THE ATTENUATION OF SOUND IN SOME DILUTE ^{3}He-^{4}He MIXTURES

A.M.R.VAN IERSEL

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PROEFSCHRIFT

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Aan mijn Moeder
Aan de nagedachtenis van mijn Vader
Aan mijn Vrouw

Teneinde te voldoen aan het verlangen van de Faculteit der Wis- en Natuurkunde volgt hieronder een kort overzicht van mijn academische studie.

Na het beëindigen van mijn gymnasiale opleiding in 1942, ving ik in 1945 aan met de studie in de Wis- en Natuurkunde te Leiden. In 1948 legde ik het candidaatsexamen, letter A, af. Na een onderbreking van twee jaar, doorgebracht in militaire dienst, hervatte ik in 1950 de studie en legde in 1953 het doctoraalexamen in de experimentele natuurkunde af. De tentamina werden mij afgenomen door Dr J. Korringa en Prof. Dr J. Droste. In 1951 kwam ik bij de groep van Prof. Dr A. F. van Itterbeek en sedert dat tijdstip heb ik, in samenwerking achtereenvolgens met Drs W. Limburg, Drs A. W. Wijnhoven, Drs H. van Ee en Drs R. David, gewerkt aan het ontwikkelen van een apparatuur geschikt om de absorptie van geluid te meten. In 1954 kwam ik in dienst van de stichting FOM.

In 1957 kwam Prof. E. W. Guptill van Dalhousie University, Halifax, voor een jaar op het Kamerlingh Onnes laboratorium en in een bijzonder prettige samenwerking hebben wij de experimenten verricht die in dit proefschrift zijn beschreven.

De heer H. Nater wist immer een oplossing voor de mechanische problemen; veel hulp kreeg ik van de cryogeentechnicus Th. Nieboer en van de glasblazer H. Kuipers. De electrotechnicus F. Juynboll bouwde de in Hoofdstuk III beschreven electronische apparatuur. De „corner reflector", beschreven in Hoofdstuk III, werd gemaakt door de glasslijper H. van Zanten. Het Engels werd gecorrigeerd door Dr W. Zimmerman.

CONTENTS

INTRODUCTION

According to the theory of Landau the thermal energy of liquid helium consists of two kinds of excitations of the liquid: phonons and rotons. The phonons are quantized sound waves, the rotons can be described as being in a way the smallest possible vortex rings.

If one knows the energy of an excitation as a function of its momentum then, as is known, the thermodynamical functions for helium II can be calculated.

For a phonon this relation is simply $E = cp$ where c is the velocity of ordinary sound. This velocity has been measured extensively. It has the rather low value of about 240 m/s, and below 1.0°K there is little variation.

Phonons and rotons can be thought of as particles, that is to say - with a slight expenditure of extra energy and by a proper combination of running waves with definite amplitude and phase relationships - the excitations are localized and the thermal disturbances of the liquid are confined to a small region in space of atomic dimensions or, in the case of phonons, of the dimensions of the wavelength of sound. One has a picture then of a gas of quasi-particles, which interact only weakly except close to the λ-point. The interactions do not influence markedly the thermodynamical properties of helium II at the lower temperatures, but they determine, as in a gas of molecules, the transport properties of the liquid such as its viscosity.

When one looks at the attenuation of sound in pure liquid ^{4}He one notes that only above 3°K the classical viscous and thermal conduction losses account for the attenuation found experimentally.

In this thesis we restrict ourselves to the temperature region form 1°K to T_λ. The theory which explains the observed attenuation in this region has been given by Khalatnikov prior to any measurements below 1.4°K. His prediction that the attenuation should increase by a factor of 20 or more when the temperature is lowered has been confirmed by the experiments of Chase. The origin of the high attenuation is a peculiar one. The *number* of excitations per volume element has to vary with the period of the sound wave. Otherwise the sound is propagated under conditions far from equilibrium and losses occur. Khalatnikov has shown that it takes a finite time to produce or annihilate excitations. This *relaxation time* varies strongly with temperature and when it becomes comparable with the period of the sound wave a large relaxation loss results.

In the present work we experimented with mixtures of ^{3}He and ^{4}He of low ^{3}He concentration and studied the effect of the ^{3}He impurity on the attenuation of sound in the above temperature region.

Before describing the experiments we give a brief outline of Khalatnikov's theory, in Chapter II. This, however, can not be done without a general knowledge of the theory of helium II to which we pay attention in Chapter I. In Chapter III and IV then follows a description of the experiments. A discussion of the results is given in the last Chapter.

Chapter I

THEORETICAL CONSIDERATIONS

This Chapter is meant as an introduction to the theory of Khalatnikov and is devoted to a sketch of the general theory for helium II.

In section 1 we review the theory of F. London briefly. As is known this theory led to the concept of the two fluid model by Tisza.

In section 2 we outline the excitation theory of Landau. His interpretation of the two fluid model differs thoroughly from that as proposed by Tisza. At the same time we introduce Feynman's calculations for the energy of an excitation as a function of its momentum because these calculations clarify the picture a great deal and give the necessary theoretical support to the original hypotheses of Landau. These hypotheses were quite ad hoc assumptions and seemed not to pay any tribute to F. London's idea about the importance of the type of statistics for the problem of helium II.

Attention is paid in section 3 to the theory of Landau and Khalatnikov for the transport properties of helium II, especially to the interactions between the elementary excitations and to the phonon mean free path length.

We will not speak about rotating helium II, nor about the friction between normal and superfluid.

1. *The early theory of F. London and Tisza's two fluid model*

As is known, helium stays liquid under its own vapour pressure down to the absolute zero of temperature. It can be solidified only by applying a pressure of about 25 atm. F. London [1], after observations by Simon [2], pointed out that this results from the role the quantum mechanical zero-point energy plays in liquid helium. We give his explanation here to stress the fact that helium is a liquid with ample spacing between the atoms.

Because of the small mass of a helium atom the zero-point kinetic energy is large, and since this energy decreases with increasing volume it acts as if there is a strong repulsive force between the atoms. On the other hand the attractive forces are only weak and therefore the minimum in total energy occurs for a volume which is about three times as large as is calculated classically.

The result is that not all the atoms are on sites of minimum potential energy. The structure of condensed helium is not crystalline.

Consistent with this picture, no difference in structure is found between helium I (above 2.17°K) and helium II (below 2.17°K), each possessing a x-ray diffraction pattern typical of an ordinary and simple liquid (Keesom and Taconis [3]), Reekie and Hutchinson [4]). The two-atom probability function $p(\mathbf{r}_1, \mathbf{r}_2)$, which is the probability per cm^3 of finding an atom at $\mathbf{r}_2$ if it is known that one is at $\mathbf{r}_1$, has a distinct maximum at a wave number of about 2 Å^{-1} which corresponds to a nearest neighbour distance of 3.16 Å.

Though helium is not crystalline, the liquid must have a high degree of order in an other sense. Many experiments indicate that between 2.2 and 1°K, liquid helium looses almost all the entropy characteristic of the ordinary liquid state [1].

In 1938 F. London [1,5]) proposed that the transition, at T_λ, of liquid helium into a state with "superfluid" properties might be understood from the degeneracy phenomenon in a perfect Bose-Einstein gas, viz. below the critical temperature, T_c, a finite fraction of the atoms "condenses" into the state of lowest momentum and energy. The degeneracy phenomenon, in reality, is modified by the interatomic forces, and one must take into account the obvious fact that the transition occurs in a liquid. However, London argued, liquid helium has gas-like properties, for instance the viscosity coefficient increases with temperature as in a gas. Since the interatomic distances in helium are large, the passage of a helium atom from one place to another is not hindered by interlocking force fields as in a liquid of higher density.

Not much attention had been paid to the Bose-Einstein degeneracy, partly because all other gases are condensed well above the condensation temperature. For a gas of helium atoms: $T_c \simeq 3$°K.

