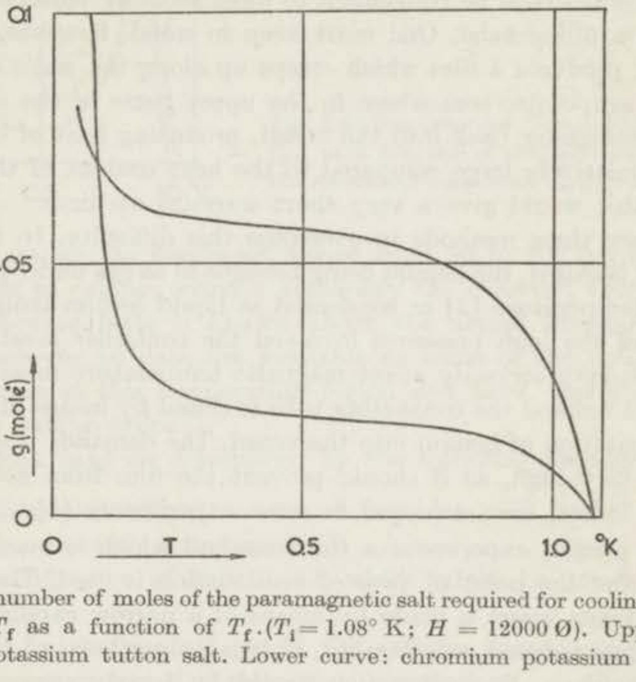

Fig. 2.3. The number of moles of the paramagnetic salt required for cooling one mole of liquid to T_f as a function of T_f. ($T_i = 1.08°$ K; H = 12000 Ø). Upper curve: copper potassium tutton salt. Lower curve: chromium potassium alum.

Data on the entropy of the salts published by DE KLERK [4] and CASIMIR *et al.* [13] were used for the calculation of the curves. For the copper salt a $1/T^2$ dependence was assumed. The LANDÉ-factor was again taken equal to 2.18 in the latter case.

2.3 *Heat insulation.* In all experiments below 1° K the thermal insulation is a very important problem. Usualy the vessel or sample is suspended in some way in an evacuated space with provisions for high heat resistance of the mechanical connections with the outer space. The usual precautions against radiation must also be taken by silvering the glass parts of the apparatus and inserting radiation-traps in pumping tubes. All this was done in the present case.

In an experiment involving liquid helium below 1° K the situation is somewhat more complicated because the helium has to be brought in. In this case it would be convenient to have an inner vessel connected by means of a filling tube. One must keep in mind, however, that liquid helium II produces a film which creeps up along the walls of this tube. The film evaporates somewhere in the upper parts of the cryostat, the vapour condensing back into the vessel, producing heat of condensation which is relatively large compared to the heat content of the vessel. In practice this would give a very short warming-up time.

There are three methods to overcome this difficulty. In the first one the vessel is closed, the helium being brought in as gas under high pressure at room temperature [2] or condensed at liquid helium temperature [3]. Because of the high pressures involved the container must be a metal one which may seriously affect magnetic temperature measurement. In the second method the connecting tube is closed by means of a plug after the condensation of helium into the vessel. The demands made upon this plug are very high, as it should prevent the film from going through. This has indeed been achieved in some experiments [4].

In the present experiment a third method which is much more convenient from the point of view of construction is used. The connection with the inner vessel is made by means of a narrow capillary. The film creep is thus reduced considerably, as it is proportional to the perimeter of the capillary. Recondensation would be largely prevented by the resistance of the capillary to gas flow and by pumping the upper part of the filling tube with a diffusion pump. A high resistance offered by the capillary to gas flow is of course essential, because the vessel itself at low temperature acts as an ideal high vacuum pump. The heat leak of the apparatus would then mainly be determined by a small residual condensation of the vapour. As will be shown, however, the actual situation in the capillary may be more complicated.

The heat leak, defined as the amount of energy increase of the vessel per second without extra heating, can easily be deduced from the experimental results. A typical example is shown in fig. 2.4. This plot shows a minimum at about 0.5° K. The increase towards low temperature might be attributed to magnetic heating due to the primary a.c. field used in the temperature measurements (see section 2.4). Unfortunately only one amplitude (about 15 Ø) of this field was employed, but a similar though much larger effect was observed in recent experiments on $CuRb_2(SO_4)_2 \cdot 6H_2O$.

In that case it could be reduced by lowering the a.c. measuring field.

Taking this into account there still remains the explanation of the heat inflow of about 12 erg/sec at low temperatures and its appreciable increase from 0.5° to 0.9° K (the value of the heat leak did not reproduce very well in this latter range; its maximum varied from 50 to 200 erg/sec). It is rather difficult to explain without ambiguity the observed values of the heat leak, because little is known about the actual situation in the capillary. Moreover no data are available on some of the possible contributing factors in this temperature region and have, therefore, to be estimated by extrapolation.

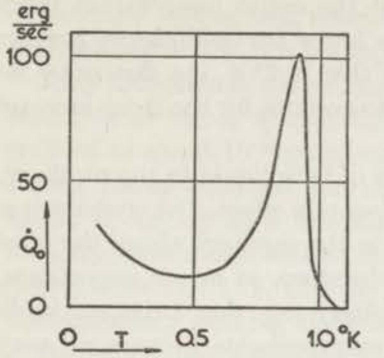

Fig. 2.4. The observed "natural" heat influx ($\dot{Q}_0$) as a function of temperature for one of the measuring runs with $CuK_2(SO_4)_2 \cdot 6H_2O$.

The heat leak may be determined by the following processes:

a. the recondensation effect,
b. the heat conduction of the glass,
c. the heat conduction of bulk liquid which might be present in the capillary,
d. conduction by the residual gas in the vacuum space,
e. conduction by means of the electrical connections.

The length of the capillary (kl) is 7 cm, its inside diameter $\approx$ 0.02 cm, its outside diameter $\approx$ 0.2 cm. Furthermore, jk $\approx$ 6 cm and the distance from the lower end of the capillary to the level of the liquid in the vessel is of the order of 1 cm (see fig. 2.1). The pressure in the pumping tube is between 10^{-3} and 10^{-2} cm mercury.

Firstly the conduction along the glass is considered. The coefficient of heat conductivity in this temperature region is not known, but it can be expected to vary as T^3 at very low temperatures. Bijl [6] finds above 1.3° K a $T^{1.3}$ dependence. A reasonable compromise seems to be to accept a T^3 law below 1° K and to adapt its magnitude to Bijl's extrapolated value for soft glass at that temperature. The result is $\lambda_{\text{glass}} = 3 \times 10^3\, T^3$ erg/deg cm. In this way one finds for the glass leak $\dot{Q}_0 \approx 4$ erg/sec, when the temperature of the vessel (T_2) is 0.5° K and that of the bath (T_1) is 1.1° K. This value does not change very much with temperature.

Secondly the recondensation effect is considered. Using Poiseuille's law on the gas flow through the capillary and taking the vaporization heat to be 20 joule/g, one finds for the heat input $\dot{Q}_0 \approx 2 \times 10^6 \frac{p}{T} \frac{\mathrm{d}p}{\mathrm{d}T}, \frac{p}{T} \frac{\mathrm{d}p}{\mathrm{d}T}$ taken at any point of the capillary; p is expressed in cm mercury. It can be seen that the recondensation effect is only able to account for the observed heat leak at low values of T_2, if the entire temperature jump from the bath to the vessel occurs in the lower few millimetres portion of the capillary. At temperatures higher than 0.7° K the flow may be somewhat larger but by no means enough to account for the steep increase of the heat leak.

An alternative possibility is the presence of bulk liquid in the capillary. This may be due to the capillarity or the fountain effect. The contribution of the former to the height of the helium in the capillary above the level in the vessel is only 0.4 cm [7] and is, therefore, of minor importance, unless some constriction occurs in the capillary (e.g. due to impurities in the liquid). The latter is, however, not very probable because no very large deviations between runs on different days were observed.

A fountain height may exist, if the temperature gradient is large enough. At low values of T_2 a temperature gradient is probably located between the helium in the vessel and the lower end of the capillary. The film contact would in that case act as a super leak connection and the capillary might at any rate partly be filled with liquid. The temperature gradient is mainly determined by the heat transport through the glass. At temperatures above 0.6° K, however, heat transport from the liquid in the vessel to the capillary mainly occurs by means of the vapour and the gradient can only be very small.

Secondly if the heat conduction of the liquid in the capillary itself is small enough, a temperature gradient might be set up and a fountain height maintained in it. Explicitly if at a certain point $\mathrm{d}T/\mathrm{d}x < (\mathrm{d}T/\mathrm{d}x)_f$, the latter being the reciprocal of the differential fountain height, the situation may be stable. The level of the liquid can not rise above the point where the equality sign holds good. The heat conductivity of the liquid is approximately known; below 0.7° K it is roughly proportional to T^3 (the constant of proportionality can be estimated, see section 4.3). Above that temperature one may perhaps use a formula derived by KEESOM, SARIS and MEYER [15] from their experimental data. It yields

$$\lambda_{\underline{\mathrm{He}}} = 0.6 \times 10^7 \, T^5 \, (\mathrm{d}T/\mathrm{d}x)^{-2/3}.$$

Working along these lines one finds that with values of T_2 smaller than 0.7° K it is just possible that the observed heat leak is compatible with a partly or wholly filled capillary. Above that temperature the temperature gradient which would exist in the liquid appears to be too small for the maintenance of a fountain height. For instance with $T_2 = 0.8°$ K a heat flow of 200 erg/sec would be necessary to keep the capillary full. The quantitative agreement of these evaluations with the experimentally observed heat leak may perhaps be improved by a slight adjustment of the parameters (e.g. the inner diameter of the capillary or the magnitude of the heat conductivity of glass).

If the explanation involving liquid in the capillary is true, the small leak below 0.6° K is determined by the heat resistance of the glass just below the capillary; the much larger heat leak above this temperature is determined by the heat resistance of the liquid in the capillary. The fast decrease above 0.9° K is perhaps due to the decrease of the fountain height.

The conduction of the exchange gas might also account for the increased heat leak at higher temperatures. Before demagnetization the pressure is reduced to about 10^{-6} cm of mercury. Most of the residual gas is adsorbed during demagnetization, but above 0.6° K it probably begins to evaporate again. According to a formula of KNUDSEN [16] a pressure of 10^{-7} cm mercury gives a heat leak of the order of 150 erg/sec, when the bath has a temperature of 1.1° K and the inner vessel is at a temperature of 0.7° K. Unfortunately nothing definite can be said about the actual pressure.

The conduction along the electrical wires can be completely neglected. They were made of superconducting material (lead or tantalum).

The conclusion which has to be drawn from all this is that the data available are not sufficient to obtain a clear picture of the actual circumstances responsible for the observed heat leak. Firstly one has the explanation involving liquid in the capillary. Secondly the heat leak can be explained by a combination of the recondensation effect, heat conduction along the glass and the effect of the exchange gas. Moreover the phenomenon *d* may easily also contribute to the leak in the first mentioned possible situation. No definite solution of the problem can be given until more data become available on the course of the temperature at different crucial points.

One final remark should be made on the influence of pumping on the capillary. As is seen, in both explanations this makes not much difference especially at low temperatures. With the recondensation effect the resistance of the capillary proves to be so large that in any case only a small amount of gas may pass through it. At higher temperatures the leak may only be slightly diminished. In the other explanation only a small decrease of the temperature at the upper end of the capillary may occur by a reduction of the vapour pressure. No explicit observations were actually made with and without pumping, but a rough determination gave about a 50 % increase of the heat leak in the latter case.

2.4 *Temperature measurement.* The temperature was measured magnetically, i.e. by determination of the susceptibility of the paramagnetic salt. This was done by means of a Hartshorn mutual inductance bridge which is a common method followed in many Leiden experiments and has been described in numerous papers and theses [8, 9]. Here it will be briefly reviewed. The bridge consists of two sets of coils each forming a mutual inductance. One of these is fixed around the salt in the cryostat, the other is variable. By adjusting the latter the current in the secondary circuit can be reduced to zero, the primary current being constant. In this simple way, however, it is not possible to get an exact zero reading, because usually the difference of the phases of the voltages of the two parts of the secondary circuit is not exactly 180°. This difficulty is overcome by introducing into the secondary circuit a small voltage, which is in phase with the primary voltage thus having a shift of 90° with respect to the inductance voltage in the secondary. A vibration galvanometer with pre-amplifier is used as a null detector. For further details of the bridge may be referred to D. Bijl's Leiden thesis [9]. The circuit diagram is shown in fig. 2.5. The frequency used was about 200 Hz.

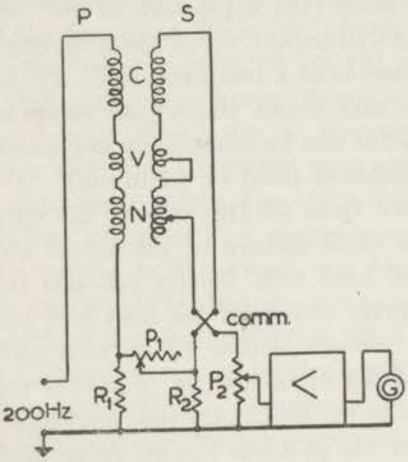

Fig. 2.5. The mutual inductance bridge. P = primary circuit; S = secondary circuit; C = cryostat coils; N = calibrated mutual inductance; V = calibrated variometer for continuous adjustment; R_1, P_1, R_2 = phase compensation circuit; comm = commutator for adjusting the sign of the phase; P_2 = sensitivity potentiometer; G = vibration galvanometer.

The primary coil in the cryostat is wound on the helium dewar; thus it is immersed in liquid hydrogen. The advantage of this is a considerable reduction of its resistance. The secondary coil is wound on the outer surface of the vacuum jacket of the inner cryostat. It consists of 3 sections, the middle one surrounding the salt, and two outer ones each having half the number of turns of the middle coil wound in the opposite direction. In this way the empty coil has a coefficient of mutual inductance near zero (the constant A of formula 2.07 is reduced), moreover, the pick-up from stray external fields is reduced considerably.

The coefficient of the mutual inductance of the cryostat coils is

$$M = A + B' \frac{d\sigma}{dH_u} \tag{2.05}$$

A and B′ being constants, H_u the external field and σ the magnetic moment of the salt. Further

$$\frac{d\sigma}{dH_u} = \frac{B}{T^* - \delta} \tag{2.06}$$

B being the Curie constant, T^* the "magnetic" temperature and δ a constant which depends not only on the magnetic properties of the salt but also on the external shape of the sample. The latter contribution to δ can be calculated exactly, if the shape chosen is an ellipsoid. For a sphere the contribution is zero. With the paramagnetic salts used here for all temperatures above 0.2° K $T = T^*$ holds good. At lower temperatures this is no longer true, but the difference is small for the temperatures in the present experiments which were never extended much below 0.1° K. If necessary, corrections were made by making use of the data from the literature. The magnitude of corrections will be given with the results.

So the measured coefficient of mutual inductance becomes

$$M = A + \frac{B''}{T^* - \delta}. \tag{2.07}$$

The constants A and B'' are determined by plotting M against $1/T - \delta$ for a few temperatures in the normal helium region. T is found by measuring the vapour pressure of the bath and using the agreed vapour pressure – temperature data of the van Dijk–Shoenberg tables [10]. The geometrical part of δ is calculated, the part of it due to the properties of the salt itself taken from the literature.

2.5 *The performance of an experiment.* Before starting an experiment the inner part of the apparatus had to be freed from all kinds of material such as air and water which could block the capillary during the cooling process. Straightforward pumping was not advisable, because the water in the crystal of the salt might be pumped off. Moreover, it proved not to be very effective, probably resulting only in the closing of the capillary by the solidification of water. Washing the system several times with pure helium gas proved to serve the purpose nicely. Afterwards it was pumped at liquid air temperature for a few hours. At this temperature the vacuum space was filled with helium gas exerting a pressure of about one centimetre of mercury.

The cryostat was disconnected before filling and transported to the helium liquefier where the helium was also condensed into the inner vessel. After condensation the bath was filled up again. The cryostat was then installed in the experimental room and the pressure of the bath reduced in a few steps calibrating the magnetic thermometer simultaneously.

After reaching the lowest pressure (about 0.02 cm mercury) the magnet which was movable on rails was pushed around the lower part of the cryostat and the field turned on. About 5 minutes were needed for re-establishment of temperature equilibrium after which the interspace was evacuated down to a pressure of 10^{-6} cm. The pumping tube connecting the interspace was then closed and that of the inner vessel opened. Finally the field was turned off, the magnet pushed away and the experiment could be started. Measurements were always made during the warming-up of the apparatus which lasted usually about one hour, of course depending on the amount of extra heating. The temperature was followed continuously.

2.6 *The specific heat experiments.* In the specific heat measurements the choice of the salt is very important because of the large heat capacity it may contribute to the total heat capacity of the calorimeter at the lower temperatures. The term of the specific heat which counts in this respect is that arising from magnetic and electric interactions and is proportional to $1/T^2$. Iron ammonium alum which was used in earlier experiments [11, 2] gives a contribution which is too large a correction for temperatures below 0.5° K. For this reason copper potassium tutton salt was chosen in the later experiments. This salt has no specific heat arising from electrical splitting, in fact its specific heat per mole is only 1/20 of that of the iron alum. As has been shown in section 2.2 the cooling effect of the copper salt is much less, but the salt correction can still be reduced by a factor 5, if the best salt-helium ratio is chosen. Fig. 2.6 gives the relative values of the heat capacities of the salt and of helium as a function of temperature in one of the experiments.