The specific heat of the ideal Bose-Einstein gas shows the embryo of the observed λ-peak. From 0°K to T_c the specific heat depends on temperature as $T^{3/2}$. This is far different from the experimental facts but it is not surprising that the ideal Bose-Einstein gas theory - which takes into account only single particle excitations and neglects all collective excitations which spring from the interaction between the atoms - is too rough an approximation.

Many attempts have been made, after the promising proposition of F. London, to modify the excited levels of the particles in a Bose-Einstein gas, aiming at a better fit of the calculated thermodynamical quantities with the experimental data.

London's idea that liquid helium, in spite of the strong interaction between the atoms, should show some characteristics of the Bose-Einstein degeneracy proved especially fruitful in inspiring Tisza [6]) to develop the so-called two fluid theory for liquid helium II. The idea that some of the atoms are "condensed" and have zero momentum and energy led him to divide the mass of the fluid into two parts:

$$\rho = \rho_n + \rho_s \qquad (1,1)$$

Tisza thus postulated that helium II, on a macroscopic scale, should be composed of two mutually interpenetrating fluids. One, the superfluid, should contain all the condensed atoms. The total contribution of the condensed atoms to the density is ρ_s; the normal fluid consists of the excited, moving atoms with total density ρ_n. From theoretical considerations Tisza inferred a zero entropy for the superfluid *. The viscosity of the superfluid is supposed to be zero as well.

With the help of the above assumptions the "super" heat conductivity of helium II can be explained. In the presence of a temperature gradient the number of normal atoms has a gradient as well (at 0°K all atoms are "condensed") and there will be a flow of normal atoms or fluid against grad T. To keep the total density constant, the superfluid will flow opposite to the normal fluid. The normal fluid carries the heat, and thus by internal convection helium II has a high thermal conductivity.

Both fluids then, in general, may have their own velocity field and the total momentum density is supposed to be:

$$\underline{j} = \rho_n \underline{v}_n + \rho_s \underline{v}_s \qquad (1,2)$$

In Tisza's theory the flow of the superfluid is a purely mechanical and reversible process and in first approximation the two fluids exert no frictional forces on each other.

The two fluid theory has been developed without any further reference to the degeneracy in a gas of bosons. H. London 8), Gorter 7) and de Groot, Jansen and Mazur 9) laid the termodynamical foundations for and generalized the two fluid theory. We will not speak about the different assumptions one can make for the relation between the entropy and the normal fluid density (both functions of p and T). This relation follows unambiguously from Landau's theory.

2. *Landau's excitation theory. The calculations of Feynman*

In 1941, Landau 10) gave quite another version of the two fluid model. According to his theory, the thermal energy of the liquid does not consist of single particle excitations but, principally, of collective excitations of the liquid as a whole. The division into "condensed" or "normal" atoms and "superfluid" atoms is not made.

In an attempt to find the stationary energy levels Landau proceeded by quantizing hydrodynamics directly. He came to the conclusion that in any liquid the states of potential (rot $\underline{v} = 0$) motion and those of vortex (rot $\underline{v} \neq 0$) motion are separated by an energy gap, Δ. As Feynman 11) has shown this conclusion is definitely not

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* London called the superfluid "virtually one quantum state" and thus denied it any entropy. Gorter 7) did not make this assumption; he suggested that the superfluid might have entropy and that there should be an entropy of mixing then as well. In Landau's excitation theory the entropy of the superfluid is zero per force.

true in general, but depends on the type of statistics. For a liquid obeying Bose statistics, like ^{4}He, it does apply.

It is equally important that every weakly excited state of liquid He II, for temperatures not too close to the temperature of the lambda-transition: 2.17°K, can be considered as an aggregate of a number of elementary excitations, which belong to one of two groups:

phonons, with dependence of energy on momentum

$$E = cp \tag{1,3}$$

and

rotons for which, with an accuracy up to terms of the second order in their momentum, the relation

$$E = \Delta + (p-p_o)^2/2\mu \tag{1,4}$$

is valid. Δ is the energy gap or the minimum roton energy, μ is the effective roton mass. That the minimum occurs at a non-zero momentum, p_o, was a third hypothesis inferred from Peshkov's [12] second sound measurements.

Landau thought the rotons to be quantized vortex motions of the liquid. Calculations of Feynman and Cohen [13] have corroborated this view. The phonons are quantized sound excitations [1,14]. They are the eigenvalues of the Hamiltonian for the liquid in irrotational movement when terms of third and higher order in the phonon creation and annihilation operators are neglected.

From the experimental values for the specific heat it follows that Δ/k is large, about 9°K, and liquid helium at temperatures up to a few tenths of a degree below T_λ is indeed weakly excited. Thus the interaction energy of the excitations can be neglected. The assembly of excitations can be thought of as a gas of Bose "particles". Their total number is not specified but has to be found from the condition that the liquid is in thermal equilibrium. Minimizing the entropy under the condition that the total energy is constant gives for the distribution function $n(\underline{p})$:

$$n = [\exp(E/kT)-1]^{-1} \tag{1,5}$$

where n is the number of excitations per volume element of phase space.

With this distribution function and the energy versus momentum spectrum (1, 3 and 4) the thermodynamical functions for helium II can be calculated. A good fit with the experimental data was obtained with the roton Δ, p_o and μ as adjustable parameters.

We give the equilibrium number densities of phonons, N_{ph}, and rotons, N_r, here:

$$N_{ph} = \int n \frac{4\pi p^2}{h^3}\, dp = (48\pi/5)\,(kT/hc)^3 \simeq 2\ 10^{19} T^3\ \text{cm}^{-3} \tag{1,6}$$

$$N_r = 2p_o^2(2\pi)^{3/2}(\mu kT)^{1/2}\exp(-\Delta/kT)/h^3 \simeq 5.1\ 10^{22} T^{1/2}\exp(-\Delta/kT)\ \text{cm}^{-3} \tag{1,7}$$

The theory of Landau seemed rather dubious. Especially the nature of rotons was far from understood. The positive energy gap, Δ, was an assumption under which Landau proved the existence of superfluid flow. Suppose that the liquid at 0°K is in potential motion with a flow velocity, $\underline{v}_s$, relative to the walls of a vessel. If the liquid is slowed down by friction with the wall this implies the transformation of mechanical flow energy into heat. According to Landau, in the liquid an excitation has to be created by interaction with the wall. The liquid gets a total momentum $\underline{p}$ due to the excitation. By a Galilean transformation [10] the energy of an excitation relative to a fixed coordinate system is

$$E = E(\underline{p}) + \underline{p} . \underline{v}_s \qquad (1,8)$$

where $E(\underline{p})$ depends on $\underline{p}$ according to (1,3) or (1,4). From the condition $E < 0$ and the energy spectrum for $E(\underline{p})$ it follows that $\underline{v}_s$ must exceed a certain velocity before an excitation can be created or friction occurs.

The situation has been clarified by the work of Feynman [15] who developed an atomic theory for liquid helium. He made it acceptable indeed that the phonons are the only low lying energy states of the liquid. This appears to be a consequence of the validity for ^{4}He of Bose-Einstein statistics.

We review here briefly the calculations of Feynman [16] of the energy of an excitation. He finds the approximate energy levels of the liquid with the help of the variational principle and thus chooses a trial wave function, Ψ. He writes Ψ as

$$\Psi = F \, \Phi \qquad (1,9)$$

where Φ is the ground state wave function for the liquid and F has still to be specified. The function Ψ depends on the 3 N coordinates of all the atoms or is a function of the configuration ($\underline{r}_1$, $\underline{r}_2$, .. $\underline{r}_n$, ..) of the atoms. The properties of Φ we know qualitatively. It is a symmetric function (Bose statistics) with respect to the coordinates of all the N atoms. Its amplitude is zero if somewhere in the liquid two atoms overlap and its amplitude is high for any configuration which on a microscopic scale has a nearly uniform density.

The reader is referred to the original article [16] to appreciate why

$$\Psi = \sum_i f(\underline{r}_i) \, \Phi \qquad (1,10)$$

where $f(\underline{r})$ is a function of the position of an atom and the sum extends over all the N atoms, is a reasonable trial function. The trial function (1,10) originally was meant to describe those states (rotons) which are orthogonal to the phonon states (large scale density fluctuations) and to the ground state Φ. It turns out, however, that the ultimate form of Ψ (see below (1,12)) is an exact eigenfunction for the phonon states and, in general, gives an upper limit for the energy for all wave vectors $\underline{k}$.