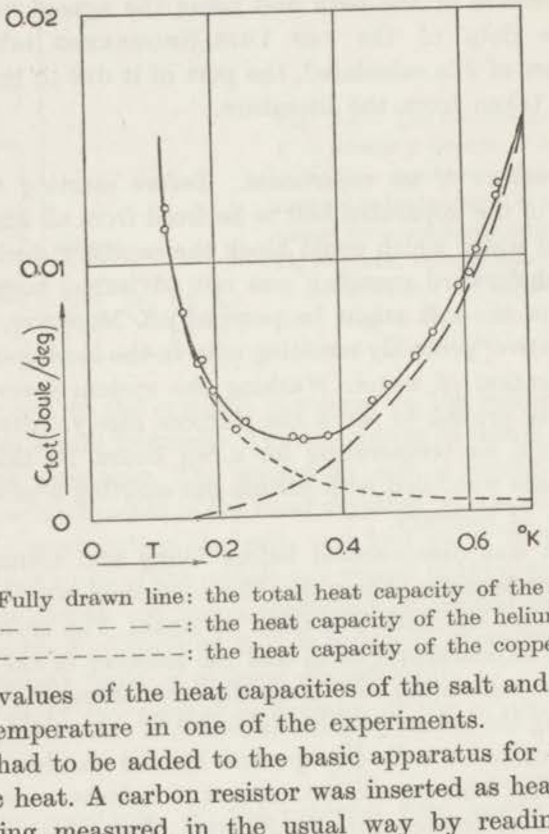

Fig. 2.6. Fully drawn line: the total heat capacity of the calorimeter.
— — — — — —: the heat capacity of the helium liquid.
- - - - - - - - - -: the heat capacity of the copper salt.

Not much had to be added to the basic apparatus for a measurement of the specific heat. A carbon resistor was inserted as heater, the energy developed being measured in the usual way by reading voltage and current. Heat was applied for 10, 20 or 30 sec., automatically controlled by a clock device which has been described elsewhere [12].

An experiment was performed in the following manner. After demagnetization the temperature was measured by means of a bridge reading at an average rate of 7 points per minute. In this way the "natural" warming up was observed. Every few minutes a heating period was inserted. The heating curves which were the result of this procedure did not show any aftereffects, so it can be concluded that temperature equilibrium was re-established within a few seconds. Fig. 3.1 gives a typical example of such a curve. The amount of helium in the inner vessel was determined after the experiment by collecting it into a calibrated volume and measuring the pressure and temperature after the apparatus attained room temperature. The amount of helium pumped away by the diffusion pump during an experimental day was negligible.

2.7 *The heat pulse experiments* (*cryogenic*). For the heat pulse experiments there is no sense in choosing a salt with a small specific heat. It is even of some advantage to have a salt with a large specific heat, because in that case a heat leak has less influence on the heating-up time, at least below about half a degree Kelvin. Potassium chromium alum was preferred, also because the magnetic temperature scale is the best known for this salt.

The measuring tube which contained two flat resistors, one for generating the heat pulse into the liquid (heater) and another for detecting the signal (thermometer) was fixed in the inner vessel of the cryostat. The tube was made of a caseine product called "lorival". Its construction is shown in fig. 2.7a. Over the top and bottom of the cylindrical tube tightly fitting covers were pushed.

Several types of resistors were tried. A carbon resistor made by painting aquadag on a circular paper sheet proved to be the best among the lot. The leads were provided for by fine platinum wire tightly wound round two small symmetrical wings of the sheet (see fig. 2.7b).

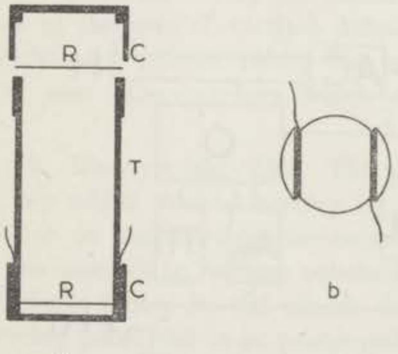

Fig. 2.7. a. Construction of a tube for the propagation of heat pulses. T = tube; C = covers (the upper one is lifted); R = resistor (heater or thermometer). b. Construction of a home-made carbon resistor.

In some experiments commercial resistor strips cut in the same general shape were used. The leads were soldered to the coppered wings.

A third kind of thermometer resistor was made by winding a phosphorbronze wire (45 μ) on a flat ring in zigzag fashion. The corresponding heater made of constantan wire was of a similar construction. The metal thermometer, however, was rather insensitive and its small resistance inpracticable. Later on the constantan heater was also used in combination with an aquadag thermometer.

Thermometers and heaters were fixed on the rim of the lorival tube by pressing on the covers.

Tubes of different lengths and diameters were used; a review is given in table 4.1 (chapter IV). In the tube and covers many holes were bored to provide for the entry of the liquid and for maintaining a good thermal contact during the experiments.

Since tubes had to be exchanged several times and the dismounting and re-assembling of the cryogenic apparatus was a very troublesome job, especially for the glassblower, the outer wall of the vacuum jacket was made of a separate piece. It could be connected to the main part of the apparatus by means of a conical ground joint, lubricated with silicon grease. This connection proved to be vacuumtight even at low temperatures and when it was in direct contact with liquid helium II. This arrangement simplified the problem of the exchange of the measuring tube to a large extent. At the same time the secondary coil of the mutual inductance which was wound on the separate piece, could be kept intact during such an exchange.

2.8 *The heat pulse experiments (electronic equipment).* In this section after a general survey of the electronic equipment some details of the essential parts are given. Fig. 2.8 contains the block diagram to which will be referred in the following lines.

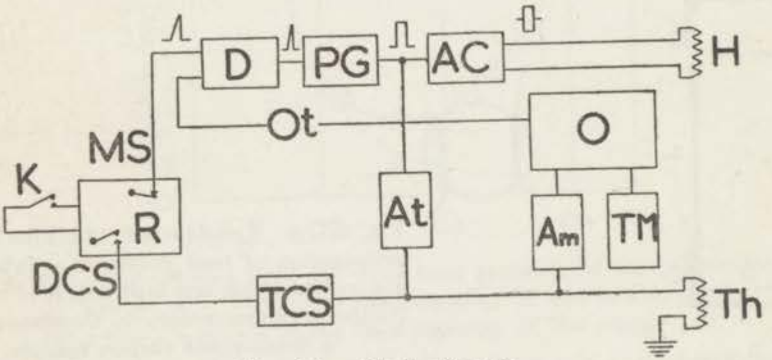

Fig. 2.8. Block diagram.

K = key; R = relays; DCS = direct current switch; MS = mercury switch; D = delay; PG = pulse generator; AC = attenuator and earth-point adjustor; H = heater; At = attenuator for testing the amplifier; TCS = thermometer current source; Th = thermometer; Am = amplifier; Ot = oscilloscope trigger; O = oscilloscope; TM = time markers generator.

2.8.1. Principle and general set-up. Since it was necessary to keep the heat input as low as possible the method employed was a so-called single pulse method as contrasted to the usual one in which the pulse is repeated periodically. For the same reason the direct current which is needed for the detection of the change of resistance of the thermometer was only switched on for a few seconds. For the sake of convenience all switching was automatised in such a way that a simple pressing of a key was sufficient for starting and completing the succession of events for one observation. By pressing this key (K) a relay was operated which switched on the direct current through the thermometer. After a delay of a few seconds the necessity of which will arise later, a second relay which broke a mercury switch was brought into operation. This latter produced a positive pulse of an arbitrary shape the first purpose of which was to start the horizontal motion of the beam on the oscilloscope. The normal position of this beam was just to the left of the screen. The second task of the pulse was to excite the pulse generator. Before this occurred, however, the pulse was delayed by a delay (D) which could be adjusted from 50 to 500 μsec. This was necessary to introduce the signal which coincided with the heating pulse at a convenient spot on the screen.

The pulse generator (General Radio 869A) produced a rectangular 50 V pulse over a resistance of 20 kΩ. The pulse width could be adjusted from 0.3 to 70 μsec; in the experiments described 10, 20 or 40 μsec pulses were almost exclusively employed. Before being put on the heater the 50 V pulse could be attenuated. At the same time an arrangement was made for adjusting the earthpoint of the pulse voltage, the purpose of which will be described separately.

The receiver part of the apparatus consisted mainly of an amplifier (Am) which carried the signal from the thermometer to the oscilloscope. After amplification by the oscilloscope-amplifier this signal was fed into one of the sets of vertical deflection plates of the Cossor double beam oscilloscope (model 1035). The second beam was used for either 10 or 100 μsec time markers which were produced by a crystal calibrator.

2.8.2. Delays (fig. 2.9). The sequel of events was started by pressing a key which worked a relay S_1. The first purpose of this relay was to switch on the thermometer current. As the thermometer current needed a few seconds to become constant (this was mainly due to the influence of the a.c.filter in the circuit described below), the generation of the driving pulse had to be postponed for this period. This was accomplished by the circuit D_1 in fig. 2.9. Its main parts are a Wheatstonebridge and a thyratron. R_1 is a resistor with a large negative temperature coefficient (Philips N.T.C.). After the closing of the circuit by one of the contacts S_1, this resistor is heated by the current and consequently decreases in value. By this the bridge balance is changed which results in a delayed switching off of the current through the thyratron. This current works a relay with a mercury switch (S_2) which is broken. The voltage of point P is suddenly raised to about 20 V and a pulse with a sharp onset is generated. The use of a switch with mercury-mercury contact was essential because this proved to be the only switch which broke the contact instantaneously, thus producing only one single pulse.

A similar delay D_2 of somewhat larger time constant served for switching off the thermometer current again and bringing the apparatus in its initial position.

A third delay of much shorter time constant had to be introduced between the time of triggering the oscilloscope and the time of the heater-pulse.

The main part of the circuit is a so-called monostable multivibrator, consisting of two triodes B_{2a} and B_{2b}. B_{2a} is originally open, but is closed by the positive pulse introduced at point P. At the same time B_{2b} is opened. After a period, depending on the fixed product R_2C_2 and on the grid bias of B_{2a}, which can be adjusted by the potentiometer (in fig. 2.9 below on the left), the original situation is re-established. The result is a positive square pulse with variable length at point Q. After differentiation by means of C_3R_3 the signal is inverted by B_{3a} and the result is a delayed narrow positive pulse which can be used for driving the pulse generator.

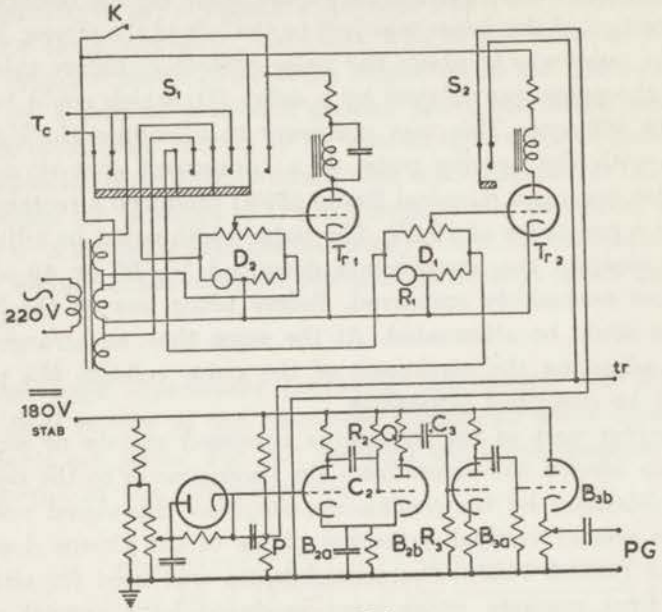

Fig. 2.9. Delays.

K = key; T_c = to thermometer d.c.switch; S_1 = first relay; S_2 = mercury switch; Tr = thyratron (PL21); R_1 = N.T.C. resistance; tr = oscilloscope trigger connection; PG = pulse generator. For further details see the text. The double-triodes B_2 and B_3 are 6SN7 valves.

2.8.3. Attenuator and pick-up compensation. Since the pulse generator was not equipped with a calibrated attenuator at its output, this had to be built separately. It was combined with an arrangement for reducing the pick-up on the thermometer circuit, that occurred mainly in the cryostat. Efficient screening could perhaps be attained, but the method used here was much more convenient.

The simplified equivalent circuit is drawn in fig. 2.10. As the coupling proved to be capacitive, it is represented as such. CD is the thermometer resistance, AB the resistance of the heater. As will be shown, the essential thing was the adjusting of the earthpoint in the heater, the thermometer being earth-connected at one of its ends. One can now consider the Wheatstonebridge ABCD. No pick-up will occur from AB on CD, when the bridge is balanced. This is approximately realised, if $R_{\mathrm{AD}}/R_{\mathrm{DB}}=C_{\mathrm{BC}}/C_{\mathrm{CA}}$, because the parallel capacitances of R_{AD} and R_{DB} have large impedances compared to those resistances and can be neglected.

Fig. 2.11 shows the practical arrangement of the earth point adjuster. Normally no current passes through the triodes owing to the large negative bias of the grids. It flows only during the time of the pulse from the pulse generator. This results in a positive pulse on the cathode of the upper valve and a negative pulse of equal magnitude on the anode of the lower one. These pulses may be attenuated in two identical attenuators one section of which is shown. The last part of the circuit consists of four 500 Ω potentiometers mounted along one axis and connected in such a way that the cathode resistance of T_1 and the anode resistance of T_2 are maintained at 250 Ω, while at the same time the impedance the heater "sees" in looking backward is a constant 500 Ω. The adjusting of the earthpoint was performed by means of these potentiometers.

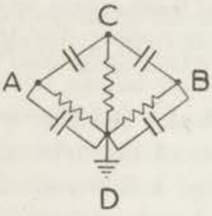

Fig. 2.10

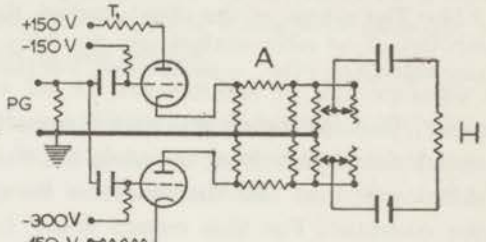

Fig. 2.11

Fig. 2.10. Simplified diagram, showing the capacitive coupling between heater and thermometer.

Fig. 2.11. Combined earthpoint-adjuster and attenuator.
PG = pulse generator; T_1, T_2 = EL 34 triodes; A = double attenuator (only one section is shown); H = heater.

The double attenuators provided for attenuation in steps of 4 db from 0 to 60 db by means of two switches in series, each having four positions: the first one for 0, 4, 8 and 12 db, the second for 0, 16, 32, 48 db. The attenuation was performed by double π sections which were at in- and output adapted to $2 \times 250\ \Omega$.

The pick-up compensation as described proved to be a very pleasant solution of difficulties with pick-up at the start of the investigation, when sometimes the pick-up was so large, that it even blocked the amplifier by overloading. By adjusting the earthpoint the pick-up signal could be eliminated almost completely, if the intercapacities of the wiring in the cryostat were not very unsymmetrical. What was left of the pick-up signal conveniently served as zero point for the time measurements.

2.8.4. **Thermometer current circuit** (fig. 2.12). To keep the current constant a high resistance in series with the thermometer was

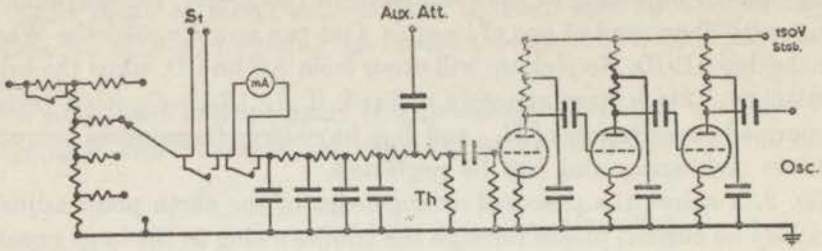

Fig. 2.12. The circuit of the direct current through the thermometer and the pre-amplifier. S_1 = relay contact (see fig. 2.9); Th = thermometer; Aux.Att. = auxiliary attenuator; Osc. = oscilloscope. For the amplifier 6AK5 valves were used.

necessary. This resistance was at the same time part of a RC-filter which prevented a.c. signals from entering the thermometer. A disadvantage of this filter was that the thermometer current needed a few seconds to become constant. For this reason delay 1 had to be inserted.

With a potentiometer the current could be adjusted from 10 μA to 1 mA in steps by a factor of about 1.5. By means of a series resistance the current could still be reduced by 1/5. The magnitude of the current could be read on a small μA meter.

2.8.5. **Amplifier** (fig. 2.12). The amplifier used was a three stage one of common design with a bandwidth ranging from 300 Hz to about 30 kHz, amplification decreasing only slowly beyond that. The amplification was about 3×10^3 which could be increased up to a factor 10^6 by using the oscilloscope amplifier. The noise level was of the order of 5 μV, mainly originating from the so-called flicker effect at low frequency in the first amplification valve. No special attempt was made to reduce this. To test the performance of the amplifier the pulse from the pulse generator could be fed directly into the input after large attenuation. The original purpose of this arrangement was also to have a signal on the screen simultaneous with the heater pulse. In practice, however, the pick-up signal proved to be more convenient for this.

As will become clear later, signals with long and almost horizontal tails had to pass through the amplifier undistorted. To test the performance of the amplifier in this respect a so-called step function was introduced at the input, i.e. on the thermometer, and the output observed on the screen of the oscilloscope. The decay proved to be approximately exponential with a decay time of 45×10^{-4} sec. When the correction is small, it is even possible, if necessary, to correct the observed signals for this decay. If the observed signal is represented as a function of time by $y = m(t_0)$, the corrected signal is

$$y(t_0) = m(t_0) + \frac{1}{D}\int_0^{t_0} m(t)\,\mathrm{d}t, \tag{2.08}$$

D being the decay time. Actually this correction had to be introduced in many cases.

2.8.6. Screening and grounding. Since the voltage level of the thermometer signals was low and the bandwidth of the amplifier had to be rather large, a lot of trouble could arise from pick-up of stray fields. In this respect contributions may be expected from the 200 Hz field used in the temperature measurement, the mains (50 Hz and overtones) and radio-frequency fields. The latter might cause a very annoying broadening of the line on the oscilloscope.