The application of the variational principle leads to the equation

$$E \int p(\underline{r}_1-\underline{r}_2)\, f(\underline{r}_2)\, d^3\underline{r}_2 = -(\hbar^2/2m)\,\nabla^2 f(\underline{r}_1) \qquad (1,11)$$

Here $p(\underline{r}_1-\underline{r}_2)$ is the two-atom probability function which can be calculated from neutron scattering data [4)]

The solution of (1,11) is:

$$f(\underline{r}) = \exp(i\,\underline{k}\,\underline{r})$$
$$\Psi = \sum_i \exp(i\,\underline{k}\,\underline{r}_i)\,\Phi \qquad (1,12)$$

and the energy $E(\underline{p})$ of an excitation is

$$E(\underline{p}) = \hbar^2 k^2/2m\,S(k) = p^2/2m\,S(k) \qquad (1,13)$$

where $S(k)$, the structure factor, is the Fourier transform of $p(\underline{r})$:

$$S(k) = \int \exp(i\,\underline{k}\,\underline{r})\, p(\underline{r})\, d^3\underline{r} \qquad (1,14)$$

$S(k)$ has a distinct maximum for $k \simeq 2\text{Å}^{-1}$, thus a minimum in the energy spectrum is found. However, since Ψ (1,10) is a bad trial function for the states of higher energy, this minimum lies much higher than the experimentally found Δ.

The wave function (1,12) represents a collective motion, though it may look like the wave function with wave vector $\underline{k}$ for the motion of one particle only. This is the case only if $k \gg 2\pi/a$ where a is the nearest neighbour distance. But for smaller wave numbers, because of the correlation in position implied by the ground state wave function Φ, the motion of one atom implies the motion of others [16)].

By a combination of plane waves we can make wave packets or localized excitations, with a slight expenditure of extra energy depending on the dimensions of the packet. The approximate wave function for a localized excitation is

$$\Psi = \sum_i g(\underline{r}_i)\,\exp(i\,\underline{k}\,\underline{r}_i)\,\Phi \qquad (1,15)$$

where $g(\underline{r})$ is some kind of Gaussian which peaks at a place $\underline{r}_0$ in the liquid.

Furthermore it can be shown that if $F = \sum_i \exp(i\,\underline{k}\,\underline{r}_i)\,\Phi$ gives one excitation with energy $E(k)$, then

$$F = \sum_i \exp(i\,\underline{k}_1\underline{r}_i) \sum_j \exp(i\,\underline{k}_2\underline{r}_j)\,\Phi \qquad (1,16)$$

represents a state with energy $E(k_1) + E(k_2)$ + terms of the order $1/V$, where V is the volume. F does not change if we interchange $\underline{k}_1$ and $\underline{k}_2$, so the "particles" behave like Bose particles which interact only weakly and may be created or destroyed.

On a suggestion of Cohen and Feynman [17)] the excitation spectrum of helium II was measured by Yarnell et al. 18). They measured the change in energy and momentum of a neutron when scattered inelastically by liquid helium. The predominant process is the creation of a single roton. The results are given in figure 1.

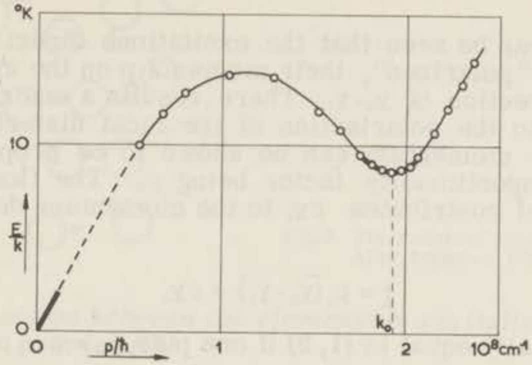

Fig. 1. The energy versus momentum spectrum of liquid helium, at a temperature of 1.1°K. From the experiments of Yarnell et al. 18).

The shape of the excitation curve does not alter much with temperature except that the sound velocity which determines the slope of the phonon branch and the roton minimum energy Δ (decreasing with increasing temperature) vary a bit. Cowan and Bendt 19) have used the experimental data of Yarnell et al. in computing thermodynamical functions for helium II. The agreement with the experimental values for the specific heat, the density of the normal fluid and the second sound velocity is, in general, excellent. Below 0.7°K there is a deviation of the "experimental values" for c_2 from the calculated ones but Gorter 20) has pointed out that at the lowest temperatures mean free path effects in the rather rarified phonon gas must become appreciable so that the concept of a second sound wave is no longer well defined.

Landau's hypothesis for the energy versus momentum relation for the excitations has thus been fully confirmed.

The picture of the "particles" comprising the thermal energy of the liquid offers a simple explanation of the two fluid model. Landau took the decisive step of assigning the inertial mass of the phonon and roton field to the "normal fluid". One must remember, however, that an excitation as such is a disturbance in the fluid and has no mass of its own. The normal density, ρ_n, is not the average of some quantity which can be assigned to the individual atoms. It appears to have meaning only for the whole group of excitations in, or nearly in, thermal equilibrium 10,16). If one supposes that the "gas" of excitations can drift relative to the background fluid, then it is not difficult to calculate the normal fluid density.

Landau showed that the energy of an excitation when the fluid moves as a whole with velocity $\underline{v}_s$ is $E = E(\underline{p}) + \underline{p v_s}$ (1, 8) (see also 16)) where $E(\underline{p})$ is the energy of the excitation in the fluid at rest. The equilibrium distribution function for the excitations when the gas of excitations moves as a whole with velocity $\underline{v}_n$ is:

$$n = [\exp(E-\underline{p}\underline{v}_n)/kT -1]^{-1} = [\exp (E(\underline{p}) + \underline{p}(\underline{v}_s-\underline{v}_n))/kT -1]^{-1} \qquad (1,17)$$

From this it can be seen that the excitations direct themselves. They become "polarized", their momenta $\underline{p}$ on the average pointing in the direction of $\underline{v}_n-\underline{v}_s$. There results a macroscopic momentum due to the polarization of the local disturbances in the fluid and this momentum can be shown to be proportional with $\underline{v}_n-\underline{v}_s$, the proportionality factor being ρ_n. The flow of the undisturbed fluid contributes $\rho\underline{v}_s$ to the momentum density, thus in total

$$\underline{j} = \rho_n(\underline{v}_n-\underline{v}_s) + \rho\underline{v}_s \qquad (1,18)$$

which is formally equal to (1, 2) if one puts $\rho_s = \rho - \rho_n$.

Feynman and Cohen [13] have given a detailed picture of a roton by choosing a better trial function. It can be shown that (1, 12) represents a state with a total momentum $\underline{p}$ or that the vector sum of all the velocities of the particles, due to an excitation, is equal to $\underline{p}/m$, where m is the mass of a ^{4}He atom. Such a unidirectional velocity field does not conserve mass. Therefore another trial function is used which ensures the necessary backflow. The energy of an excitation (1, 11) contains a negative correction term then and the fit with the experimental Δ becomes much better: $\Delta_{theor} = 11.5^{o}K$ whereas $\Delta_{exp} = 8.58^{o}K$.

We can say that a localized roton induces a velocity field in the whole liquid. Feynman and Cohen describe the roton classically as the smallest possible vortex ring with diameter of the order of the atomic spacing. The atoms can slip through only one after the other. They return outside the ring with a low velocity. Figure 2 shows this schematically.

Quantum mechanical effects are the cause of important differences with this classical picture.

a) The wave vector can not be smaller than $2\pi/a$ since if the pattern is shifted by one atomic distance, a, the wave function, being symmetric, must have returned to its original value. b) To localize a disturbance as described classically a small extra energy is required. The lowest value for the energy of a roton results if it is in quantum mechanical sense "everywhere" in the liquid. c) The vortex ring does not drift. This is because it has its smallest possible size. There is no force which tries to contract it. Classically this force is balanced by the forward motion of the ring [13].

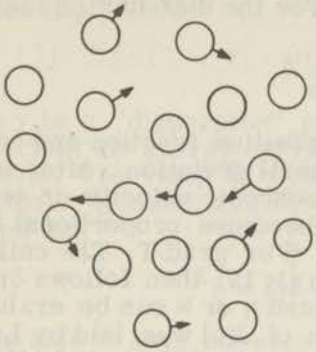

Fig. 2. The motion of the atoms in a roton. After Feynman [11]).

3. *The interaction between the elementary excitations. Phonon mean free path length*

The transport coefficients of helium II are determined by the net transport of momentum or energy by rotons and phonons in the case of a macroscopic gradient. The viscosity coefficient, η, has been analyzed in detail by Landau and Khalatnikov [21]), the coefficient of thermal conductivity, $\varkappa$, by Khalatnikov [22]). Both coefficients rise steeply with decreasing temperature. This is because the number of rotons, as given by (1, 7), is diminishing rapidly when the temperature is lowered and consequently the phonon mean free path length is rapidly becoming longer.

The viscosity coefficient has been measured in several ways. There is fair agreement with the theoretical values when the experimental value was obtained either with the rotating viscometer or by the surface damping of second sound. The damping of an oscillating disk, however, gives experimental values which become too high at the lower temperatures. But in this case the product $\rho_n\eta$ is measured and since ρ_n is very small spurious damping affects the value of η greatly.

The coefficient of thermal conductivity, $\varkappa$, is mainly responsible for the dissipation of energy in a second sound wave. It has been measured indirectly by subtracting from the observed attenuation the contributions of other types of attenuation. We note that in the case of a temperature gradient in helium II heat flows reversibly - for not too high velocities of the two components - by convection, the excitations having a mean group velocity, $\underline{v}_n$, different from zero. This type of conduction is not given by $\varkappa$. The coefficient $\varkappa$ relates a temperature gradient in the gas of excitations to a diffusion of thermal energy due to collisions between the excitations. Thus $\varkappa$ denotes a "normal" thermal conductivity.

As in gas kinetic theory, Landau and Khalatnikov proceed from a Boltzmann equation, one for each type of excitations: phonons and rotons.

$$\partial n/\partial t + \dot{\underline{r}}\ \partial n/\partial \underline{r} + \dot{\underline{p}}\ \partial n/\partial \underline{p} = J(n) \quad n = n(\underline{r}, \underline{p}, t) \qquad (1, 19)$$

where J(n) is the collision integral. For the distribution function, n, they write in the usual way

$$n = n_o + \delta n \qquad (1,20)$$

where n_o is the local equilibrium distribution function and δn, for a small macroscopic gradient, is a small deviation. After insertion of n_o (dependent on the local macroscopic velocity or temperature) the left-hand side of (1, 19) becomes proportional to the gradient du/dz (in the calculation of η) or grad T. The collision integral is proportional to δn. From (1, 19) then follows δn as a function of the macroscopic gradient and η or $\varkappa$ can be evaluated.

The foundation for the calculation of J(n) was laid by Landau and Khalatnikov in their first paper, on the differential effective cross sections for collisions between the elementary excitations. The cross sections follow from the quantum mechanical transition probability for the transition from a state with two excitations, say 1 and 2, to a final state with excitations 3 and 4 - the laws of conservation of energy and momentum being fulfilled. The time proportional transition probabilities are in turn proportional to $|H'_{AF}|^2$ where H' is the perturbing or interaction energy and H'_{AF} denotes the matrix element between the initial, A, and final, F, state. The interaction energy between two elementary excitations is known only roughly. We review the assumptions of Landau and Khalatnikov for these interaction energies.

a) *Phonon-phonon interaction. The 4-phonon process*

The perturbing energy, in principle, is known. The Hamiltonian of the liquid, H_{op}, for irrotational flow, is developed into powers of the phonon creation ($\rho_{\underline{p}}^*$) and annihilation ($\rho_{\underline{p}}$) operators:

$$H_{op} = H_o + H' = H_o + V_3 + V_4 \qquad (1,21)$$

The phonons are solutions of H_o which equals

$$H_o = (c^2/\rho_o) \sum_{\underline{p}} (\rho_{\underline{p}} \rho_{\underline{p}}^* + \rho_{\underline{p}}^* \rho_{\underline{p}}) = \sum_{\underline{p}} (n_{\underline{p}} + 1/2)\, cp \qquad (1,22)$$

where $n_{\underline{p}}$ is the number of phonons with momentum $\underline{p}$. The interaction energy, H', is of the third or higher order in the phonon operators.

Difficulties arise with the 4-phonon process when the phonons collide at a small angle. The mean contribution to H'_{AF} comes from V_3 in the second approximation of the perturbation theory:

$$H'_{AF} = \sum_{i=I}^{VI} \frac{(V_3)_{Ai}(V_3)_{iF}}{E_A - E_i} \qquad (1,23)$$

where i means an intermediary state. When the angle between the colliding phonons goes to zero, for some intermediary states the denominator E_A-E_i becomes zero as well *if* the energy of a phonon depends linearly on its momentum: E=cp.

Landau and Khalatnikov, to avoid the divergence of the matrix element H'_{AF}, write

$$E = c(p - \gamma p^3) \qquad (1,24)$$

where γ is a "dispersion" parameter. With (1,24) they express the fact that the energy curve (fig. 1) bends over towards the roton minimum. The value of γ they infer from experimental data for the roton Δ, p_0 and μ and for the velocity of sound. In this way it is found that *

$$\gamma \simeq 2.8\ 10^{37} g^{-2} cm^{-2} s^2 \qquad (1,25)$$

It is safe to say, however, despite the uncertainty concerning the exact value of γ, that the differential effective cross section for phonon-phonon collisions has a sharp maximum when the colliding phonons move nearly parallel. This in turn leads to a rapid exchange of energy between phonons moving in the same direction.

The time t_{ph} which characterizes the attainment of energy equilibrium in the phonon gas is determined mainly by the above small angle scattering and is equal to [21)]

$$1/t_{ph} \simeq 10^5\ T^7\ x\ (x + 6)^3 \quad x = pc/kT \qquad (1,26)$$

where x is the energy, in units of kT, of those phonons which originally were not present in equilibrium number.

The key assumption of Landau and Khalatnikov in calculating the *phonon* part of the kinetic coefficients, which they afterwards justify, is that the time t_{ph} is much smaller than the time which characterizes the viscosity or the thermal conductivity (see below).

In connection with the 4 phonon process we note that Landau and Khalatnikov [21)] say that the 3 phonon process is impossible since the laws of conservation of energy and momentum have to be fulfilled. In this process two parallel phonons combine to form one moving in the same direction (c.f. H. A. Kramers [24)]). Indeed, if the dispersion parameter γ is introduced, the 3 phonon process is impossible.

b) *Roton-roton interaction. The 4-roton process*

Of the interaction energy between two rotons nothing was known at the time Landau and Khalatnikov wrote their article. They assumed a δ-function interaction:

$$V_{int} = V_0\ \delta(\mathbf{r}_1 - \mathbf{r}_2) \qquad (1,27)$$

---

* This value of γ is of no consequence for the phonon specific heat for instance. We calculated γ from the maximum in the energy curve [18)] and found $\gamma \simeq 7.1\ 10^{37}\ g^{-2} cm^{-2} s^2$.

Thus the rotons are treated as "hard spheres". As in a gas where the viscosity coefficient is independent of pressure, the roton part of the viscosity, due to roton-roton collisions, is independent of the roton number density, i.e. of temperature. From experimental data for η a value of

$$V_o \simeq 1.1\ 10^{-38}\ \text{erg cm}^3 \qquad (1,28)$$

is found for the roton-roton interaction energy.