The 200 Hz pick-up could be compensated easily by introducing a small counter signal in the thermometer circuit, using a magnetic coupling with the circuit of the primary coils of the bridge.

By the use of co-axial lines for all connections to the cryostat and careful earthing, the other effects could also largely be eliminated. A large amount of experimenting was required to get adequate earthing; loops had to be avoided by bringing all earth-connections to one point only. This point was found by trial.

2.8.7. Recording of observations. An observation was recorded by photographing the screen of the oscilloscope. A Southern camera was used; no shutter was necessary because of the single stroke operation of the oscilloscope. The films were developed afterwards and were stored for further inspection. For the purpose of making observations the pictures were enlarged by projecting on a white wall. Time measurements could easily be made by comparison with the time-markers and the shape of the signals could be analysed.

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CHAPTER III

THE SPECIFIC HEAT

In this chapter the evaluation of the specific heat from the experiments will be described. The corrections required are investigated in some detail. The data obtained on the specific heat are presented and discussed. Finally values of the entropy, computed by the integration of the experimental data, are given and discussed.

3.1. *Determination of the heat capacity of the calorimeter.* As was mentioned in section 2.6, the heating curves, resulting from a plot of the temperature against time, were of a very simple shape. In practice it was more convenient to plot the directly measured coefficient of mutual inductance *n*; of course, this makes no difference. *n* was expressed in number of turns of the secondary of the calibrated mutual inductance which was used for the compensation. No appreciable after-effects were found in these curves (see fig. 3.1) and the temperature course before and after the heating period follows practically straight lines. Also, except for the lowest temperatures, the slopes before and after are almost parallel. Consequently averaging of these slopes gives a reliable determination of the "heat leak" during the heating period.

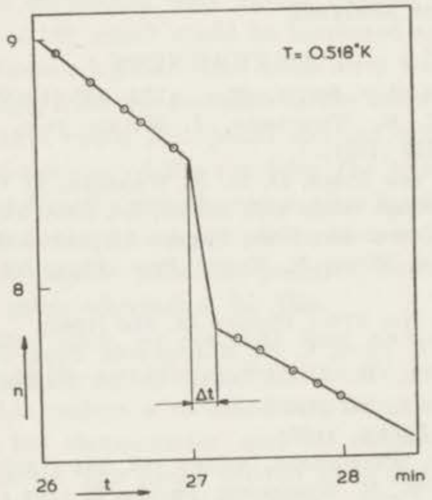

Fig. 3.1. A typical example of a heating point.

The temperature increase during a heating period was usually of the order of a few hundredths of a degree. For this reason the heat leak and the heat capacity could always be considered as linearly dependent on n and T during the heating period, although the heat capacity may change rapidly with temperature.

The experiment gives directly the quantity $\mathrm{d}n/\mathrm{d}t$ and $(\mathrm{d}n/\mathrm{d}t)_0$, the latter being an average of the slopes of fore- and after-period. The difference gives the change due to heating only:

$$(\mathrm{d}n/\mathrm{d}t)_{\mathrm{cor}} = \mathrm{d}n/\mathrm{d}t - (\mathrm{d}n/\mathrm{d}t)_0. \tag{3.01}$$

The temperature calibration yields

$$n = A + \frac{B''}{T-\delta} \tag{3.02}$$

A, B'' and δ being known constants. No appreciable deviations from this Curie-Weiss law occur in the region above 0.2° K. So

$$\left(\frac{\mathrm{d}n}{\mathrm{d}t}\right)_{\mathrm{cor}} = -\frac{B''}{(T-\delta)^2}\,\frac{\mathrm{d}T}{\mathrm{d}t} = -\frac{(n-A)^2}{B''}\,\frac{\mathrm{d}T}{\mathrm{d}t}. \tag{3.03}$$

If the heat input per second $= \mathrm{d}Q/\mathrm{d}t$, the heat capacity is equal to

$$c_{\mathrm{tot}} = \frac{\mathrm{d}Q}{\mathrm{d}t}\Big/\frac{\mathrm{d}T}{\mathrm{d}t} = -\frac{B''}{(n-A)^2}\,\frac{\mathrm{d}Q}{\mathrm{d}t}\Big/\left(\frac{\mathrm{d}n}{\mathrm{d}t}\right)_{\mathrm{cor}} \tag{3.04}$$

all quantities on the right being known. There is, however, one difficulty concerning the value of δ in the calibration formula 3.02. The part of δ arising from the shape correction is 0.002°. There is, however, a much larger contribution arising from the difference between the field working on the magnetic ions of the salt and the external field H_e. Since it was not possible to determine δ with sufficient accuracy from the calibration curve, the value of the latter contribution, represented by Θ, had to be taken from the literature. This will be discussed in the next section.

3.2. *Corrections*

3.2.1. The salt correction. The correction arising from the heat capacity of the paramagnetic salt becomes quite large at low temperatures, its contribution being proportional to $1/T^2$. In fact, as has been shown in section 2.6, it sets a lower limit to the temperature range for the determination of the specific heat of helium by the method employed. In fig. 2.6 the heat capacities of the calorimeter, the salt and the helium are plotted together against the temperature. It is clear from this picture that with the present set-up no reliable values of the specific heat of helium can be obtained below 0.25° K.

Since the construction of the apparatus did not allow of large variations in the amount of helium, it was not possible to determine the salt correction experimentally. Therefore, its value together with the value of Θ had to be taken from data in the literature. This gave rise to a difficulty, because the various values published were not consistent. Some of them are given in table 3.1. The quantity b is again the coefficient in the $1/T^2$ law which is valid for the specific heat of the salt in the temperature region concerned.

TABLE 3.1

Data on $CuK_2(SO_4)_2 \cdot 6H_2O$

Author	$\Theta(°K)$	b/R per mole	Temp. region
De Klerk	0.052	6.8×10^{-4}	$T < 1° K$
Garrett	0.035	6.1×10^{-4}	$T < 1° K$
Benzie and Cooke	0.035 ± 0.005	6.0×10^{-4}	$1° K < T < 4° K$
Broer and Kemperman		6.4×10^{-4}	liquid air

It is seen that there are roughly two sets of data, one with somewhat larger values of Θ and b then the other. The uncertainty of these constants has, however, not a very large influence on the resulting specific heat values of the liquid. Above 0.7° K this influence is even negligible: the salt correction is very small and the temperature scale is only slightly affected, since it is adapted to the real temperature in the calibration region. Below 0.7° K the final results actually depend on the choice of the constants. On one hand the extrapolation of the temperature from the calibration points gives a temperature scale depending on the value of Θ used. On the other hand the correction of the heat capacity of the salt depends on the chosen value of b. Fortunately, however, both effects largely compensate each other, as the larger value of Θ corresponds to the larger value of b.

An illustration of this "compensation" is given in the following lines. It was not possible to deduce the salt data with sufficient accuracy from measurements of the heat capacity in the region below 0.25° K, where the helium contribution is negligible. It was not even possible to make a decisive choice between the sets of data given in table 3.1. All combinations fit equally well. The reason of this can be seen by calculating from the experimental data the heat capacity at some specific point, using first the low value of Θ, then the high value. In the first case a smaller value of T (i.e. a larger value of $1/T^2$) results than that in the second case. Consequently, since dQ/dT is not appreciably affected, the first calculation gives a smaller coefficient of the $1/T^2$ term than the second one. This is just what should be expected from the corresponding b-values.

On the whole the value of De Klerk should perhaps be given preference, because a salt sample from the same source was used (impurities may affect the data). Moreover, there was just an indication that his Θ value agreed somewhat better with the temperature calibration of the present experiments. For the sake of completeness calculations were also carried out on the basis of Garrett's data (see fig. 3.2). All further discussion is, however, based on de Klerk's values.

3.2.2. The vapour correction. If only part of the volume of the calorimeter is filled with the liquid, a correction is necessary for the heat of evaporisation and the heat capacity of the vapour. This correction was only small in the present case, because 90 % of the calorimeter was usually filled with the liquid. The correction which can easily be calculated amounts to

$$(3.05)\qquad (\mathrm{d}Q/\mathrm{d}t)_{\mathrm{vap}} = 13300\left\{\frac{c_{\mathrm{vap}}}{RT}\left(\frac{\mathrm{d}p}{\mathrm{d}T}-\frac{p}{T}\right)+\frac{3}{2}\frac{p}{T}\right\}\ \text{ergs/deg cm}^3;$$

c_{vap} is the heat of evaporation per mole, p is expressed in cm of mercury. This formula is valid only, if the pressure is so low that the vapour can be considered as an ideal gas. The ratio of this correction to the heat capacity of the liquid is, of course, decisive. The evaluation shows an increase with temperature which is not very large. Although the experiments were extended up to 1.8° K this ratio never exceeded 1.5 %.

There remains to be considered the role played by the volume above the capillary in the evaporation process. In the temperature region below 1° K the contribution can be completely neglected, but at higher temperatures this volume might have an appreciable influence.

The experimental conditions for the measurements in the upper temperature region differ in some respects from those below 1° K. The helium bath was always maintained at about 1.05° K. The heat leak has, therefore, a negative sign. The pumping tube of the calorimeter is now closed to prevent the pumping away of helium from the calorimeter. Only the part of the upper pumping tube at liquid helium temperature may give an appreciable contribution to the correction of formula 3.05, the density of the vapour at higher temperatures being too small. As is well-known, the temperature in a cryostat rises sharply just above the bath, so the level of the bath determines the volume under consideration. Two cases have now to be considered which were actually observed in the course of the experiments.

In the first case there is a good thermal contact between the bath and the pumping tube. This occurs, when the bath level is high enough. In this case the vapour pressure in the pumping tube cannot exceed its saturation value (about 0.02 cm of mercury). The helium in the calorimeter may evaporate and the gas would flow through the capillary. This current is wholly governed by the difference of pressure according to POISEUILLE's law. The negative heat leak arising from this evaporation process increases rapidly with the temperature; it amounts to 2×10^4 erg/sec at 1.8° K. It gives, however, straight lines for the fore- and after-period of a heating point and consequently the average heat flow from the calorimeter can easily be evaluated. An extra correction for the heat of evaporation is, therefore, not required. Its effect is completely accounted for by correcting for the "heat leak". The influence of the helium condensed in the pumping tube can be neglected. It will probably return into the calorimeter by means of the capillary and in this way increase the negative heat leak, but this is only a small effect. The amount of helium in the calorimeter is certainly not changed appreciably during the experiments. The heat leak of 2×10^4 erg/sec occurring at 1.8° K corresponds to a mass transport of vapour of only 0,0006 gr/min, the content of the calorimeter being about 1.8 gr.

The second case occurs with a low bath level (below point j in fig. 2.1). Now there is no equilibrium between the pressure in the tube and the bath. The negative heat leak is much smaller (see table 3.2), because the low pressure in the upper tube is not maintained at the saturated vapour pressure of the bath and consequently the pressure difference can be much smaller than in the first case. At a heating point the relative change of this pressure difference during the heating, however, is much larger. An estimate of the gas flow through the capillary shows that during the time of the heating an appreciable part of the pressure equalisation should take place. Consequently it is possible only for a part of the correction due to evaporation to be included in the heat leak evaluated by means of averaging the slopes of the fore- and after-periods. The other part cannot be corrected for, because the exact temperature in the upper tube is unknown. On the other hand, since probably only a small portion of the upper tube tube is at low temperature, the average density of its vapour content is low and consequently the amount of gas to be transported through the capillary for attaining pressure equilibrium is small too. Thus the correction involved is probably negligible. To give a specific example: if the part of the upper volume at low temperature is 2 cm^3, its total contribution to the heat content of the calorimeter would be about 4 % at $T = 1.6°$ K. About one fourth of the amount of vapour responsible for this passes the capillary during a heating period of 30 sec. The incorrectible part would, therefore, give an error of 1 %. Probably the situation is even more favourable in a real case.

The argument with respect to the second case is certainly not completely conclusive. The best proof of the reliability of the experimental results is perhaps the absence of any systematic difference between the measured specific heat values in both cases. The first case usually occurred with the first series of measurements on a measuring day. In the latter two or three series the second case was observed.

3.2.3. Other corrections. The calorimeter consists of glass and contains, apart from paramagnetic salt and helium, a few platinum wires. The heat capacities of the glass and the wires can be completely neglected compared to the joint contribution of the salt (about 14 g) and the liquid helium (1.7 to 1.8 g). Accurate values of the specific heat of glass are not known, but the temperature dependence is almost certainly a third power law with an absolute value of the coefficient well below 1 % of that of helium in the "phonon" region (the Debije temperature is certainly larger than 100° K).

TABLE 3.2

Two series of measurements.

T °K × 10^3	ΔT °K × 10^3	$\mathrm{d}Q/\mathrm{d}t$ $\frac{\text{erg/sec}}{10^3}$	$(\mathrm{d}Q/\mathrm{d}t)_0$ $\frac{\text{erg/sec}}{10^3}$	Δt sec	c_{tot} joule/deg	c_{salt} joule/deg	c_{vap} joule/deg	c_{liq} joule/deg	C $\frac{\text{joule}}{\text{g deg}}$
125	3.3	0.039	0.038	10	0.0123	0.00117			
150	5.3	0.039	0.035	10	0.0076	0.0081			
229	35.1	0.040	0.023	30	0.0035	0.0035			
324	13.2	0.040	0.023	10	0.00313	0.0017		0.0014	0.00082
377	13.3	0.040	0.020	10	0.00309	0.00128		0.0018	0.00106
448	36.3	0.162	0.021	10	0.00526	0.00091		0.0044	0.0026
518	26.0	0.159	0.019	10	0.00621	0.00068		0.0055	0.0033
558	22.3	0.158	0.018	10	0.00718	0.00057		0.0066	0.0039
592	19.5	0.156	0.021	10	0.00894	0.00052		0.0084	0.0050
622	15.1	0.156	0.023	10	0.0105	0.0005		0.0100	0.0059
656	12.4	0.154	0.029	10	0.0135	0.0004		0.0130	0.0077
687	19.0	0.151	0.038	20	0.0156	0.0004		0.0152	0.0088
726	14.8	0.151	0.046	20	0.0208	0.0003	0.0001	0.0204	0.0120
764	29.3	0.851	0.061	10	0.0295	0.0003	0.0002	0.0290	0.0172
817	36.0	0.852	0.080	20	0.0480	0.0003	0.0003	0.0474	0.0279
860	26.4	0.852	0.096	20	0.0649	0.0002	0.0005	0.0642	0.0378
901	32.3	2.79	0.091	10	0.0874	0.0002	0.0007	0.0865	0.0503
962	60.0	2.77	0.091	30	0.140		0.001	0.139	0.082
1015	41.1	2.75	0.071	30	0.202		0.002	0.200	0.118
1052	31.8	2.74	0.017	30	0.259		0.002	0.257	0.151
1080	22.8	6.87	−0.063	10	0.301		0.002	0.298	0.175
1099	19.8	6.86	−0.075	10	0.346		0.003	0.343	0.208
1118	17.7	6.88	−0.130	10	0.388		0.003	0.385	0.226
1142	27.0	12.22	−0.30	10	0.453		0.004	0.449	0.264
1165	27	12.19	−0.31	10	0.456		0.004	0.452	0.270
1185	24	12.17	−0.47	10	0.501		0.004	0.497	0.296
1212	43	12.15	−0.57	20	0.573		0.005	0.568	0.332
1240	36	12.20	−0.51	20	0.674		0.006	0.668	0.398
1265	25	19.0	−0.49	10	0.753		0.006	0.747	0.445
1284	24	18.9	−0.48	10	0.801		0.007	0.794	0.473
1303	21.5	19.1	−0.5	10	0.891		0.007	0.884	0.526
1321	19.5	19.0	−0.5	10	0.979		0.008	0.971	0.579
1344	40	43.1	−0.4	10	1.069		0.008	1.06	0.632
1380	35	43.1	−0.6	10	1.23		0.01	1.22	0.725
1409	32	43.1	−0.8	10	1.35		0.01	1.34	0.800
1438	28	43.0	−0.6	10	1.52		0.01	1.51	0.902
1473	50	43.1	−0.7	20	1.74		0.01	1.72	1.03
1516	42	43.1	−0.8	20	2.03		0.01	2.01	1.20
1553	38	43.1	−0.8	20	2.30		0.02	2.28	1.36
1592	54	43.0	−0.8	30	2.38		0.02	2.37	1.41
1627	26	76.1	−1.0	10	2.89		0.02	2.87	1.71
1660	49	76.0	−1.6	20	3.09		0.02	3.07	1.83
1700	43	76.3	−1.8	20	3.55		0.02	3.52	2.12
1730	30	121.0	−1.1	10	4.06		0.03	4.03	2.40
1769	55	121.4	−2.0	20	4.39		0.03	4.36	2.60
1817	49	122.2	−3.0	20	4.99		0.03	4.96	2.96
1860	47	136.0	−1.0	20	5.76		0.04	5.72	3.41
1922	48	152.3	−3.0	20	6.34		0.04	6.30	3.76

TABLE 3.3

Measuring points below $T = 0.75°$ K.

	T °K × 10^3	C j/gdeg × 10^4
A I	414	17.2
	476	26.3
	524	34.0
	566	43.6
	614	53.1
	673	95.1
	737	127
A II	306	6.4
	401	17.0
	475	25.2
	523	35.8
	575	47.0
	649	63.0
	750	150
B I	281	4.9
	357	11.1
	423	19.0
	467	23.5
	501	29.8
	546	32.5
	595	55.3
	654	71.8
B II	274	4.4
	323	8.0
	395	14.7
	428	19.8
	460	25.5
	491	28.1
	515	33.4
	536	37.1
	559	45.0
	585	49.0
	606	53.3
	627	67.2
	652	72.1
	678	90.3
B III	248	3.7
	286	6.6
	337	8.4
	434	20.3
	465	28.3
	497	29.6
	528	35.4
	554	51.0
	586	50.7
	627	70.1
	673	90.1
	716	122
C I	324	8.2
	377	10.6
	448	25.6
	518	32.6
	558	38.9
	592	49.5
	622	59.0
	656	76.6
	687	88.3
	726	120
C II	246	4.3
	319	8.5
	402	15.1
	476	25.2
	552	38.4
	611	52.5
	660	73.5
	693	102
	739	137

3.3. *Results.* The results are shown in the tables 3.2, 3.3 and 3.4 and in the figures 3.2 and 3.3. Table 3.2 contains a review of two series of measurements, the first one for temperatures below, the second one for temperatures above the temperature of the bath. Apart from the actual results, the heat input per second, the average heat leak per second and the corrections for the salt and the vapour, discussed in section 3.2 are also given. The second column contains the temperature increase during

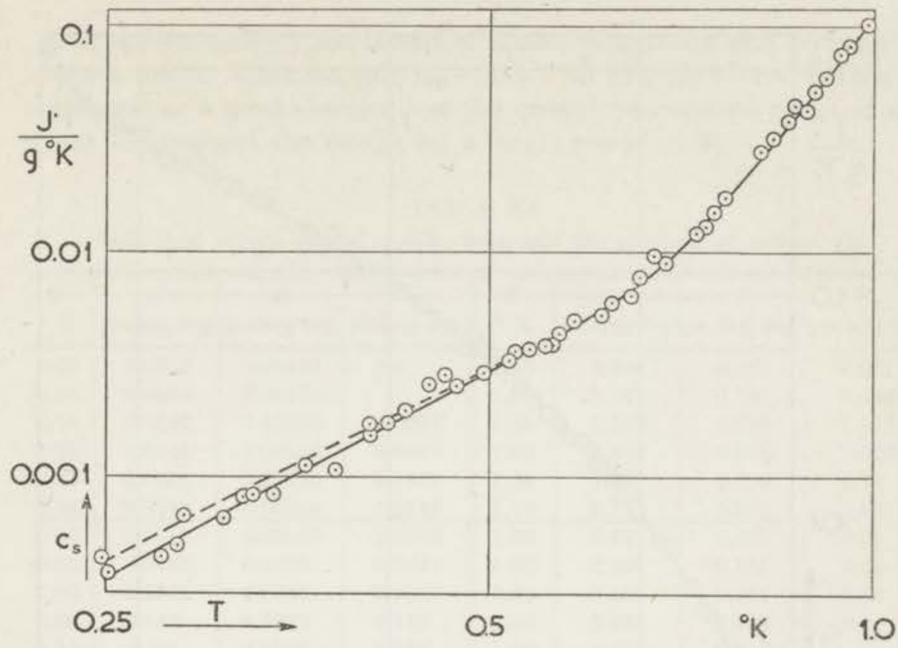

Fig. 3.2. The specific heat below 1° K as a function of temperature.
Full line: calculated with de Klerk's data of Θ and b.
Dotted line: calculated with Garrett's data.

heating, the fourth column the time of heating. These two series give an idea of the relative magnitude of heat leaks, external heat input and different corrections. They include only about 20 % of the total amount of data obtained.

In table 3.3 the results of measurements below 0.75° K are compiled. Fig. 3.2 shows a logarithmic plot of the specific heat as a function of temperature for the lower region. Fig. 3.3 represents the same semi-logarithmically for the higher region. Again only part of the measured points are plotted to avoid confusion. The full line corresponds to the "smoothed" values which are tabulated in table 3.4. For temperatures above 1.8° K the data of Keesom and Keesom [5, 6] were used.

The graphs give a good impression of the accuracy. Deviations above 0.75° K may amount to 2 to 3 %. In the lower temperature range this is somewhat larger, due to the relatively large influence of the salt correction and the heat leak. Moreover, some small systematic differences between the results of different series were found, due to some uncertainty in the dependence of n on temperature. The temperature scale is after all an extrapolation of the curve through the calibrated points in the normal helium temperature region. Therefore, rather small ambiguities in the absolute value of the temperature may affect the results more seriously as the temperature dependence of the phenomenon under consideration is very large. The scattering of the points between 0.75 and 0.4° K amounts to 5 to 10 %; below 0.4° K the scattering increases rapidly, the salt correction becoming predominant.

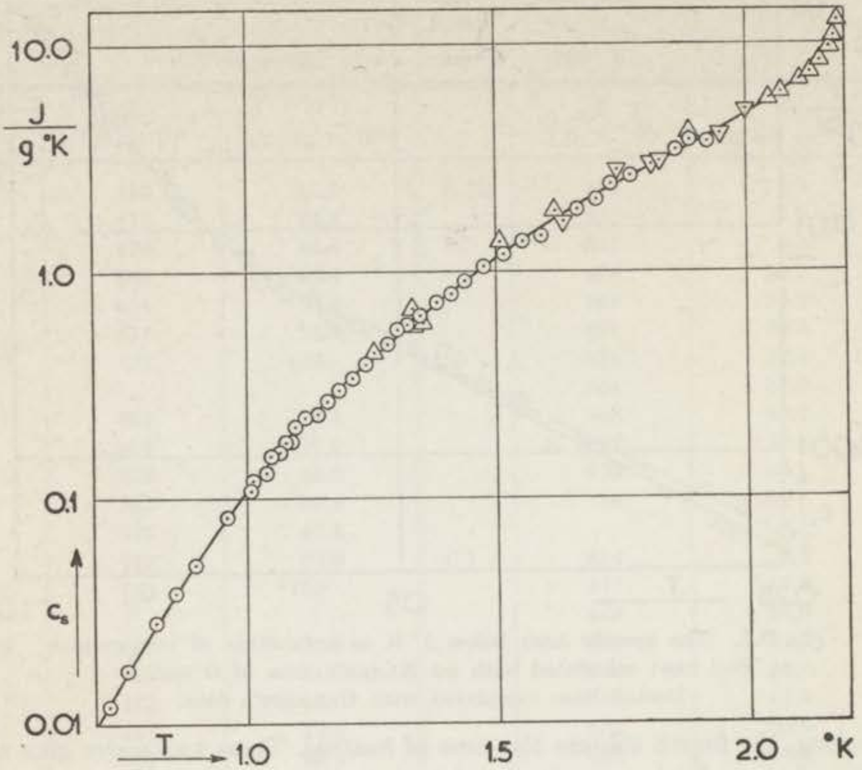

Fig. 3.3. The specific heat in the temperature region between 0.75° K and the λ-point (1949 temperature scale).

⊙ measuring points from the present investigation.

△ points of Keesom and Miss Keesom.

▽ points of Keesom and Clusius.

3.4. *Comparison with other experiments.* Several experimental data in the temperature region attainable by pumping on liquid helium had already been published. The points of Keesom and Clusius [7] and Keesom and Keesom [6] are plotted in fig. 2.3. There are only a few points available between 1.25 and 1.9° K which are in reasonably good agreement with the data of the present investigations. These results had partly to be recalculated to the 1949 temperature scale of the vapour pressure of helium. The scattering of Keesom and Keesom's points above 1.9° K is much less, so they could be used for computing the smoothed values of the specific heat given in table 2.4.

Other published data are those of Keesom and Westmijze [8] in the temperature region from 0.6 to 1.6° K. Their values are represented by the formula $c_{sat} = 9.6 \times 10^{-2} T^6$ joule/g deg; these values are on the average about 8 % too low. The authors did not claim, however, a great accuracy.

The same applies for the values compiled by Gorter *et al.* [9], because they used the Keesom and Westmijze formula in the region between 1.0 and 1.6° K.

The agreement with the results of HULL, WILKINSON and WILKS [10] is much better. Their formula $c_{\mathrm{sat}} = 10.0 \times 10^{-2} T^{6.2}$ (0.6 – 1.4° K) can be considered as a good average over the quoted temperature range, if one wants to represent the results by a single power of T.

TABLE 3.4

Smoothed values of the specific heat and the entropy of helium II.

T ° K	C joule/g deg	S joule/g deg	C_r joule/g deg	T ° K	C joule/g deg	S joule/g deg	C_r joule/g deg
0.60	0.0051	0.00169	0.0	1.45	0.944	0.162	0.872
0.65	0.0068	0.00215		1.50	1.127	0.197	1.048
0.70	0.0098	0.00276	0.0017	1.55	1.330	0.238	1.243
0.75	0.0146	0.00358	0.0047	1.60	1.572	0.284	1.476
0.80	0.0222	0.00475	0.0102	1.65	1.83	0.336	1.72
0.85	0.0343	0.00644	0.0199	1.70	2.11	0.395	2.00
0.90	0.0510	0.00885	0.0338	1.75	2.46	0.461	2.33
0.95	0.0743	0.0122	0.0541	1.80	2.80	0.535	2.66
1.00	0.1042	0.0168	0.0807	1.85	3.19	0.617	3.04
1.05	0.142	0.0227	0.115	1.90	3.63	0.709	3.47
1.10	0.191	0.0304	0.160	1.95	4.27	0.812	
1.15	0.250	0.0402	0.215	2.00	4.95	0.929	
1.20	0.322	0.0523	0.282	2.05	5.82	1.061	
1.25	0.410	0.0672	0.364	2.10	6.92	1.215	
1.30	0.516	0.0853	0.464	2.15	8.61	1.40	
1.35	0.634	0.1069	0.576	2.18	11.6	1.53	
1.40	0.780	0.132	0.716	2.186	14.3	1.57	

The more recent experiments of HERCUS and WILKS [11] which were published after the original publication of the experiments discussed here [12] gave, however, values about 10 % higher in the temperature range from 1° to 2° K.

The results of experiments by DOKOUPIL *et al.* [13] which were performed in the Leiden laboratory, using a completely different technique, are once more in good agreement.

3.5. *Comparison with theory.* According to the theory of excitations the specific heat should be built up out of two contributions, one due to the phonons, the other due to the so-called rotons. As was stated in chapter I, this division makes only sense for temperatures which are not close to the λ-point, since in that region the density of the excitations becomes too large. In fact, the applicability of the theory of excitations breaks down near the λ-point.

The phonon part of the specific heat is proportional to T^3. Assuming that no transverse waves are possible in liquid helium, it is only dependent on the velocity of sound v_{I}:

$$c_{\mathrm{ph}} = \frac{16}{15} \pi^5 \frac{\varkappa^4}{h^3} \frac{1}{\varrho} \frac{T^3}{v_{\mathrm{I}}^3}. \tag{3.06}$$

The roton part of the specific heat, assuming the energy spectrum of a roton to be represented by formula 1.26 is given by

$$(3.07)\quad c_r=(2\pi)^{-3/2}\frac{1}{\varrho}\frac{\varkappa^{3/2}}{h}\left(\frac{p_0}{\hbar}\right)^2\left(\frac{\mu}{m}\right)^{1/2}m^{3/2}\left(\frac{\Delta}{\varkappa}\right)^2\left[T^{-3/2}+\frac{\varkappa}{\Delta}T^{-1/2}+\frac{3}{4}\left(\frac{\varkappa}{\Delta}\right)^2T^{1/2}\right]e^{-\Delta/\varkappa T},$$

m indicating the mass of one helium atom.

The roton contribution has a very large temperature dependence, so it is to be expected that at low enough temperatures only the phonon contribution will survive.

Returning now to the experimental results, the inspection shows that this actually occurs below 0.6° K. The data in that region are in good agreement with a T^3 law and can be represented by

$$(3.08)\qquad c_{\rm ph}=0.0235\ (\pm\ 0.0015)\times T^3 \text{ joule/g deg.}$$

Using a value of $v_{\rm I}=237$ m/sec, obtained by extrapolating the measurements on the velocity of first sound [14, 15] in the normal helium region, one gets from formula 3.06

$$c_{\rm ph}=0.0212\times T^3 \text{ joule/g deg.}$$

The agreement is not bad; the difference, however, just exceeds the accuracy limit claimed for the specific heat experiments. On the other hand, it is not to be expected that there is a large error in the measurements on the velocity of sound. The extrapolation to lower temperatures cannot introduce any large uncertainty either, since just above 1° K the velocity is already practically constant. If there is any ambiguity here, it points to an even lower value of the specific heat. So the difference has to be explained by a systematic error in the present experiments or by an unforeseen complication in the theory. The first supposition can only be justified by making new measurements. These are actually in progress in the Leiden laboratory. The ambiguity of the data on the salt mentioned in section 3.2 is certainly not responsible for the deviation.

As to the second supposition, one is tempted at first sight to suppose a dispersion of the velocity of sound at the high frequency of the "thermal" phonons. (The measurements of ATKINS and CHASE were performed at 1.4×10^7 Hz; the average frequency of the thermal phonons at 0.5° K is of the order of 10^{10} Hz). The velocity of sound corresponding to the experimental value of the specific heat is 226 m/sec. It should be pointed out, however, that this supposed dispersion has the wrong sign. In normal sound the dispersion, due to relaxation effects, gives an increase of the velocity with the frequency. On the other hand, some kind of "resonance" dispersion may perhaps occur, if the phonon wave length is no longer very large compared with the atomic distance (see fig. 1.1).

Turning now to the higher temperatures, where the specific heat should be represented by $c=c_{\rm ph}+c_{\rm r}$, one can compare $c_{\rm r}$, found by subtracting $c_{\rm ph}$ from the total specific heat, with formula 3.07. The experimental values of $c_{\rm r}$ are also compiled in table 3.4.

Formula 3.7 has the form $c_r = Bf(T)\, e^{-\Delta/\varkappa T}$, B being a constant and $f(T)$ a function which changes only slowly with temperature (practically $f(T) = T^{-3/2}$). The main dependence on temperature arises from the exponential factor. By plotting the experimental values of $c_r f(T)$ against $1/T$ a straight line should be obtained. This is actually the case in a large region: between 0.8° and 1.6° K the deviations are less than 4 %. Below 0.8° K the roton specific heat seems to decrease even faster with temperature. Because of the uncertainty of the phonon specific heat, not much importance should be attached to this observation. The former is still a major contribution in this region. Above 1.6° K the dependence on temperature is also somewhat faster, probably due to the influence of the neighbourhood of the λ-point as discussed at the beginning of this section. From the slope of the straight line found in this way, the magnitude of $\Delta/\varkappa$ can be deduced. It is found to be 9.1° K (± 0.15) (see also section 4.7) [1].

The constant B still contains two unknown quantities: the "effective roton mass" μ and p_0, the momentum of a roton of the smallest energy. For an evaluation of μ and p_0 yet another function connecting these two quantities has to be found from experiments. The specific heat data lead to $(p_0/\hbar)^2 \times (\mu/m)^{1/2} = 2.4 \times 10^{16}$. In these calculations the difference between the specific heat at constant volume considered in theory and the actually measured specific heat at the saturated vapour pressure can be completely neglected.

The conclusion is that there exists a reasonable agreement between the experiments and the predictions from the theory of excitations. The latter gives in any case an adequate description of the specific heat below 1.6° K. Experiments seem to show a somewhat sharper bend in the curve of figure 2.2 at $T = 0.6°$ K than was expected from the theory, but the accuracy of the measurements is not sufficient in this region to make a definite statement.

3.6. *The entropy.* The entropy can be calculated from the specific heat by means of

$$S(T) = \int_0^T \frac{c}{T}\, \mathrm{d}T. \tag{3.09}$$

The results of this integration can be found in table 3.4. The smoothed values of the specific heat were used.

The integration of formula 3.8 yields for temperatures below 0.6° K:

$$S_{\mathrm{ph}} = 0.0078\ (\pm\ 0.0005) \times T^3 \text{ joule/g deg.}$$

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[1] In the original publication of these experiments [12] a value of about 8° K was given. The difference with the present value is due to the use of LANDAU's first proposal of the energy spectrum of a roton: $\varepsilon_r = \Delta + p^2/2\mu$, which at the time seemed to be the best representation. The main term of $f(T)$ in that case is $T^{-1/2}$ and this just causes the difference.

The previously published entropy data [9], based on a similar calculation by making use of other measurements of the specific heat show deviations of the same order as those mentioned in section 3.4.

The computed entropy data can now be compared with the entropy calculated from measurements on the fountain effect and the mechano-caloric effect, provided the formulae of H. LONDON are accepted. At the same time, this comparison furnishes a test to those formulae. H. LONDON's formula for the fountain effect is

$$\mathrm{d}p/\mathrm{d}T = \varrho S \tag{3.10}$$

and for the mechano-caloric effect

$$Q = ST. \tag{3.11}$$

In the latter Q is the heat production in a vessel per unit mass of superfluid helium flowing from the vessel.

Consider now first the entropy calculated from KAPITZA's measurements on the mechano-caloric effect [16]. The values are about 4 % higher at temperatures below 1.5° K and 10 % higher above this temperature than the present results.

The experiments of MEYER and MELLINK [17] yield values in agreement between 1.4 and 1.7° K, but a few percent higher below this region and up to 10 % lower above it. The scattering of their points is, however, rather large especially near the λ-temperature.

New experiments on the fountain effect by PESHKOV [18] in the temperature region from 0.82° to 1.59° K are in excellent agreement with the entropy values given in table 3.4.

Finally the recent indirect measurements of the fountain effect by VAN DEN MEYDENBERG *et al.* [19] are within the limits of accuracy of both the experiments in agreement with the present data.

3.7. *Conclusion.* It should be noticed that results of experiments on the specific heat of helium II may depend very much on the temperature scale used for the vapour pressure of helium. This is due to the rapid change of the specific heat with temperature. Particularly in the region between 1.5° and the λ-point an uncertainty still exists. For instance, the scale proposed by KISTEMAKER [20] brings down the λ-temperature by 0.014 degree Kelvin compared to the 1949 scale of VAN DIJK and SHOENBERG. At 1.5° K the two scales agree, in the intermediate zone the differences change about linearly ¹).

The effect on the specific heat and the entropy results mainly in a

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¹) Since this text has been written, a new temperature scale was accepted at the Paris conference on Low Temperature Physics in September, 1955. In the temperature region under discussion this scale is practically in agreement with the Kistemaker scale. The data given in table 3.4 for temperatures larger than 1.8° K must, therefore, now definitely be recalculated.

shift of the temperatures of the values mentioned. The values themselves are hardly effected (less than 0.5 %).