Cohen and Feynman [17)] have calculated the roton interaction resulting from the following process: a roton with momentum $\underline{P}_1$ emits a virtual phonon $\underline{p}$ which in turn is absorbed by a roton $\underline{P}_2$. They have found indeed a δ-function interaction and the value of V_o comes close to (1,28). However, they are led to a velocity dependent interaction as well. This part of the roton-roton interaction may be small because the mean group velocity of rotons is small. One may well imagine that it is quite probable that the interaction between rotons depends on the angle between their momenta as well.

c) *Phonon-roton interaction. Scattering of a phonon by a roton*

The roton is treated as a particle in the velocity, $\underline{v}$, and density, ρ, fields which extend over the whole liquid volume and which are a consequence of the presence of the phonon.

Firstly, the velocity field $\underline{v}$ results in an interaction energy $-\underline{P}.\underline{v}$ where $\underline{P}$ is the momentum of the roton.

Secondly, the roton parameters depend on density, and the density fluctuations in the sound field cause a contribution to the interaction energy of

$$\frac{\partial\Delta}{\partial\rho}\rho' + \frac{1}{2}\left[\frac{\partial^2\Delta}{\partial\rho^2} + \frac{1}{\mu}\left(\frac{\partial p}{\partial\rho}\right)^2\right]\rho'^2 \qquad (1,29)$$

The assumption has been made that $P \simeq p_o$. Khalatnikov shows that the term with $\partial\Delta/\partial\rho$ can be neglected if $(1/\mu c^2)(\partial\Delta/\partial\rho)^2 \ll \partial^2\Delta/\partial\rho^2$.

Thus for the scattering of a phonon by a roton the values of $\partial\Delta/\partial\rho$, $\partial p_o/\partial\rho$ and $\partial\mu/\partial\rho$ and of the second derivative $\partial^2\Delta/\partial\rho^2$ have to be calculated from experimental data. As we will see in Chapter II, these derivatives enter into the final expression for the attenuation of sound in liquid helium as well.

From the dependence on pressure of the velocity of ordinary sound (Atkins and Stasior [25)]) the phonon part of the thermal expansion coefficient, α_{ph}, can be calculated. Subtracting α_{ph} from the experimental value of α (Atkins and Edwards [26)]) one gets α_r, the roton part of the expansion coefficient which is related to the roton part of the entropy by $\alpha_r = -\rho\ \partial\sigma_r/\partial p$.

Atkins and Edwards, after some calculation, get

$$(\rho/\Delta)\ \partial\Delta/\partial\rho = -0.57$$
$$(\rho/2\mu)\ \partial\mu/\partial\rho + (2\rho/p_o)\ \partial p_o/\partial\rho = -0.38 \qquad (1,30)$$

The dependence on pressure of the velocity of second sound (Peshkov and Zinovieva 27)), together with α, gives 28)

$$(\rho/p_o)\,\partial p_o/\partial\rho = +0.26 \longrightarrow (\rho/\mu)\,\partial\mu/\partial\rho = -1.8 \qquad (1,31)$$

These values deviate considerably from those given by Khalatnikov in his review article 22); he gives $(\rho/p_o)\,\partial p_o/\partial\rho \simeq 1/3$ and $(\rho/\mu)\,\partial\mu/\partial\rho \simeq 0$. The influence of the values of these derivatives on the scattering cross section for the scattering of a phonon by a roton can not be neglected. They appear as well in the final expressions for the attenuation of ordinary and second sound. We think that the values of Atkins 28) are to be preferred, since Khalatnikov made use of less accurate data for $\partial u_1/\partial p$ and α.

Finally, the derivative $\partial^2\Delta/\partial\rho^2$ can be calculated approximately from the temperature dependence of the velocity of ordinary sound.

Khalatnikov finds 22)

$$(\rho^2/\Delta)\,\partial^2\Delta/\partial\rho^2 \simeq -5 \qquad (1,32)$$

The total interaction energy between a phonon and a roton becomes 21):

$$V_{int} = -\tfrac{1}{2}(\underline{P}\underline{v} + \underline{v}\underline{P}) + \frac{\partial\Delta}{\partial\rho}\rho' + \tfrac{1}{2}\left[\frac{\partial^2\Delta}{\partial\rho^2} + \frac{1}{\mu}\left(\frac{\partial p}{\partial\rho}\right)^2\right]\rho'^2 \qquad (1,33)$$

With this interaction energy, the differential effective cross section for the scattering of a phonon by a roton through an angle ψ is obtained. It is proved that the scattering of a phonon by a roton is analogous to the scattering of a light by a heavy particle. The reason is that the momentum of a roton is much larger than the momentum of a phonon.

We do not relate here the results of Landau and Khalatnikov for the kinetic coefficients; roughly, as has been remarked already, they rise steeply with decreasing temperature.

We must pay attention, however, to the mean free path lengths, because if the mean free path length of an excitation becomes comparable with the wavelength of sound the macroscopic hydrodynamic equations, which govern the propagation of sound, are no longer valid. Generally, the mean free path length of the excitations becomes longer with decreasing temperature.

For rotons the situation is clear. They are "heavy" particles and are scattered by rotons only. If we take their interaction energy to be (1,27) with V_o given by (1,28), the roton mean free path length is

$$l_{rot} \simeq 1.65\ 10^{-9}\exp(\Delta/kT)\ \mathrm{cm} \qquad (1,34)$$

At 1°K, with $\Delta/k = 9^{\circ}$K, $l_{rot} \simeq 10^{-5}$ cm. The wavelength of 10 MHz sound is $\lambda_{sound} \simeq 2\ 10^{-3}$ cm. At 1°K the number of rotons is about half the number of phonons (from 1,6 and 1,7). Below

$1^{\circ}K$ the number of rotons is rapidly decreasing. So we need not be concerned about the roton mean free path length becoming too large.

For phonons, however, the situation is complicated. It is even difficult, if not impossible, to formulate the problem.

Landau and Khalatnikov define a mean free path length by calculating a relaxation time in a situation when in some respect the gas of phonons is out of quasi-equilibrium in the sense defined below. This time, τ_{rel}, is multiplied - supposing that the collision time is of the same order of magnitude - with the group velocity of the phonons, i.e. with the velocity of sound. Thus, in general:

$$l_{ph} = \tau_{rel} \cdot c \qquad (1,35)$$

One relaxation time we know already (1, 26). It characterizes the attainment of *energy equilibrium* in the gas of phonons and is governed by small-angle phonon-phonon scattering. So, when the phonons run nearly parallel

$$l_{ph} = t_{ph} \cdot c \text{ (small angle phonon-phonon scattering)}$$

$$\simeq 2\ 10^{-1}/T^7\, x\, (x+6)^3 \text{ cm} \quad x = pc/kT \qquad (1,36)$$

Landau and Khalatnikov calculate a second relaxation time which characterizes the viscosity. At $t = 0$ there is supposed to be a deviation from the distribution $n = n_0 + \delta n$ (1, 20) that is to say δn does not have its stationary value pertaining to the velocity gradient present. The relaxation time determines how fast, by phonon-phonon and phonon-roton collisions, the stationary distribution is reached. This time, call it τ_η, is governed by large-angle phonon scattering. Small-angle scattering does not contribute to the transport of momentum. Thus a second mean free path length is defined by [21)]

$$l_\eta = \tau_\eta \cdot c \text{ (large angle phonon-roton and phonon-phonon scattering)} \qquad (1,37)$$

Still another relaxation time is obtained by studying how fast a deviation of (1, 20) relaxes in the case of a macroscopic temperature gradient. And the phonon mean free path length which characterizes the thermal conductivity becomes [22)]:

$$l_\varkappa = \tau_\varkappa \cdot c \text{ (phonon-roton scattering)} \qquad (1,38)$$

Khalatnikov shows that a pure phonon gas has no thermal conductivity, the coefficient $\varkappa_{ph}$ is zero. This consideration implies that $l_\varkappa$ is related to phonon-roton scattering only. The mean free path length characterizing the viscosity, l_η, above $0.9^{\circ}K$, is determined mainly by phonon-roton scattering as well, and the result is that l_η and $l_\varkappa$ do not differ much at temperatures above

0.9°K. Below 0.9°K, however, since $l_{\varkappa}$ is still determined by phonon-roton scattering only, while l_{η} is decreased by phonon-phonon scattering, it follows that $l_{\eta} \ll l_{\varkappa}$.