On the other hand, this can only have an influence on the data of **Keesom** and **Keesom** and those of **Keesom** and **Clusius**, which were used to fill the gap between 1.8° K and the λ-point. The magnetic thermometer used in the experiments described in this chapter was usually calibrated against the vapour pressure at about 2.6°, 1.4° and 1.08° K and not in the disputed region. So these results are independent of such a change of scale.

A few final remarks may perhaps be made on the reliability of the data of the present experiments. The agreement with other reliable experiments is usually good. A serious difference exists, however, with the results of **Hercus** and **Wilks** [11]; this situation has yet to be cleared. The indirect measurements on the fountain effect of **van den Meydenberg** *et al.* [19] and the latest experiments of **Peshkov** [18] give a good support for the calculated entropy values as well as for the validity of H. **London**'s formula for the fountain effect. The older direct measurements on this and the mechano-caloric effect show poor agreement. The mutual agreement of these experiments, however, is still worse. The conclusion is that there is as yet no reason to distrust the values of the entropy based on the experiments described in this chapter, nor in fact the formulae of H. **London**.

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CHAPTER IV

THE HEAT PULSE EXPERIMENTS

In this chapter the results of the experiments concerning the propagation of heat pulses in the liquid are given and discussed. Section 4.1 contains a mere statement of the results. In section 4.2, 4.3, 4.4 and 4.5 they are analysed in more detail for different temperature regions and discussed with respect to experimental and theoretical considerations. Section 4.6 contains the result of a preliminary experiment in which atmospheric helium was used instead of helium from wells. In connection with it the possible influence of extremely small amounts of ^{3}He is discussed. Section 4.7 is reserved for some final remarks and conclusions.

To avoid confusion the nomenclature used in the description of the results is first given here. The experiment always concerns the behaviour of a heat pulse after travelling through the liquid over a certain distance. The path length is indicated by z, the diameter of the tube through which propagation takes place is called d. The zero point of the time of propagation is always put at the start of the transmitting signal. Because in general the pulse changes its shape while passing through the liquid, the time of propagation cannot be defined unambiguously. t_1 indicates the time of arrival of the front of the detected signal, t_3 the time of reaching its half-height and t_4 the time corresponding to its top. The corresponding "velocities", i.e. z/t are called u_1, u_3, u_4 respectively. The pulse energy is indicated by ε and the pulse width by τ.

4.1. *Review of experimental results.* Table 4.1 summarizes the different tubes used in the experiments. An indication of the construction of the heaters and the thermometers is given and lengths and diameters are tabulated. The most reliable measurements were made on tubes 8, 9 and 10; therefore, the main part of the discussion will be devoted to them. Deviating results obtained in experiments with other tubes will be mentioned at the end of this section.

Fig. 4.1 contains eight typical photographs taken from the total amount of a few thousand. In all the pictures the time-scale is shown on the second beam. The distance of the main time markers is 100 μsec, in some of the pictures the subdivision of 10 μsec markers is also shown. In the upper two photographs the time scale is three times less than that in the

TABLE 4.1.

Measuring tubes

A. = aquadag on paper
C.R.S. = commercial resistance strip (carbon)
Const. = constantan wire
Ph.br. = phosphorbronze wire

No.	heater	thermometer	length cm	diameter cm
1	A.	A.	2.99	0.40
2	C.R.S.	C.R.S.	6.02	0.40
3	C.R.S.	C.R.S.	1.53	0.40
6	Const.	Ph.br.	3.0	0.80
7a	Const.	A.	1.60	0.95
7b	Const.	A.	4.72	0.95
8	A.	A.	1.60	0.95
9	A.	A.	3.05	0.95
10	A.	A.	6.4	0.95

remaining ones. The transmitting time is indicated by the pick-up signal at the left on the main beam.

Fig. 4.2 depicts for comparison the shapes of signals for the three main tubes. The fully-drawn lines are direct reproductions from the photographs. The dashed lines were produced in the following way. First the signals from the three tubes at the same temperature were adjusted to equal energies of the input pulse and also as far as possible to the same thermometer-sensivity. The amplifier correction, mentioned in 2.8.5, was then made. This is an important correction especially in the case of the very long tail signals of the 6 cm tube. The amplitudes obtained in this way should not be taken too seriously, because the thermometers used were not always the same, moreover, their sensitivity is by no means linearly dependent on the direct current. The table underneath figure 4.2 gives the data on the input-pulses.

Fig. 4.3 gives the results of the time measurements for the three tubes as a function of temperature. The points plotted are only a part of the total number obtained. To avoid confusion they have been omitted altogether for the 3 cm tube. Because of the poor accuracy the curves for the t_4 values below about 0.5° K have not been drawn.

All the data summarized in the first part of this section are obtained from pulses with "normal" energies and pulsewidths. Normal means in this respect as small as possible but well defined above the noise level. Of course this depends very much on the temperature and the tube length, thermometer sensitivity, specific heat and pulse-spreading changing within large limits. Table 4.2 contains a review of these "normal" values.

A general survey of the results gives the following picture.

4.1.1. $T > 1.0°$ K (region A).

This is the region of normal second sound, i.e. the pulses remain well-bunched, apart from some aftereffects

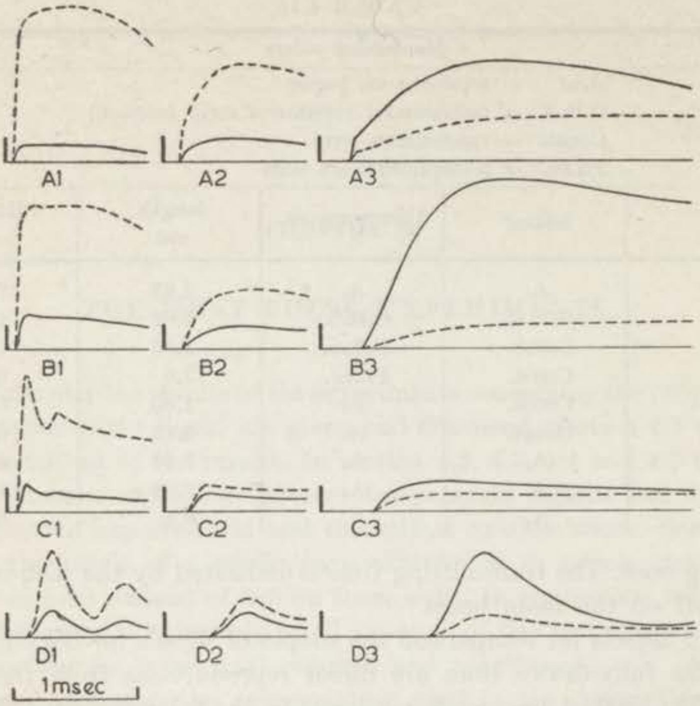

Fig. 4.2. Shape of the detected signal at different temperatures (T) and different tube lengths (z).
Fully-drawn line: actually observed signal; dotted line: normalized signal (see text).

ε = energy of actual input pulse.
τ = width of input pulse.
ε_n = energy of normalized pulses.

	1			2			3			
	z = 1.60 cm			z = 3.05 cm			z = 6.4 cm			
	T °K	ε erg	τ μsec	T °K	ε erg	τ μsec	T °K	ε erg	τ μsec	ε_n erg
A	0.172	0.146	20	0.154	0.162	20	0.156	0.88	20	1.2
B	0.402	1.26	20	0.370	9.55	20	0.417	11.5	40	15
C	0.506	1.27	20	0.548	19.3	40	0.532	11.6	40	18
D	0.691	3.34	20	0.674	9.9	20	0.675	11.9	40	30

perhaps due to spurious reflections. No appreciable difference is found between the pulse shapes of the three tubes.

4.1.2. 1.0° K $> T >$ 0.7° K (region B). Going to lower temperatures one observes firstly a shift of the signal to the left on the oscilloscope screen, i.e. the velocity is increasing as should be expected from other experiments. Moreover the pulsewidth is not preserved, but shows an appreciable spreading (see fig. 4.1. e, f, g, h). There is a marked dependence of the amount of spreading on the tube-length. A good parameter for

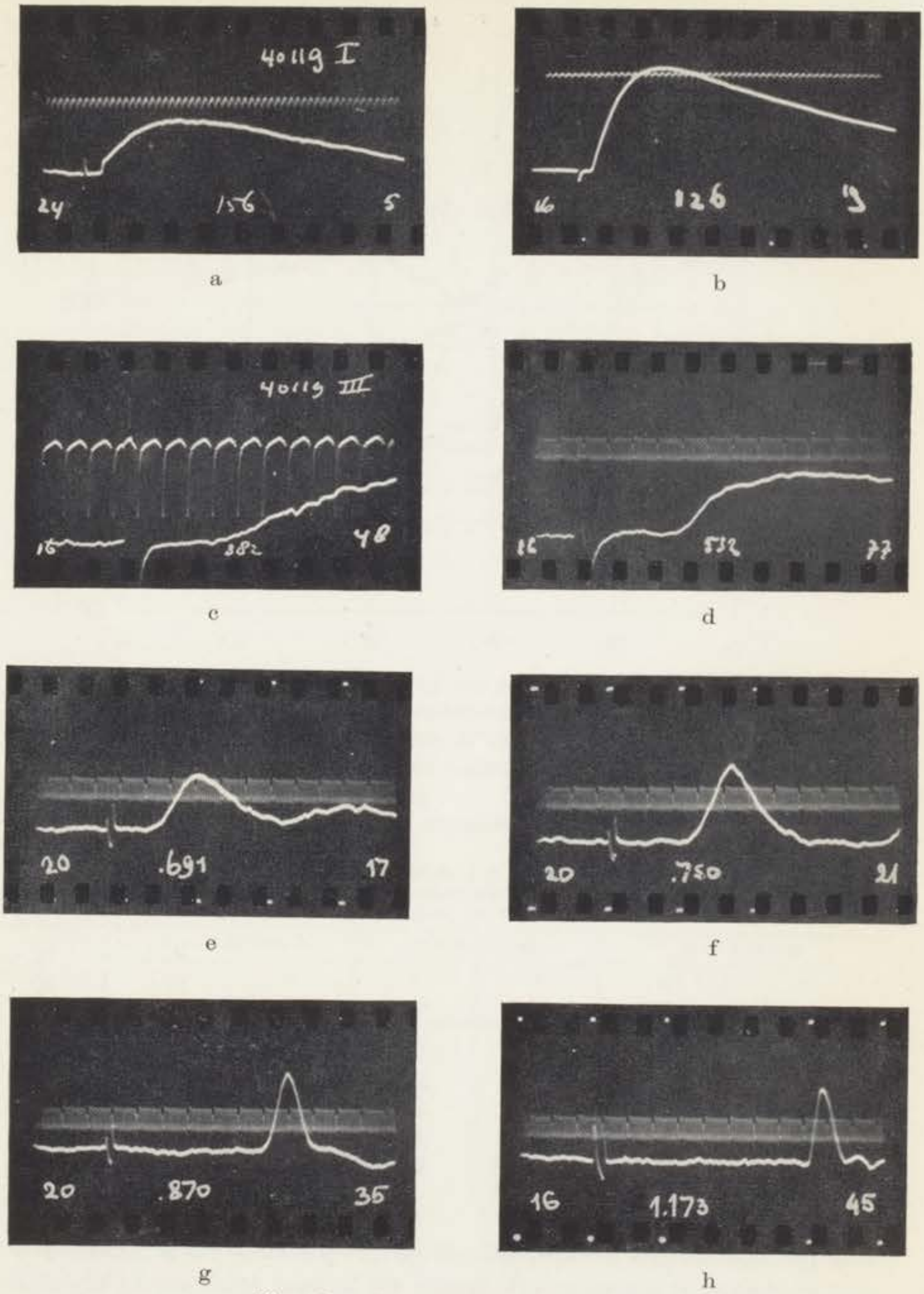

Fig. 4.1. Some typical photographs.

a. tube 10 ($z = 6.4$ cm); note the sharp edge at the start.
b. tube 7b ($z = 4.72$ cm); constantan heater, no sharp edge.
c. tube 10 ($z = 6.4$ cm); front of the very flat signal.
d. tube 10 ($z = 6.4$ cm); typical example of the intermediate region.
e, f, g, h. tube 8 ($z = 1.60$ cm); the pulse shape in the higher temperature region.
The number in the centre of each picture gives the temperature belonging to it in millidegrees Kelvin.

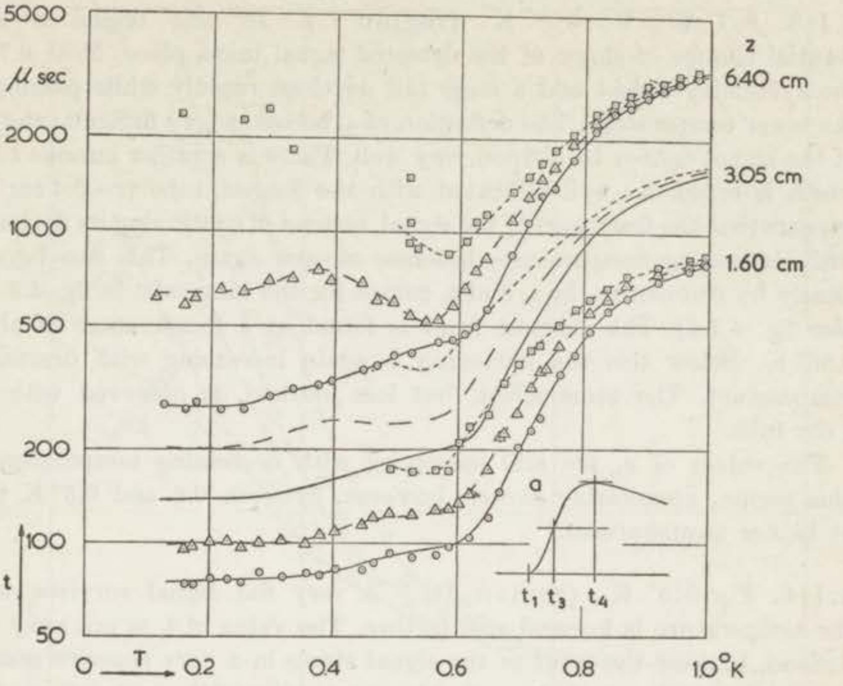

Fig. 4.3. Time-interval measurements for the tubes 10, 9 and 8 (z = 6.4, 3.05 and 1.60 respectively).

⊙ ——————— start of the signal (t_1)

△ — — — — half height (t_3)

⊡ - - - - - - - top (t_4)

3a. Review of the measured time points.

TABLE 4.2

Normally used pulse energies (ε) and pulse widths (τ)

T °K	z = 1.60 cm		z = 3.05 cm		z = 6.4 cm	
	ε erg	τ μsec	ε erg	τ μsec	ε erg	τ μsec
0.15	0.15	10 to 20	0.15	20	0.5	20
0.3	0.5	20	1.3	20	3.5	20 to 40
0.5	1.3	20	3.4	20	7.0	40
0.7	3.4	20	3.4	20	18	40
0.9	3.2	20 to 40	6.4	20 to 40	30	40 to (70)
1.1	10	40	19	40	35	40 to (70)

this spreading is the value of $t_4 - t_3$; i.e. the difference of time between top and half-height of the front part of the signal. For constant temperature this time interval is within the limits of accuracy proportional to $z^{1/2}$ (see fig. 4.6).

In earlier publications [1, 2, 3] the velocity of second sound was supposed to be equal to $u_1 = z/t_1$. The value of u_1, however, appears to be dependent on the length of the tube. This is clearly seen in fig. 4.4; u_1 is plotted as a function of T for the 6 cm and the 1.6 cm tube. A unique velocity has to be defined in another way (see section 4.2).

4.1.3. 0.7° K $> T >$ 0.5° K. (region C). In this region a substantial change of shape of the detected signal takes place. Near 0.7° K the symmetry is lost and a large tail develops rapidly while passing to the lower temperature. The definition of t_4 becomes very difficult; the top of the signal cannot be defined very well. There is a rather curious effect which is especially well indicated with the longest tube ($z = 6.4$ cm). It appears that the front part of the signal, instead of continuing its flattening with decreasing temperature, becomes steeper again. This can be seen clearly by comparing the t_3 and t_1 curves for the long tube in fig. 4.3 (see also fig. 4.1.d). The steepest front is found at a temperature of about 0.55° K. Below this the flattening is again increasing with decreasing temperature. The same effect, but less marked, is observed with the 3 cm tube.

The values of u_1 are still increasing with decreasing temperature in ahis region, appreciably slower, however, between 0.6 and 0.5° K than tt higher temperatures.

4.1.4. $T <$ 0.5° K. (region D). A very flat signal survives when the temperature is lowered still further. The value of t_1 is not very well defined, because the front of the signal starts in a very concave manner (see fig. 4.1.c). The result is a rather large spreading of the measuring points. In this region the introduction of the amplifier correction is very essential (see fig. 4.2.B3). The 1.6 cm tube is not very suitable for analysis, because the direct signal is probably intermingled with the first *vice versa* reflection (just a trace of a second top can be seen in fig. 4.2.B1). The results of the other two tubes indicate that t_4 is probably proportional to z^2, although the accuracy is rather poor. The time of the first top of the shortest tube also fits rather well in this assumption.

At about 0.36° K with the 3 cm tube and 0.22° K with the 6 cm tube a new effect appears. The front of the signal becomes very sharp, so t_1 is again well defined (see fig. 4.1.a). For the 1.6 cm tube no clear demarcation between a sharp edge and the rest of the signal can be observed. The main part of the signal is rather sharp, so that the sharp edge may not easily be distinguishable from it.

The corresponding u_1 values are

$z = 1.60$ cm $u_1 = 215$ m/sec
$z = 3.05$ cm $u_1 = 220$ m/sec
$z = 6.4$ cm $u_1 = 231$ m/sec

It should be noted here that these values are very near to the velocity of first sound.

4.1.5. The influence of large energies. Some experiments have been carried out with larger energy input (between 20 and 200 times larger than the "normal" values). The following deviations from the above mentioned phenomena were observed:

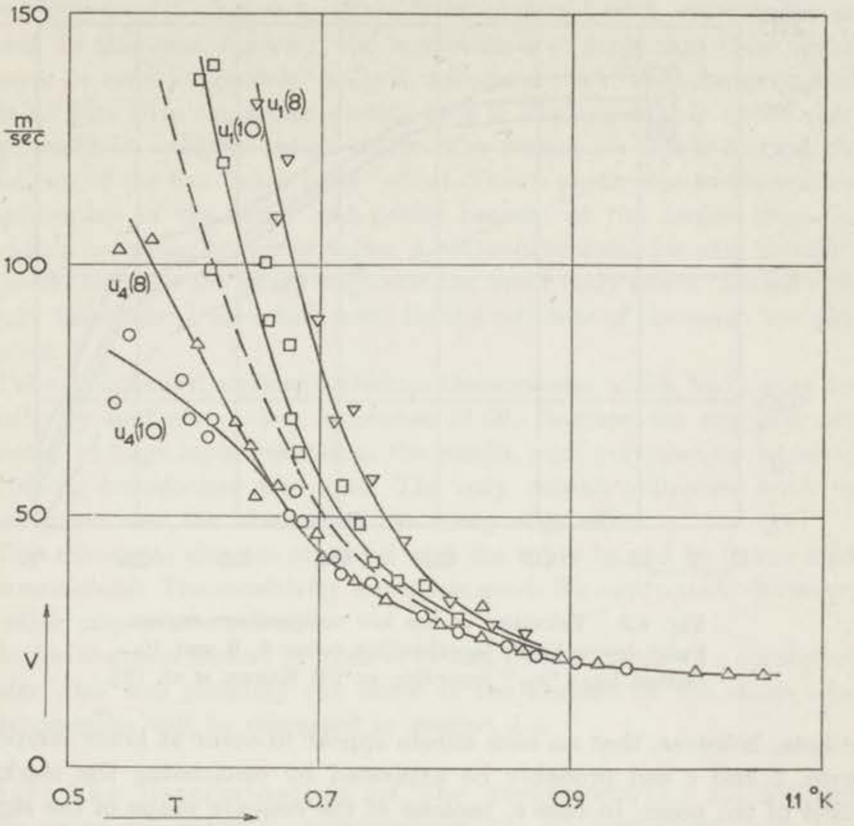

Fig. 4.4. Velocities in the high temperature region.
▽ tube 8: u_1.
△ tube 8: corrected velocity of the top.
☐ tube 10: u_1.
○ tube 10: corrected velocity of the top.
Dotted line: v_{II} according to De Klerk *et al.* [3].

a: an increase of u_1 with increasing energy for $T > 0.9°$ K.
b: the existence of slightly larger u_1-values below 0.6° K, especially between 0.4° K and 0.6° K.
c: the appearance of the sharp edge in region D at somewhat higher temperatures (up to a few hundredths of a degree higher).

Moreover, for very large energies (more then 100 times the "normal" values) were observed:

d: an increase of the velocity of the sharp edge nearer to the expected value of v_I (237 m/sec) (temperature region D).
e: an increase of the ratio of the amplitude of the sharp edge to the maximum amplitude of the signal (region D).

a is probably a large amplitude effect of the kind also noted by Osborne [4]; a detailed discussion is out of the scope of this exposition. It is curious

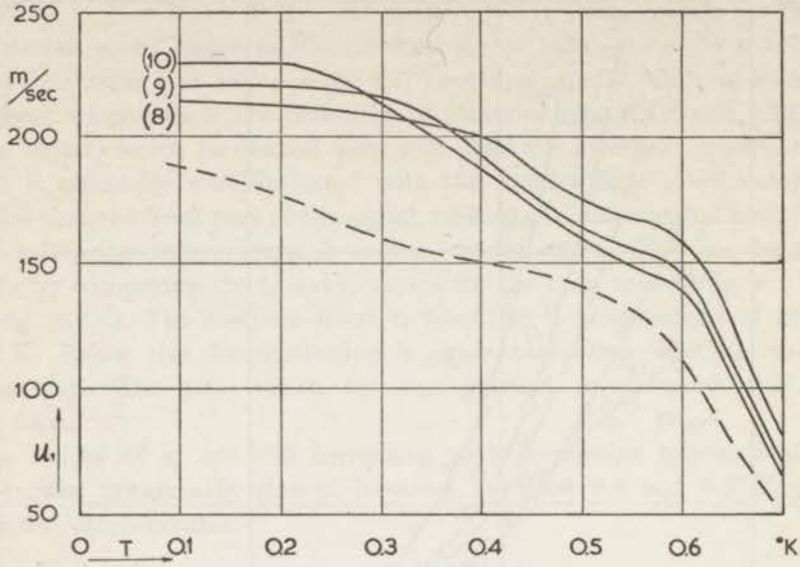

Fig. 4.5. Velocities in the low temperature region.
Fully-drawn lines: u_1-values for tubes 8, 9 and 10.
Dotted line: "v_{II}" according to De Klerk *et al.* [3].

to note, however, that no such effects appear to occur at lower temperatures. *b* and *c* can probably be explained by considering the marking effect of the noise. In case *b*, because of the concave shape of the signal front, with small input energy (i.e. relatively large noise level) the start of the signal is suppressed by the noise. The same applies to the very small sharp edge signal at its first appearance in case *c*. *e* is probably due to non-linearity of the amplifier for large input voltages; *d* will be discussed in section 4.5.

4.1.6. The other tubes. The first three tubes (no. 1: $z = 3$ cm, no. 2: $z = 6$ cm, no. 3: $z = 1.5$ cm) had about the same lengths as the tubes mentioned above. The diameters were, however, about twice as small. With tubes 2 and 3 the spreading occurring below 0.5° K was observed to be about twice as large as that of the corresponding tubes 10 and 8. The results were, however, not very reliable, probably because of the relatively small sensitivity of the commercial resistance strips used. In the experiments on tube 2 the sharp edge occurred in exactly the same manner as in tube 10. At higher temperatures the results from tube 2 were not very satisfactory. The signals were much more spreaded than in the other tubes and the sensitivity partly for this reason not very good so that pulses of moderate energy did not give adequate results.

Tube 1 was investigated with an amplifier not suitable for low frequency amplification. Therefore, the results from the temperature region with flat signals cannot be considered as reliable. By introducing the amplifier correction results similar to those later observed with other tubes are found. In this case, however, the correction is so large that these results cannot be used for further analysis. In one respect this tube gave more reliable data than the corresponding tube 9. The appearance of the sharp edge could be analysed much better. The reason for this is the relative smallness of the flat "main body" effect. This is partly due to the reduced amplification of the latter and partly because of the larger spreading probably occurring in narrow tubes. A better differentiation can, therefore, be made between the sharp edge and the main body effect. Actually the results from tube 1 have been used for the estimate of the mean free path (section 4.5).

Tube 6 contained a phosphorbronze thermometer which had a very low sensitivity and also a low resistance (2 Ω). Because the amplifier was adapted to large input resistance, the results were very meagre, although a step-up transformer was used. The only notable difference with the other tubes was the absence of the sharp edge effect.

This effect was also not observed with the tubes 7a and 7b (home-made thermometers). The sensitivity was again good. No appreciable deviations in other respect were found.

As the common feature of tubes 6, 7a and 7b was the use of a constantan heater, this was probably the cause of the absence of the sharp edge. This question will be discussed in section 4.5.

4.1.7. The determination of the temperature. All temperatures given are magnetic temperatures, i.e. no corrections have been introduced for the ellipsoidal shape of the salt or for deviations from the Curie law. The errors thus introduced are small and for the most part either well within the accuracy of the measurements or in a region of practical independence on temperature of the phenomena.

For temperatures above 0.3° K the correction is less than -0.002 of a degree, at 0.2° K it is about -0.004 of a degree; only at 0.1° K it may become of the order of -0.01 of a degree.

4.2. *The temperature range from* 0.7° K *to* 1.0° K. In this region the shape of the original pulse, although spread out substantially, preserves more or less its original character. It will be shown that the experimental results are in agreement with the supposition that second sound still exists but is rather heavily damped. It appears, however, that this absorption is probably still of the kind described in section 1.3, i.e. only relatively small deviations from local equilibrium occur in the gas of excitations and a description using common irreversible processes is adequate. According to equation 1.19 the absorption coefficient in this case is proportional to ω^2. The following analysis has first been made by Dingle [5].

When the original input pulse has approximately the shape of a δ-

function, the response can in general be written as a Fourier-integral:

$$\psi_\delta(z,t) = (2\pi)^{-1} \int_{-\infty}^{+\infty} \exp\{i\omega(t - z/v(\omega))\} d\omega. \tag{4.1}$$

The phase velocity v depends on the frequency in the way indicated by equation 1.18, i.e.

$$v^2 = v_{\mathrm{II}}^2 + i\omega\alpha. \tag{4.2}$$

In this chapter v_{II} always indicates the second sound velocity for the limit of zero frequency.

Introducing (4.2) in (4.1) and expanding in rising powers of α yields:

$$\begin{cases} \psi_\delta(z,t) = (v_{\mathrm{II}}^3/2\pi\alpha z)^{1/2} \exp\{-(t - z/v_{\mathrm{II}})^2/(2\alpha z/v_{\mathrm{II}}^3)\}\varphi(z,t) \\ \varphi(z,t) = 1 + \text{ terms of the order } (\alpha/zv_{\mathrm{II}})^{1/2}. \end{cases} \tag{4.3}$$

In first approximation this is a Gaussian curve. The top is determined by

$$t_{\max} = zv_{\mathrm{II}}^{-1}\{1 - \text{terms of the order } \alpha/zv_{\mathrm{II}}\}. \tag{4.4}$$

Returning now to the experiments and identifying the observed symmetric "pulse" with the Gaussian curve of this calculation, the first conclusion which can be drawn concerns the velocity v_{II}. As has been pointed out, the velocity u_1, accepted to be the second sound velocity by earlier investigations [3], actually depends on z. On the other hand, according to equation 4.4, the proper value of v_{II} should be equal to $z/t_{\max}$, i.e. z/t_4. Indeed z/t_4 proved to yield a unique velocity for all tubes down to 0.7° K. Below this temperature the curves for the different tubes deviate (see fig. 4.4).

In view of the very short pulses used, no attempt has been made in this and the next calculation to take into account the finite width of the heating pulse. Merely, the t_4-values used have been corrected in such a way that the zero point of time lies in the centre instead of at the start of the original pulse.

The v_{II} curve from reference [3] is also shown in figure 4.4; it was obtained from the u_1-values for a 5 cm tube. It is curious that this curve lies below the u_1-curve for the 6 cm tube in the present investigation. One would expect it to be situated just above but very near the latter. This can perhaps be attributed to a somewhat different definition of the start of the signal or to the characteristics of the amplifier employed.

The second conclusion which can be drawn concerns the value of α or of β/ω^2, in which β is the absorption coefficient at frequency ω. β/ω^2 is supposed to be independent of the frequency. According to equation 1.19

$$\beta/\omega^2 = \tfrac{1}{2}\alpha/v_{\mathrm{II}}^3. \tag{4.5}$$

A very simple calculation yields

$$(t_4 - t_3)^2/z = 2.8(\beta/\omega^2), \tag{4.6}$$

$t_4 - t_3$ being the difference of time of the top and the halfheight of the leading part of the signal. Indeed it is found that $t_4 - t_3$ is, in any case approximately, proportional to $z^{1/2}$ at constant temperature. Figure 4.6

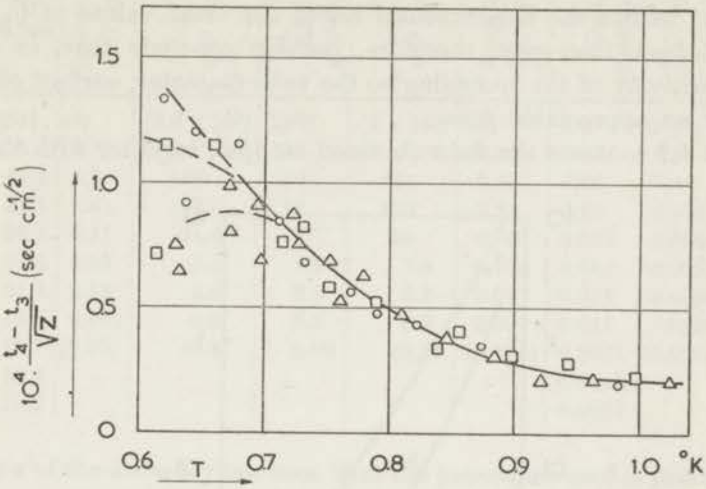

Fig. 4.6. Reduced width of pulses.
○ tube 8 ($z = 1.60$ cm).
□ tube 9 ($z = 3.05$ cm).
△ tube 10 ($z = 6.4$ cm).

gives the values of $(t_4 - t_3)/z^{1/2}$ as a function of temperature. The accuracy of this kind of measurements seems to be rather small so that not a very good accuracy can be expected for the resulting data on β/ω^2 either. The main cause for the substantial spreading of the points is that the experiments were always performed in such a way that the heating signal and the detected signal appeared both on the oscilloscope screen. For this reason especially with the longest tube the time scale had to be small. In the case of the 1.6 cm tube the time scale was usually three times as large as that employed with the two other tubes. Consequently the largest weight is given to these results. Again, below 0.7° K the values from the different tubes deviate from each other and this calculation has no more any sense.

From figure 4.6 it appears that at the high temperature end the spreading becomes constant. The width of the signal is, however, still much larger than the width of the heating pulse (20 to 40 μsec). This spreading may be due to the amplifier characteristics at high frequencies, the transfer of energy from the heater to the liquid and from the liquid to the thermometer or to an extra absorption or reflection by the surface of the walls. Probably all those effects are present but it is not easy to decide which is the most important one. Since also for this constant part the value of $(t_4 - t_3)^2/z$ appeared to be about the same for the three tubes, this part was just subtracted. The value of β/ω^2 was calculated from the corrected data on

$(t_4-t_3)^2/z$. In view of this large extra spreading it is of no use to apply a correction for the finite width of the original heating pulse.

The same calculation was made with data from tube 3 (1.5 cm length, 0.4 cm diameter). The accuracy of these early experiments was not very good, but within the experimental limits the same values of $(t_4-t_3)z^{-1/2}$ were obtained. One may, therefore, perhaps conclude that, in view of the insensitivity of the spreading to the tube diameter, surface effects do not play an appreciable role.

Figure 4.7 contains the data obtained on β/ω^2, together with the curve calculated by Khalatnikov [6]. The theory and the experiment appear to be in good agreement. It should be mentioned here that experiments on the absorption of second sound above 1° K performed by Atkins and Hart [7] and by Hanson and Pellam [8] lead to the same conclusion. The value of α can be calculated directly from β/ω^2, using also the data obtained on v_{II}.

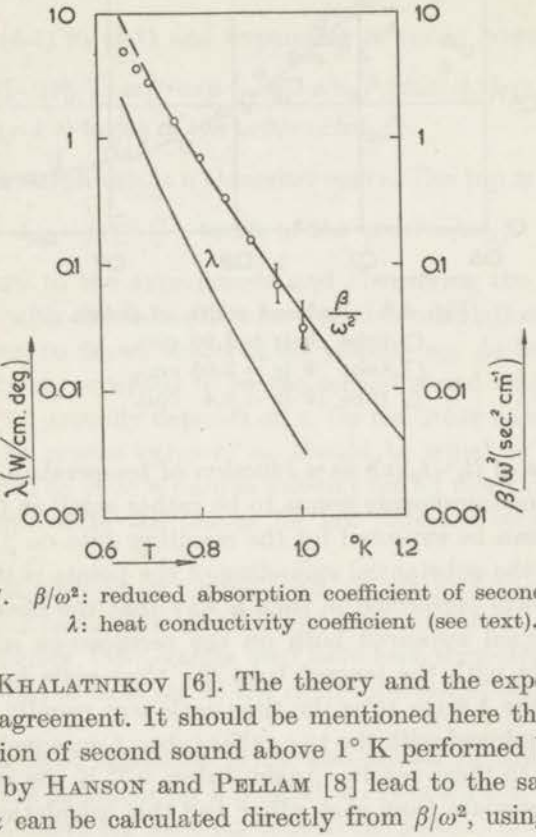

Fig. 4.7. β/ω^2: reduced absorption coefficient of second sound.
λ: heat conductivity coefficient (see text).

In table 4.3 the smoothed values of v_{II}, β/ω^2 and α are tabulated [1]).

It is now possible to investigate the validity of neglecting the higher order terms of the equations 4.3 and 4.4. α/zv_{II} had to be small compared to 1 for the evaluation of v_{II}. It proves to be equal to 0.02 and 0.07 with the 6 cm and 1.5 cm tubes respectively at $T=0.7°$ K. The square root of α/zv_{II} determines the limit set to the validity of the calculation of β/ω^2 and α. It is equal to 0.14 and 0.26 in the corresponding cases at $T=0.7°$ K.

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[1]) In the original publication [9] somewhat different data on α are given, because the correction just mentioned was not introduced.