When we compare the different mean free path lengths, we have to keep in mind that l_{ph} (1, 34) depends strongly on the energy of the phonons. For an average energy, however, of $E_{ph}/N_{ph} \simeq 3$ kT, where E_{ph} is the total phonon energy per cm^3 and N_{ph} their total number, x in equation (1, 26) equals 3 at all temperatures, and l_{ph} depends on temperature as T^{-7}. At a temperature of 0.2°K (orders of magnitude): $l_{ph} \simeq 10$, $l_{\eta} \simeq 2000$, $l_{\varkappa} \simeq 10^{15}$ cm. And at 1.0°K: $l_{ph} \simeq 10^{-4}$, $l_{\eta} \simeq l_{\varkappa} \simeq 10^{-3}$ cm.

We make two comments on these theoretical values. Firstly, the phonon mean free path length is decreased by ^{3}He impurities (c.f. Kramers [29]). Secondly, the concept of these mean free path lengths has to be modified when we consider phonon or roton creation and annihilation processes.

Phonons and rotons may be created or annihilated. Below 0.9°K the average time between *inelastic* collisions is smaller than the average time between *elastic* collisions [22,23]. One only in so many collisions is an elastic one. We may as well as for elastic collisions define a phonon mean free path length for inelastic collisions:

$$l_i = \tau_i \cdot c \text{ (inelastic collisions)} \qquad (1, 39)$$

At what temperature is "the" phonon mean free path length comparable with the wavelength of sound? At a frequency of 10 MHz $\lambda_{sound} \simeq 2.4 \times 10^{-3}$ cm. Above 0.9°K $l_i \gg l_{\eta} \simeq l_{\varkappa}$ [22,23]. It does not make sense to take l_{ph} as determining free path length because l_{ph} is restricted to the phonons moving parallel. We have found that at 1.0°K $l_{\eta} \simeq 10^{-3}$ cm. The conclusion is that it is not safe to proceed from the macroscopic hydrodynamic equations - which we will give in the next section - in describing the propagation of sound for temperatures lower than about 1.0°K.

4. *The equations for macroscopic motion*

The macroscopic thermohydrodynamic equations for the two fluids have been derived, in the following form, by Landau [10]. They are, if we neglect dissipative effects:

$$\partial\rho/\partial t + \mathrm{div}\,\underline{j} = 0$$

$$\partial S/\partial t + \mathrm{div}\, S\underline{v}_n = 0$$

$$\partial \underline{j}_i/\partial t + \sum_k \partial\Pi_{ik}/\partial x_k = 0,\ \Pi_{ik} = p\delta_{ik} + \rho_n v_k^{(n)} v_i^{(n)} + \rho_s v_k^{(s)} v_i^{(s)}$$

$$\partial \underline{v}_s/\partial t = -\,\mathrm{grad}\,[\Phi + v_s^2/2 - (\rho_n/2\rho)(\underline{v}_n - \underline{v}_s)^2] \qquad (1, 40)$$

S is the entropy per cm^3, Φ is the thermodynamic potential per gram. The total differential of Φ is $d\Phi = (1/\rho)\, dp - \sigma\, dT$ where σ is the entropy per gram.

Neglecting quadratic terms in (1, 40) the equations for the propagation of sound of small amplitude become:

$$\begin{aligned} \partial\rho/\partial t + \rho_n \operatorname{div} \underline{v}_n + \rho_s \operatorname{div} \underline{v}_s &= 0 \\ \partial \underline{j}/\partial t + \nabla p &= 0 \\ \partial S/\partial t + S \operatorname{div} \underline{v}_n &= 0 \\ \partial \underline{v}_s/\partial t + \nabla\Phi &= 0 \end{aligned} \qquad (1,41)$$

After elimination of $\underline{v}_n$ and $\underline{v}_s$:

$$\begin{aligned} \partial^2\rho/\partial t^2 &= \Delta p \\ \partial^2 S/\partial t^2 &= (S/\rho_n)(\Delta p - \rho_s \Delta\Phi) \end{aligned} \qquad (1,42)$$

Khalatnikov chooses as independent variables ρ and S. The deviations from the average values, ρ' and S', are taken proportional to $\exp[i(\omega t - kx)]$. The velocity of sound $u = \omega/k$.

The velocities of ordinary (u_I) and of second (u_{II}) sound follow from the determinental equation:

$$\begin{vmatrix} u^2 - (\partial p/\partial\rho)_S & (\partial p/\partial S)_\rho \\ u^2 - \rho_s(\partial\Phi/\partial\rho)_S & (\rho_n/S)u^2 + \rho_s(\partial\Phi/\partial S)_\rho \end{vmatrix} = 0 \qquad (1,43)$$

The term with $(\partial\rho/\partial S)_\rho$ is small since the coefficient of expansion, α_p, of helium II is very small and $(\partial p/\partial S)_\rho = T\alpha_p/c_v\chi_T$. Thus the solutions are:

$$\begin{aligned} u_I^2 &= (\partial p/\partial\rho)_S + (\partial p/\partial S)_\rho\, S/\rho = (\partial p/\partial\rho)_\sigma \\ u_{II}^2 &= (\rho_s S/\rho_n)[(\partial p/\partial S)_\rho/\rho - (\partial\Phi/\partial S)_\rho] = (\rho_s/\rho_n)\sigma^2(\partial T/\partial\sigma)_\rho \end{aligned} \qquad (1,44)$$

The coupling between the two kinds of sound is negligible [1,30].

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Chapter II

ORDINARY SOUND IN PURE ^{4}HE

The velocity and the attenuation of ordinary sound in helium II have been measured by many authors. The first section of this chapter is devoted to a review of the experimental results. In the next section we outline the theory of Khalatnikov for the attenuation of both types of sound in helium II, which is valid, as will be shown, between 1.0 and 2.0°K. Finally we make some remarks on his theory.

1. *Experimental results*

The velocity, c, of ordinary sound in liquid helium has been measured by means of the interferometric method by Findlay et al. [1]. Later, van Itterbeek and Forrez [2], with the same method, obtained more accurate data. Their results are reproduced in figure 1.

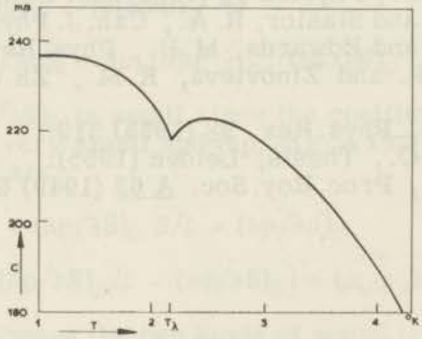

Fig. 1. The velocity of sound in liquid ^{4}He, according to van Itterbeek and Forrez [2].

The dispersion of sound, which theoretically is small, has not yet been detected.

The attenuation has been measured, with the help of the pulse method, in the frequency range from 1 MHz to 15 MHz [3-10]. Figure 2 shows the results for a frequency of 12 MHz. The part of the curve below the lambda-point is drawn from the experimental data of Chase [5] and Whitney [8].

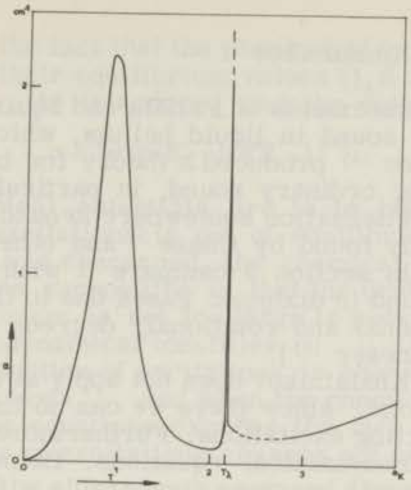

Fig. 2. The attenuation of 12 MHz sound, from the measurements of Pellam and Squire [3]), Chase [5,10]) and Whitney [8]).

Above Tλ only a few data are available [3]), and they are for a frequency of 15 MHz. In drawing fig. 2 the assumption has been made for temperatures above Tλ that the attenuation is proportional to the frequency squared.

Recently, Chase [10]) studied the propagation of sound close to the λ-point with a high degree of accuracy. The frequency used was 1 MHz.

A large and narrow maximum in the attenuation occurs at about 8×10^{-5} °K below Tλ. The velocity of sound goes through a minimum of 218 m/s at a temperature of 2×10^{-4} °K below Tλ.

The data of Chase are in fairly good agreement with a theory proposed by Khalatnikov [11]). There is a relaxation time connected with the attainment of the equilibrium value of a certain parameter which characterizes the degree of asymmetry in the helium II phase (Cf. also Kronig, Thellung and Woldringh [12])).

This relaxation phenomenon can not explain the high attenuation of sound just above the λ-point, since there is no superfluid left. Indeed, in the region from Tλ to about 3°K the attenuation of sound is higher than can be explained classically by viscous and thermal conduction losses. Pippard [13]) has developed a theory for the excess attenuation in the temperature region above Tλ by assuming that statistical temperature fluctuations in the liquid entail the possibility of small inclusions of He II in the matrix of bulk liquid which is He I. Pippard calculates that a significant fraction of the liquid consists of He II. The mechanism which causes the attenuation is as follows: on compression the inclusions decrease in temperature while the matrix of He I increases in temperature since the expansion coefficient changes sign at Tλ. There is an irreversible flow of heat from the matrix to the inclusions and sound energy is dissipated.

2. *The theory of Khalatnikov*

After the measurements of Pellam and Squire 3) on the attenuation of ordinary sound in liquid helium, which extended down to 1.4°K, Khalatnikov 14) produced a theory for both types of sound in helium II. For ordinary sound, in particular, he predicted a maximum in the attenuation somewhere around 1.0°K. This maximum was actually found by Chase 5) and others. We will sketch this theory and in section 3 compare it with the theory for the attenuation of sound in diatomic gases due to the relaxation of the internal, vibrational and rotational, degrees of freedom of the molecules (cf. Kneser 15)).

The theory of Khalatnikov does not apply at temperatures close to the lambda-point, since there we can no longer speak of a gas of weakly interacting excitations. Furthermore, Khalatnikov proceeds from hydrodynamical equations. Below 1°K the phonon mean free path length becomes comparable with the wavelength of 10 MHz sound, and thus the applicability of the theory of Khalatnikov is limited to temperatures between 1°K and 2°K. For second sound the region of validity extends downwards to temperatures of about 0.7°K.

Khalatnikov concludes from the experimental values for the attenuation 3) which are higher than the classical Stokes-Kirchhoff values:

$$\alpha = \frac{\omega^2}{2\rho c^3}\left(\frac{4}{3}\eta + (\gamma-1)\frac{\varkappa}{c_p}\right) \simeq \frac{\omega^2}{2\rho c^3}\,\frac{4}{3}\eta \quad \text{in He II,}^{*} \qquad (2,1)$$

local equilibrium but which are slow relative to the period of the sound wave. From Chapter I, section 3, it follows that energy equilibrium is attained rapidly. By elastic collisions the excitations distribute themselves over the different energy levels in a time which is very short when compared with the period of the sound wave. In fact, the time t_{ph} from equation (1,26) which characterizes the attainment of energy equilibrium in the phonon gas, and the roton collision time (compare 1, 34) are extremely small relative to the time, τ_η, which characterizes the phonon part of the viscosity coefficient.

The *equilibrium numbers of phonons and rotons per cm³* depend on pressure (or, in the case of second sound, on temperature), and Khalatnikov assumes that the slow rate of production of excitations is responsible for the observed excess attenuation.

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[*] Here ω is the angular frequency, ρ the density and c the velocity of sound; η is the viscosity-coefficient and $\varkappa$ the coefficient of thermal conductivity which determines the irreversible flow of heat in the presence of a temperature gradient. With the experimental data of Zinovieva 16) for $\varkappa$ and η, of Atkins and Edwards 17) for $\gamma = c_p/c_v$ and of Kramers et al. for C_p 18) it turns out that the losses due to thermal conduction can be neglected at temperatures above 1°K.

To account for the fact that the phonon and roton number densities do not have their equilibrium values (1, 6 and 7) a chemical potential, μ_r or μ_{ph}, is introduced and the distribution function (1, 7) becomes

$$n = [\exp(E-\mu)/kT - 1]^{-1} \tag{2,2}$$

Here the chemical potentials are to be taken per particle. When the gas of excitations is out of equilibrium, as far as the number of particles is concerned, the chemical potentials have a non-zero value. The supposition is that the deviation from thermodynamic equilibrium is not too large to prevent the use of the well-known thermodynamical identities.

The rate of production of excitations is characterized by certain kinetic coefficients, γ, and when the chemical potentials are small the following equations express the fact that in a volume element the number of excitations changes not only by flow but by production as well:

$$\begin{aligned} \dot{N}_{ph} + \mathrm{div}(N_{ph}\underline{v}_n) &= \gamma_{phr}\,\mu_r - \gamma_{phph}\,\mu_{ph} \\ \dot{N}_r + \mathrm{div}(N_r\underline{v}_n) &= -\gamma_{rr}\,\mu_r + \gamma_{rph}\,\mu_{ph} \end{aligned} \tag{2,3}$$

These two equations, together with the hydrodynamic equations (1, 41) describe the propagation of sound in helium II when the slow production of excitations is taken into account. Viscous and thermal conduction losses are omitted for the moment. After elimination of $\underline{v}_n$ and $\underline{v}_s$ there results:

$$\begin{aligned} \ddot{\rho} &= \Delta p \\ \ddot{S} &= (S/\rho_n)(\Delta p - \rho_s\,\Delta\Phi) \\ \dot{N}_r - (N_{ro}/S)\,\partial S/\partial t &= -\gamma_{rr}\,\mu_r + \gamma_{rph}\mu_{ph} \\ \dot{N}_{ph} - (N_{pho}/S)\,\partial S/\partial t &= \gamma_{phr}\,\mu_r - \gamma_{phph}\mu_{ph} \end{aligned} \tag{2,4}$$

Apart from ρ' and S', the deviations from the average density and entropy, the chemical potentials now vary like $\exp[i(\omega t - kx)]$. The pressure and the thermodynamical potential per gram, Φ, depend on two more variables, μ_r and μ_{ph}. Formally, the velocity of ordinary sound is as in (1, 44)

$$u_I^2 = (\partial p/\partial\rho)_S + (\partial p/\partial S)_\rho S/\rho \tag{2,5}$$

But now

$$\begin{aligned} (\partial p/\partial\rho)_S &= (\partial p/\partial\rho)_{S,\mu_r,\mu_{ph}} + (\partial p/\partial\mu_r)(\partial\mu_r/\partial\rho)_S + (\partial p/\partial\mu_{ph})(\partial\mu_{ph}/\partial\rho)_S \\ (\partial p/\partial S)_\rho &= (\partial p/\partial S)_{\rho,\mu_r,\mu_{ph}} + (\partial p/\partial\mu_r)(\partial\mu\ /\partial S)_\rho + (\partial p/\partial\mu_{ph})(\partial\mu_{ph}/\partial S)_\rho \end{aligned} \tag{2,6}$$

Here $(\partial p/\partial\rho)_{S,\mu_r,\mu_{ph}}$ is about equal to $(\partial p/\partial\rho)_{S,\mu_r=\mu_{ph}=0}$ and $[(\partial p/\partial\rho)_S+(\partial p/\partial S)_\rho S/\rho]_{\mu_r=\mu_{ph}=0}$ is the value of u_I^2 for very low-frequency sound.