TABLE 4.3

(Values in brackets are extrapolated)

T °K	v_{II} m/sec	$\beta/\omega^2 \times 10^9$ sec^2 cm^{-1}	α cm^2 sec^{-1}	$\alpha_\eta+\alpha_\gamma$ (Khal.) cm^{2}sec^{-1}	$\alpha_\lambda=\alpha-\alpha_\eta-\alpha_\gamma$ cm^2 sec^{-1}	$\lambda=\alpha_\lambda/\varrho c_v$ watt/cm deg	λ (Khal.) watt/cm deg	l_{ph} (exp) cm	$x \times 10^4$
0.65	(61)	(5)	(2.3×10^3)	(200)	(2×10^3)	(2)	1.3	0.3	1.2
0.67^5	53	(3.7)	(1.1×10^3)	100	(1×10^3)	(1.2)	0.65	0.15	1.75
0.7	42.8	2.9	450	40	410	0.57	0.40	0.06	3.0
0.75	34.1	1.41	112	12	100	0.21	0.20	0.019	5.6
0.8	28.1	0.71	31.4	3.7	28	0.087	0.072	0.0065	10.3
0.85	23.8	0.33	9.1	1.3	7.8	0.038	0.042	0.0023	18.0
0.9	21.1	0.15	2.8	0.5	2.3	0.017	0.018	0.0009	32.7
0.95	19.5	0.067	0.9	0.2	0.7	0.007	0.011	0.0003	50.3
1.0	18.7	(0.03)	(0.4)	0.09	(0.3)	(0.004)	0.0071	(0.0001)	82.5
1.05	18.4						0.0045		112
1.1	18.2						0.0031		161

In view of the accuracy one sees, that the procedure used is just adequate down to 0.7° K (for the evaluation of v_{II} even somewhat lower). At these temperatures the largest importance was given to the data from the longest tube (smallest high order terms). This does not conflict with the poor accuracy for this tube noted before, since the spreading is very large at $T = 0.7°$ K.

As has been explained in chapter I, α is supposed to contain contributions from first and second viscosity and from the heat-conductivity. As neither of these have been measured or even can be measured separately, for the following some conclusions of the theory have to be used. The largest contribution, according to the theory of Khalatnikov [6], is that from the heat-conductivity. His calculation gives a contributing term α_η from the first viscosity, which is about 5 % of the contribution from the heat-conductivity term (see equation 1.43). Khalatnikov's value of η is a reasonable extrapolation of measurements above 1° K. The second viscosity contribution has been deduced by Khalatnikov from the absorption coefficient of normal sound. This is a very complicated calculation which cannot be repeated here. In table 4.3 the total contribution to α from both the viscosity terms is given according to these calculations. After subtracting it from the experimental value of α the remaining part α_λ should be due to the heat-conductivity. This procedure has some sense, because the other terms are supposed to be relatively small (together up to 20 %).

One sees from equation 1.21

$$\alpha_\lambda = \lambda/\varrho c_v \tag{4.7}$$

so that λ can be evaluated. These data are also given in table 4.3 together with Khalatnikov's values. The agreement is fair. The experimental values of λ are shown in fig. 4.7.

Finally it is possible to get an estimate of the mean free path of the phonons (l_{ph}) by using the well-known formula:

$$\lambda = \tfrac{1}{3} \varrho c_{v\mathrm{ph}} l_{\mathrm{ph}} v_{\mathrm{I}}; \tag{4.8}$$

$c_{v\mathrm{ph}}$ is the phonon part of the specific heat. The data on l_{ph} are given in fig. 4.11 and in table 4.3.

4.3. *The low temperature region (main body effect).* The next part of the discussion will be devoted to the region below 0.5° K, the phenomena at intermediate temperatures being postponed until the next section. Also the sharp edge effect is overlooked for the time being.

The main shape of the detected signal shows, at any rate for the longer tubes, a very flat front and a long tail. The time corresponding to the top is roughly proportional to z^2 but, due to the large spreading of the signal, the accuracy of this determination is rather poor. Moreover, as has been pointed out, the amplifier correction has an appreciable influence on t_4. The quadratic dependence suggests a phenomenon very similar to diffusion or heat conductivity.

As has been discussed in chapter I, it may be expected that the mean free path of the phonons in this region is large compared to the diameter of the measuring tube. In that case the phonons collide mainly with the walls and not with rotons or with one another. As these collisions are probably diffuse (the mean wave length of the phonons is of the order 5×10^{-6} cm), one faces with a problem which is essentially the same as that of the heat conductivity in solid crystal rods at low temperatures, considered by Casimir [10]. For a long narrow tube this "random walk" problem can be considered as one-dimensional. The appropriate differential equation is

$$\partial T/\partial t = \tfrac{2}{3} v_{\mathrm{I}} r \, \partial^2 T/\partial z^2; \tag{4.9}$$

r is the radius of the tube (see also [11]). This is indeed a similar equation to that of the normal heat conductivity, only the temperature conduction coefficient $\lambda/\varrho c_v$ is replaced by $2rv_{\mathrm{I}}/3$. Incidentally, comparison with equation 4.8 shows that these quantities are identical if the mean free path is taken to be equal to $2r$.

With the initial condition of a heat pulse of vanishing length but finite total energy, i.e. $(\partial T/\partial z)_{z,0} = \delta(z)$ one gets the well-known solution

$$T(z, t) = A(z) t^{-1/2} \exp\left(-3z^2/8rv_{\mathrm{I}}t\right). \tag{4.10}$$

The dependence on z of the time of the maximum of this long-tailed signal is indeed quadratic:

$$t_{\mathrm{top}} = 3z^2/4rv_{\mathrm{I}}. \tag{4.11}$$

As one sees, the shape of this signal is completely determined by r and v_I which are known quantities. Therefore, it is possible to compare the theory with the experimental results.

The condition of a long and narrow tube is, however, not very well fulfilled. The best comparison can be made with the 6 cm tube (no. 10). Figure 4.8 contains the theoretical curve for this tube together with four experimental curves at different temperatures. Especially around 0.25° K the agreement is satisfactory. At lower temperatures the sharp edge disturbs the picture, while at higher temperatures (0.4° K) the front of the signal has been shifted to a larger time delay. The rear part of the curve, including the top, however, is still in close agreement. The curve at $T = 0.55°$ K shows a quite different character and fits no more with the theoretical curve.

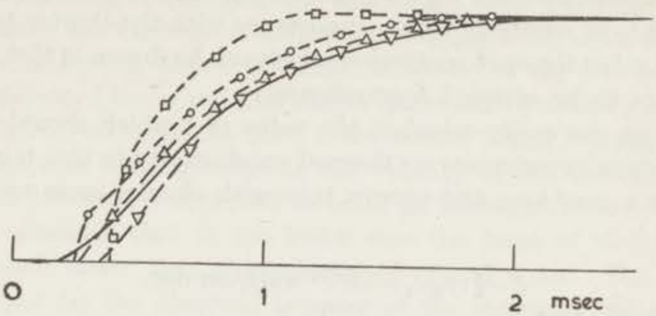

Fig. 4.8. Shape of the signal detected in the long tube ($z = 6.4$ cm).
Fully-drawn line: theoretical curve (formula 4.10).
Dotted lines: experimental results.
⊙ $T = 0.15°$ K ▽ $T = 0.40°$ K.
△ $T = 0.25°$ K ⊡ $T = 0.55°$ K.

In section 4.5 it will be shown that the mean free path of the phonons is probably not very much larger than the tube diameter. This may perhaps account for the "pushing back" of the front with increasing temperature. It is not unreasonable to suggest that a limited mean free path has indeed a relatively larger influence on the front of the signal. The phonons responsible for building up this part can only have collided a few times with the walls. By passing obliquely with respect to the walls they may easily be diverted by a collision in the liquid itself and so the front may be seriously affected. Similar collisions occur, of course, with the phonons which undergo a larger number of collisions with the wall. The collisions occurring in the liquid with these phonons are even more numerous than with the former ones, because the total path length is longer, but it is not unreasonable to suggest that the relative importance of these collisions is much less. Consequently, only a small change of shape of the rear parts of the signal may occur.

The results from the shorter tubes cannot be used for a similar analysis. Firstly, the condition of a long and narrow tube is by no means fulfilled.

This means a relatively large effect of phonons having collided only once or a few times with the walls and it is no longer a unique problem of one-dimensional "random walk". Secondly, the original signal is reflected by the thermometer and again by the heater and so on. Though these reflections are heavily damped, the detected signal may be built up out of the direct signal together with several of these reflections. Actually a second top can be traced with the 1.6 cm tube (see fig. 2, B1). Thirdly, the "sharp edge" effect is of relatively larger importance with the shorter tubes. For the shorter tubes the only conclusion which can be drawn is that equation 4.11 appears to be satisfied approximately.

Finally one can easily calculate the value of λ which should be found from stationary experiments on thermal conductivity in this temperature region. For a very long and narrow tube with circular cross section, this yields

$$\lambda = \tfrac{2}{3} r v_{\mathrm{I}} \varrho c_v = 53 r T^3 \text{ watt/cm deg.} \tag{4.12}$$

This result is, up to 0.6° K, in approximate agreement as to temperature dependence and order of magnitude with experiments of Fairbank and Wilks [12]. They used a capillary of 0.029 cm inner diameter i.e. the condition mentioned above is much better fulfilled than in the present case. λ is of course no real heat-conductivity coefficient, because it depends on the geometry of the measuring vessel. The deviations from formula 4.12 found by these authors may be due to the influence of specular reflections.

4.4. *The intermediate region* (0.5 – 0.7° K). In this region the change from second sound at the high temperature to the phenomena described in the last section at the low temperature takes place. This is just the region mentioned in section 1.7. When the temperature is decreased, the local equilibrium, characteristic in second sound, is established with increasing difficulty. This can be attributed to the rapid decrease of interaction between the excitations. Finally one may expect, that at low temperatures only some kind of diffusion of phonons is left and all trace of second sound is lost. This picture, however, may be expected to be seriously affected by the influence of the walls, because the mean free path of the phonons becomes at the same time comparable to the diameter of the experimental tube. For this reason it is not easy to give a quantitative analysis of the phenomena in this region.

The most remarkable effect is that of the relatively steep front observed with the longest tube at about 0.55° K (see fig. 4.1.d and fig. 4.8). At that temperature the rear part of the signal has already developed its long tail. The corresponding value of u_1 is about 150 m/sec. Since this is not very different from the hypothetical second sound velocity in the phonon region ($v_{\mathrm{II}} = v_{\mathrm{I}}/3^{1/2} = 137$ m/sec), it is not unreasonable to suppose that this effect represents the last trace of second sound. Moreover, a pure phonon diffusion effect without any second sound would give a signal even flatter than that occurring in the lower temperature range.

At first sight the increase of the steepness of the front of the observed signal, when the temperature is lowered from 0.7° to 0.55° K, is very curious. As has been shown, the reverse effect occurring from 1.0° to 0.7° K can be explained by the increase of damping due to irreversible effects. In a more kinetic language this means that, when the temperature is decreased, the excitations diffuse increasingly away from the travelling second sound pulse which, therefore, is spread in the forward as well as in the backward direction. The simple quantitative analysis of section 4.2 can, however, no longer be used, if the second sound velocity becomes of the same order of magnitude as the velocity of the individual phonons (v_I) which actually is expected to occur in the region below 0.6° K. It is quite conceivable that in the latter case the front of the pulse cannot travel much faster than the velocity of second sound. This may be an explanation for the observed increase of the steepness below 0.7° K. It should be mentioned, however, that in this region the quantitative analysis of section 4.2 breaks down in any case because of the large magnitude of the spreading effect, which makes the series development inadequate.

It would be very interesting to make observations in this intermediate region without the presence of the walls. When there is time enough for establishment of some kind of local equilibrium, i.e. for large enough z, one would actually expect a fairly well defined second sound pulse. For short distances no such establishment of equilibrium may be possible, in fact the diffusion effect would predominate. An increased relative sharpness of the second sound pulse with larger values of z is actually observed in the region above 0.7° K, the spreading being proportional to $z^{1/2}$ only.

This remark may furnish an explanation for the fact that the steepening of the front at about 0.55° K was well established in the 6 cm tube, but not at all in the 1.5 cm tube. The influence of the walls, however, tends to confuse the whole picture, the observed long tail is probably mainly due to that. More experiments are required for a confirmation or a rejection of the remarks of this section, especially experiments of the kind just mentioned, i.e. care should be taken that the influence of the presence of walls is reduced as well as possible.

4.5. *The sharp edge effect.* This effect, occurring at the lowest temperatures, is especially pronounced for the longest tube. With the 3 cm tube it can just be detected, while the 1.6 cm tube does not show it. As has been pointed out in 4.1.4, it is highly probable that, owing to the pretty sharp front of the main effect in the latter case, it cannot be distinguished from it.

The velocities u_1 are not the same for the three tubes, but all are very near to the expected velocity of first sound (237 m/sec). The difference of the u_1-values can be explained by assuming a constant delay of 8 ± 2 μsec. The corresponding unique velocity is

$$u_1 = 236 \pm 4 \text{ m/sec}$$

and this corresponds indeed to v_{I} within the limits of accuracy.

The cause of this constant retardation is not very clear. It is certainly not due to the electronics. Perhaps an explanation would be the combined influence of the Kapitza thermal resistances at the surfaces of the heater and the thermometer. The heat transfer from the heater to helium is characterised by a quantity having the dimension of time which is the product of this resistance (K) and the heat capacity (C) of the heater. The same occurs to the heat transfer from helium to the thermometer. If one considers only one of these effects, a time delay can never occur. The result is only a rise of temperature which is not instantaneous but shows a linear ascent. The combination of two such effects (in this case at the heater and the thermometer), however, causes indeed a horizontal tangent at the starting point. An apparent time delay is now found in the experiments, because the noise and the finite line width of the oscilloscope beam prevent the observation of the real starting point of the signal.

By extrapolation of the Kapitza resistance from the normal helium region, assuming the T^{-3}-dependence found there [13, 14] to be valid also at low temperatures, one finds at 0.2° K: $K \approx 2500$ deg/watt. The value of the heat capacity of the carbon resistance corresponding to the noted time delay should be of the order of a few hundredths of an erg per degree, which is not unreasonable.

In section 4.1.5 it was mentioned that with higher pulse-amplitudes u_1 was much nearer to v_{I}. This is in agreement with the explanation just given, because the relative influence of the noise is much reduced. Moreover, the temperature of the heater is during the time of the heat transfer almost certainly much higher than the surrounding liquid, so the Kapitza-resistance much lower than that with small pulses. Because the heat capacity of carbon is proportional to T^2 a smaller time delay should come out. It cannot, however be proved that this is the right explanation. Nothing is actually known about the Kapitza resistance at low temperatures and the heat capacity of the carbon resistors could not be determined either.

The obvious explanation of the sharp edge effect, being due to thermal phonons (i.e. with $\omega \approx \varkappa T/\hbar$) travelling directly from the heater to the thermometer, will be discussed later on in this section. It is, however, first necessary to investigate the other possible explanation ascribing the sharp edge to low-frequency (acoustical) phonons. According to this picture the heater would generate normal sound observable at the thermometer by means of a microphonic effect of the latter.

In an attempt to arrive at a decision between the two possibilities the different kinds of thermometer were used. Unfortunately, however, the results do not lead to a conclusive answer. If a constantan heater was used, the effect did not occur indeed but, as will be explained now, this is probably due to its large heat capacity. The specific heat of a metal at these low temperatures is almost exclusively due to the electrons and so proportional to T. Actually for the heater used the heat capacity was of the order of $10^{-7} \times T$ joule/deg. This means that the "KC-time", determining the heat transfer to the liquid is proportional to T^{-2}, if one assumes the same extrapolation as before of the KAPITZA resistance. The result is $KC \approx 4 \times 10^{-6} T^{-2}$, i.e. about 100 μsec at 0.2° K and even 400 μsec at 0.1° K. (The magnitude of K is taken to be twice as large as that in the former case, because the effective area is smaller by a factor 2.) This large KC-time causes the front of the signal, departing from the heater, to rise only slowly, the tangent at the start being proportional to $1/KC$, assuming an original instantaneous rise. So the sharp edge is completely drowned. Moreover, it has been proved that also the overall shape of the signal is seriously affected. It is "rounded off" appreciably, especially with the 1.5 cm tube no. 7a (see also fig. (4.1.b).

The u_1-values of these tubes in this region appear to be slightly smaller than those with the tubes no. 8, 9 and 10, but this may be attributed to the combined effect of noise and slow ascent. The conclusion of this discussion is, that the non-existence of the sharp edge in experiments on tubes with constantan heaters is no proof for a sound pulse in the case of carbon heaters. As has been pointed out earlier in this section, no direct evaluation can be made of the heat capacity of the carbon resistors. The amount of carbon and the behaviour of the paper background is unknown but in any case the temperature dependence of the heat capacity is certainly much larger (probably $\sim T^2$ for the carbon and $\sim T^3$ for the background paper). The KC-time is, therefore, only increasing with $1/T$, while going to lower temperatures. This points to a much smaller effect in the latter case, compared to that of the constantan heaters.

The observations of the sharp edge with commercially made carbon resistors show exactly the same behaviour as those made with the home-made ones. This is an argument for the phenomenon not to be due to normal sound, because these resistors might be expected to be much less liable to produce sound.

The next part of the discussion will concern the second possibility which attributes the sharp edge to the propagation of thermal phonons. It will be shown, that this hypothesis completely fits into the picture of the liquid at these temperatures.

One has to start with a consideration of the phenomena below about 0.15° K, where the edge is fully developed. Firstly, the elevation of the sharp edge is roughly proportional to the solid angle looking from the heater to the thermometer (i.e. $\sim 1/z^2$) as should be expected. Secondly, the observed ratio of the height of the sharp edge to the finally reached almost constant value of the corrected signal can be compared with the computed ratio of the temperature rise due to phonons going the direct way and the final temperature rise reached, if all energy input is used for heating the helium content of the tube. Again only a rough estimate can be given, especially because the height of the tail cannot be measured with much accuracy. In any case the first ratio is found to exceed the second one probably by a factor 3 or 4. This appears at first to be disconcerting. However, phonons which have only once or twice reflected with the walls and pass almost parallel to them, may also contribute to the sharp edge. Moreover, one has to keep in mind that an appreciable amount of heat may escape through the holes in the walls, thus tending to reduce the after-effect. This is in agreement with observations with earlier tubes (no. 1, 2 and 3), where the excess factor was still much larger. The whole surface of the walls of these tubes was perforated with many holes, while in tubes 8, 9 and 10 only four rows of holes were bored.

One must turn now to the curious phenomena of the disappearance of the sharp edge already at about 0.2° K with the 6 cm tube and at about 0.34° K with the 3 cm tube. Actually one would expect from an extrapolation of Khalatnikov's predictions the mean free path to be much larger than z in the whole region below about 0.45° K. If this was true, the sharp edge would exist everywhere below 0.45° K. Moreover, the increase of the mean free path would be so fast with decreasing temperature, that the appearance of the edge would occur at practically the same temperature for different values of z. According to Khalatnikov l_{ph} is proportional to T^9 in this region [18].

The appearance is, therefore, that in reality the mean free path is much more limited than would be expected from Khalatnikov's deductions. One can try to calculate the mean free path from the observations in the following way. At extremely low temperatures, i.e. with very large mean free path, all phonons starting from the heater within a certain solid angle will reach the thermometer along the direct way. At higher temperatures only the relative amount $F = \exp(-z/l_{\mathrm{ph}})$ will accomplish this. For F equal to 0.5

$$l_{\mathrm{ph}} = 1.65\, z. \tag{4.13}$$

As it is not possible to determine the height of the sharp edge in absolute measure, because the thermometer sensitivity changes in an unknown way with temperature, this method could not be used directly. The best procedure appeared to be to calculate the ratio of the height of the sharp edge (h_1) and the height of the signal at $t_2 = 2t_1$ (h_2). The result for the tubes 1 and 10 is shown as a function of temperature in fig. 4.9. Tube 1 (the narrow 3 cm tube) was used instead of tube 9, because the sharp edge was much better observable in the former. The reason for that has already been mentioned in 4.1.6. It may be noticed here that the larger spreading occurring with narrower tubes is in complete agreement with what has been said about the main body effect. From equation 4.11 it is immediately seen, that the signal is much flatter with small r, because t_{top} is inversely proportional to r.

Equation 4.13 was now taken to be valid at the temperature, where h_1/h_2 was reduced to one half of its value at the lowest temperature. It is clear, that the accuracy is not very good. Moreover, the procedure used here for determining the mean free path is very much open to criticism, because the heat leak through the holes in the wall will also change considerably with temperature (compare equation 4.12). This effect may be supposed to have a much larger influence on h_2 than on h_1. At any rate it can be expected, that the order of magnitude is right. One finds $l = 5$ cm at $T = 0.34°$ K and $l = 10{,}5$ cm at $T = 0.21°$ K.

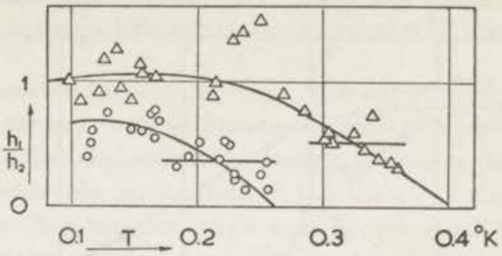

Fig. 4.9. The relative sharp edge effect as a function of temperature. △ tube 1 ($z = 2.99$ cm); ○ tube 10 ($z = 6.4$ cm).

An explanation for this curious extra-limiting of the mean free path will be discussed in the next section.

4.6. *The influence of very small amounts of* ^3He. The experimental results of section 4.5 show, that the mean free path of the phonons probably undergoes yet another limitation of its magnitude apart from the processes discussed by Khalatnikov. Firstly it may be that Khalatnikov has misinterpreted the phonon-phonon interaction processes. His treatment of this part of the theory appears to be not quite satisfactory as has already been mentioned in chapter I. On the other hand in the analogous case of phonons in a crystal one has also concluded to a very large mean free path.

Another possible explanation is that noted in the last part of section 1.7, i.e. a limitation of the mean free path due to interactions with the very small concentration of ^{3}He atoms always present in the liquid which is used in the experiment. The concentration of ^{3}He in helium from wells is probably about 1.4×10^{-7} [20]. To investigate this influence one experiment was done with atmospheric helium (^{3}He concentration about 10^{-6}). Unfortunately in this experiment the 3 cm tube 9 was used so that the sharp edge could not be observed properly. An appreciable change was, however, observed in the behaviour of u_1. Figure 4.10 shows a comparison between the results of both kinds of helium for this tube. With the atmospheric helium u_1 is seen to be practically constant between 0.35 and 0.6° K. Its magnitude in that case is about 150 m/sec. In the liquid with the smaller ^{3}He concentration such a constant region perhaps exists between 0.5° and 0.6° K, but below that temperature u_1 is increasing again. Such an increase occurs with the atmospheric helium only below 0.35°. Moreover, in that case the final v_{I} value is reached just below 0.2° K instead of in the neighbourhood of 0.3° K. All this points to a considerable decrease of the mean free path. The intermediate zone of section 4.4 appears to be extended down to 0.35° K and the sharp edge region with u_1-values very near to v_{I} is probably also shifted to lower temperatures. In view of the result of these measurements it is quite plausible, that in the case of a ^{3}He concentration which is about 10 times less, i.e. in the normal experimental helium, the mean free path will be limited in the way explained in the last section. In pure ^{4}He one may perhaps expect the sharp edge actually to occur already near 0.45° K.

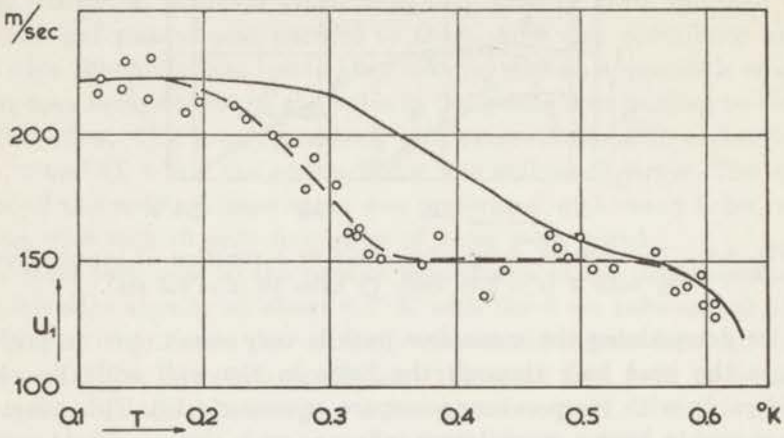

Fig. 4.10. The influence of the ^{3}He concentration on u_1.
Full line: helium from wells; dotted line and points: atmospheric helium.

From a general point of view it is evident, that an interaction of phonons with ^{3}He atoms should indeed exist. It is perhaps allowed to compare the latter in this respect with the rotons. Actually the number of rotons and the number of ^{3}He atoms in the same volume are of the same order of magnitude at about 0.5° K. The same applies for $x_{\mathrm{r}} = \varrho_{\mathrm{nr}}/\varrho$ and the ^{3}He concentration at about 0.45° K (see fig. 4.11), therefore, it is quite feasible, that somewhere near this temperature the ^{3}He atoms begin to take over the role of path-limiters from the rotons.

In contrast to that of the rotons the ^{3}He concentration is independent of the temperature. Therefore, the dependence of the mean free path on the temperature should indeed be much smaller below the temperature quoted. At first sight one would expect a temperature-dependent factor of T^4 as in the Rayleigh type of scattering of phonons by impurities. The experimental values given indicate even a smaller power of T, but it should be emphasized, that neither the accuracy not the number of experimental data (two points on the $l(T)$ curve) is sufficient to draw a definite conclusion.

One may wonder perhaps, if the heat-flush effect does not have a large influence on the actual ^{3}He concentration. Due to this effect, as has been observed in the normal helium region, the main part of the ^{3}He might be swept by the normal fluid to the coldest spot of the vessel, i.c. the paramagnetic salt. Because of the very small value of ϱ_n or, in other words, the small amount of excitations at this temperature it is not unreasonable to suppose this effect to be relatively small.

4.7. *Some final remarks and conclusions.* The expositions of the foregoing sections are, in general, well in agreement with the picture proposed in the first chapter. The behaviour of second sound in the region above 0.7° K finds a reasonable explanation in the theory of elementary excitations combined with Khalatnikov's considerations of the interactions. The proposed picture at lower temperatures is a logical extension of the latter and appears to be well in agreement with the experiments. The influence of a very small concentration of ^{3}He is probably also established, although more experiments are required to get more exact information. It would give a sufficient explanation of the deviation from Khalatnikov's original hypothesis with respect to the mean free path at temperatures

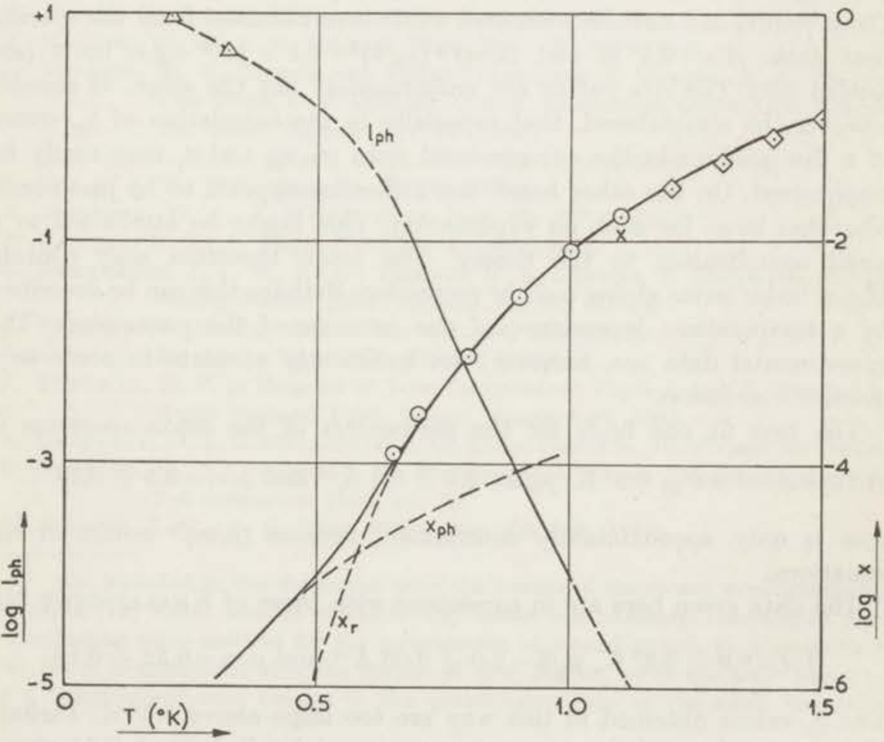

Fig. 4.11. $x = \varrho_n/\varrho$ and l_{ph} as a function of temperature.
⊙ points calculated from the present experiments;
◇ points calculated from Peshkov's experiments.
For further details see the text.

below half a degree. There is, therefore, as yet no reason to distrust his deductions for the case of pure ^{4}He.

If one accepts equation 1.17 for the velocity of second sound

$$v_{\mathrm{II}}^2 = \frac{\varrho_{\mathrm{s}}}{\varrho_{\mathrm{n}}} \frac{S^2 T}{c_v}$$

ϱ_{n} or $x = \varrho_{\mathrm{n}}/\varrho$ can be calculated from the experimental data on v_{II} and c_v. The obtained values can be compared with the theoretical predictions of the theory of excitations.

The latter yields for the phonons

$$x_{\mathrm{ph}} = \varrho_{\mathrm{nph}}/\varrho = \tfrac{1}{3}\, c_{\mathrm{ph}}/v_{\mathrm{I}}^2 = 14 \times 10^{-5} \times T^4 \tag{4.14}$$

using a value of v_{I} equal to 237 m/sec together with the experimental data on the specific heat of chapter III.

One can now compare $x - x_{\mathrm{ph}}$ with the theoretical formula for x_{r} (see equation 1.40). Proceeding in a way analogous to that used for the specific heat one finds: $\Delta/\varkappa = 8.7°$ K and $(\mu/m)^{1/2}\,(p_0/\hbar)^4 = 6.4 \times 10^{32}$ c.g.s. units. These values are within a few percent in agreement with the data calculated from the experiments between 0.8° K and 1.5° K. For the temperature region above 1.1° K the data on v_{II} of Peshkov [15, 16] were used. These results can now be combined with those obtained from the specific heat data: $\Delta/\varkappa = 9.1°$ K and $(\mu/m)^{1/2}\,(p_0/\hbar)^2 = 2.4 \times 10^{16}$ c.g.s. units (see section 3.5). The $\Delta/\varkappa$ values are unfortunately not the same. It should, however, be remembered, that especially in the calculation of ϱ_{nr} errors of a few percent in the experimental data on v_{II} and c_v may easily be emphasized. On the other hand, the difference appears to be just somewhat too large for such an explanation. This might be attributed to a small complication in the theory. The roton spectrum may contain higher order terms giving a slight correction. Perhaps this can be described by a temperature dependence of one or more of the parameters. The experimental data are, however, not sufficiently accurate to come to a definite conclusion.

The best fit one finds for the parameters of the roton spectrum is

$$\Delta/\varkappa = 9.0 \pm 0.2°\ \mathrm{K},\ p_0/\hbar = 2.0 \pm 0.1\ \text{Å}^{-1} \text{ and } \mu/m = 0.3 \pm 0.15 \tag{4.15}$$

μ/m is only approximately determined, because $(\mu/m)^{1/2}$ occurs in the equations.

The data given here are in agreement with those of Khalatnikov [6]:

$$\Delta/\varkappa = 8.9 \pm 0.2°\ \mathrm{K},\ p_0/\hbar = 2.0 \pm 0.05\ \text{Å}^{-1} \text{ and } \mu/m = 0.32 \pm 0.13.$$

The ϱ_{n}-values obtained in this way are too large above 1.2° K. Perhaps there the values of the parameters proposed by Feynman [17] give a better agreement.

In figure 4.11 x, calculated by means of the equations 4.14 and 1.40, using the data on the rotons 4.15, is plotted as a function of T. The points shown are calculated from the experiments as explained in this section. In the same figure are also given the data obtained on the phonon mean free path.

The conclusion which can be made from the work of this chapter is that many of the observed phenomena can be explained rather well in a qualitative way by means of the theoretical considerations put forward in chapter I, together with the supposition of the influence of a small ^{3}He impurity in the lowest temperature region. Some phenomena have even found a more or less quantitative foundation, but in this respect quite an amount of work remains to be done. Also, more experiments are required on the influence of ^{3}He; these have actually been started in the Kamerling Onnes Laboratory ¹).

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¹) Not included in the discussion were the results of the recent experiments by Osborne [19] which became available only at the last moment. This author used a continuous wave method for the propagation of second sound. In a qualitative way there is agreement with the results of this chapter, although there seems to be a disagreement with respect to the quantitative data on the mean free path.

SAMENVATTING

In dit proefschrift worden twee reeksen experimenten beschreven, die tot doel hadden nadere gegevens te verkrijgen over het gedrag van vloeibaar helium in het temperatuurgebied beneden 1° K. In de eerste groep experimenten werd de soortelijke warmte gemeten. Aangezien deze metingen werden uitgevoerd tussen 0.25 en 1.8° K, konden ook nieuwe gegevens verkregen worden over de soortelijke warmte in het heliumgebied boven 1° K. De voortbeweging van korte warmtepulsen in de vloeistof werd onderzocht in de tweede groep experimenten.

In hoofdstuk I wordt een overzicht gegeven van de theoretische achtergrond van deze experimenten. Het twee-fluida model wordt besproken, waarbij bizondere aandacht wordt besteed aan het z.g. second sound en de absorptie van deze onder invloed van irreversible processen. Voorts wordt een korte samenvatting gegeven van de theorie van de elementaire excitaties (fononen en rotonen), welke, althans in dit temperatuurgebied, een adequate basis vormt van het twee-fluidamodel. Tenslotte wordt de theorie van KHALATNIKOV besproken; op grond van beschouwingen over de wisselwerkingen tussen de elementaire excitaties blijkt het mogelijk een nadere analyse te geven van de irreversibele processen in helium II.

Hoofdstuk II bevat een overzicht van de gebruikte meetopstellingen en experimentele methoden. Enkele daarmee samenhangende problemen worden bovendien aan een nader onderzoek onderworpen.

In hoofdstuk III worden de resultaten van de soortelijke warmte experimenten uiteen gezet en gediscussieerd aan de hand van de theorie van de elementaire excitaties. Geheel in overeenstemming met deze blijkt de soortelijke warmte in het laagste temperatuurgebied (beneden 0.6° K) evenredig te zijn met T^3 en dus geheel bepaald door de fononen. Als men de temperatuur laat toenemen, gaat de soortelijke warmte van de rotonen, welke evenredig is met $\exp(-\Delta/\varkappa T)$, de overheersende rol spelen. Theorie en experiment stemmen ook hier binnen de meetnauwkeurigheid goed overeen.

Hoofdstuk IV bevat allereerst een opsomming van de experimentele gegevens, die verkregen werden uit de waarnemingen aan de warmtepulsen. Deze worden daarna uitvoerig besproken en voorzover mogelijk getoetst aan de theorie. Het blijkt, dat boven 0.7° K de warmtepulsen zich gedragen als gedempte second-sound-pulsen. De experimenteel gevonden demping stemt goed overeen met de voorspellingen van KHALATNIKOV. Beneden 0.5° K kan het gedrag van de pulsen niet meer verklaard worden met behulp van second sound. De gevonden resultaten wijzen erop, dat de vrije weglengte van de fononen in dit gebied groot wordt ten opzichte van de doorsnede van de meetbuis. Dit moest ook op grond van de theorie van KHALATNIKOV verwacht worden. Er zijn echter sterke aanwijzingen, dat de vrije weglengte toch minder snel toeneemt met afnemende temperatuur dan deze theorie voorspelt. Als een mogelijke verklaring van dit verschijnsel wordt de invloed genoemd van zeer geringe hoeveelheden ^{3}He. Voorlopige resultaten van nieuwe experimenten zijn hiermee in overeenstemming. Het gebied tussen 0.5 en 0.7° K tenslotte vormt een overgang tussen beide bovengenoemde gevallen. De resultaten van de metingen zijn daar nog weinig toegankelijk voor een quantitatieve analyse.