The derivatives of the chemical potentials, $\partial\mu_r/\partial\rho$, $\partial\mu_r/\partial S, \ldots$, follow from (2, 4) through which the values of μ_r and μ_{ph} are related to the excess density and entropy in the sound wave:

$$(i\omega\, \partial N_r/\partial\mu_r+\gamma_{rr})\mu_r - \gamma_{rph}\mu_{ph} = -i\omega[\rho'\,\partial N_r/\partial\rho+S'\,\partial N_r/\partial S-S'(N_r/S)]$$

$$-\gamma_{phr}\mu_r + (i\omega\,\partial N_{ph}/\partial\mu_{ph}+\gamma_{phph})\mu_{ph} = -i\omega[\rho'\,\partial N_{ph}/\partial\rho+S'\,\partial N_{ph}/\partial S - S'(N_{ph}/S)] \tag{2,7}$$

The equations (2, 7) determine $\partial\mu_r/\partial\rho$, $\partial\mu_r/\partial S, \ldots$ as a function of frequency.

Finally, the derivatives $\partial p/\partial\mu_r$ and $\partial p/\partial\mu_{ph}$ are obtained as follows: the thermodynamical identity

$$dE = TdS + \Phi d\rho + \mu_{ph}\, dN_{ph} + \mu_r\, dN_r \tag{2,8}$$

expresses that the energy per cm^3, E, when the gas of excitations is slightly out of thermodynamic equilibrium, is a function of the chemical potentials per particle, μ_r and μ_{ph}. In equilibrium $\mu_r = \mu_{ph} = 0$. The non-equilibrium pressure is obtained by differentiating the total energy with respect to the volume, V, keeping the total entropy: SV, mass: ρV and number of particles: NV constant. Thus:

$$\begin{aligned} p &= -\frac{\partial(EV)}{\partial V} = -E-V\left(T\frac{\partial S}{\partial V} + \Phi\frac{\partial p}{\partial V} + \mu_{ph}\frac{\partial N_{ph}}{\partial V} + \mu_r\frac{\partial N_r}{\partial V}\right) = \\ &= -E+TS+\Phi\rho+N_{ph}\mu_{ph} + N_r\mu_r = \\ &= -(E-N_{ph}\mu_{ph}-N_r\mu_r) + TS+\Phi\rho \end{aligned} \tag{2,9}$$

The differential of the function $E-N_{ph}\mu_{ph} - N_r\mu_r = \varepsilon$ is from (2, 8)

$$d\varepsilon = TdS + \Phi d\rho - N_{ph}\, d\mu_{ph} - N_r d\mu_r \tag{2,10}$$

Differentiating $p = -\varepsilon + TS + \Phi\rho$ with respect to μ_{ph} or μ_r we get from (2, 10):

$$\begin{aligned} (\partial p/\partial\mu_{ph})_{\rho,S} &= -\,\partial\varepsilon/\partial\mu_{ph} + S\partial T/\partial\mu_{ph} + \rho\partial\Phi/\partial\mu_{ph} = \\ &= N_{ph} - (\partial N_{ph}/\partial S)_{\rho,\mu} - \rho(\partial N_{ph}/\partial\rho)_{S,\mu} \\ (\partial p/\partial\mu_r)_{\rho,S} &= N_r - (\partial N_r/\partial S)_{\rho,\mu} - \rho(\partial N_r/\partial\rho)_{S,\mu} \end{aligned} \tag{2,11}$$

The derivatives, again, are taken for $\mu_r = \mu_{ph} = 0$. To calculate them, they are expressed as functions of ρ and T.

We do not relate Khalatnikov's calculations in more detail. Before giving his result for the attenuation, we explain which processes are mainly responsible for the production of excitations.

a) *The 5 phonon process*

In the 5 phonon process the inelastic collision of 2 phonons results in 3 phonons. The interaction energy is given by the terms V_3, V_4, . . in the Hamiltonian for potential flow (1, 21). It proves too difficult to calculate the differential effective cross section for this process. The probability for the creation of a phonon is again largest when the colliding phonons move nearly parallel. However, it is possible to calculate the temperature dependence of the kinetic coefficient, Γ_{ph}, in the equation:

$$\dot{N}_{ph} = -\Gamma_{ph}\mu_{ph} \tag{2, 12}$$

which determines the change in the total number of phonons per cm^3 as a result of the 5 phonon process when the chemical potential of the phonons, μ_{ph}, is not zero. It turns out that:

$$\Gamma_{ph} = a\ T^{11} \tag{2, 13}$$

where a is a temperature independent coefficient which can be found from experimental data on the attenuation of ordinary sound.

Khalatnikov neglects the production processes between rotons. These are highly improbable indeed. Because of the law of conservation of energy, for the 3 roton process the condition $(p_1-p_o)^2/2\mu \geq \Delta$ and for the 5 roton process the condition $(p_1-p_o)^2/2\mu + (p_2-p_o)^2/2\mu \geq \Delta$ must be fulfilled. The number of rotons which have such a high energy is very small.

b) *Transformation of a phonon into a roton or vice versa*

When two rotons collide one of them can be transformed into a phonon. The momentum of a phonon is always smaller than the momentum of a roton (Chapter I, figure 1). The law of conservation of momentum implies that the above process can take place only if the rotons collide at large angle. Moreover, the phonon must have a high energy. Of the interaction between a roton and a phonon of high energy nothing is known. Khalatnikov assumes that a high-energy phonon differs from a roton only by its smaller momentum and that the interaction energy, as in (1, 27), is given by

$$V = V_o\delta(\mathbf{r}_1-\mathbf{r}_2) \tag{2, 14}$$

The coefficient Γ_{PhR} relates the change in N_{ph} and N_r, due to the above process, to the chemical potentials:

$$\dot{N}_{ph} = -\Gamma_{PhR}\,(\mu_{ph}-\mu_r) \qquad (2,15)$$

$$\dot{N}_r = -\Gamma_{PhR}\,(\mu_r-\mu_{ph})$$

The temperature dependence of Γ_{PhR} is found to be:

$$\Gamma_{PhR} = b\,\exp(-2\Delta/kT) \qquad (2,16)$$

Again, b is temperature independent and has to be calculated from experimental data.

Certainly, it is doubtful whether the interaction is given by (2,14) but even for another interaction the coefficient Γ_{PhR} will probably turn out to be proportional to N_r^2, and the temperature dependence of Γ_{PhR} will be dominated by the temperature dependence of the roton number density; i.e. Γ_{PhR} will vary like $\exp(-2\Delta/kT)$. The coefficients Γ are related to the coefficients γ (2,3) in a simple way.

At a temperature of 1.8°K the production processes are much faster than the density changes in the sound wave. But when the temperature is lowered the actual number of excitations more and more falls out of phase with the number for $\mu_r=\mu_{ph}=0$. A high relaxation loss results. (See figure 2.)

Two relaxation times, Θ_{ph} for the 5 phonon process and Θ_{PhR} for the transformation of a roton into a phonon, are defined by Khalatnikov. Above 0.9°K they are numerically of the same order of magnitude as the time which characterizes the phonon viscosity or the phonon thermal conductivity (1,37 and 1,38).

For low frequencies, defined by the conditions:

$$\omega\Theta_{ph} \ll 1, \quad \omega\Theta_{phr} \ll 1 \qquad (2,17)$$

the excess attenuation for ordinary sound is shown to be

$$\alpha_{I,\mathrm{rel.}} = \frac{\omega^2}{2\rho c^3}\left[\frac{1}{\Gamma_{ph}}\left(\frac{\partial p}{\partial \mu_r}+\frac{\partial p}{\partial \mu_{ph}}\right)^2_{\rho,S} + \frac{1}{\Gamma_{PhR}}\left(\frac{\partial p}{\partial \mu_{ph}}\right)^2_{\rho,S}\right] \qquad (2,18)$$

The numerical values of the derivatives $\partial p/\partial\mu_r$ and $\partial p/\partial\mu_{ph}$ can be calculated when one knows the dependence on pressure of the parameters of the energy curve; i.e. when one knows $\frac{1}{2}\frac{\rho}{\mu}\frac{\partial\mu}{\partial\rho}+\frac{2\rho}{p_0}\frac{\partial p_0}{\partial\rho}$ and $\frac{\rho}{\Delta}\frac{\partial\Delta}{\partial\rho}$ (see Chapter I, section 3).

And thus from the experimental values of α_I at two temperatures the coefficients a and b in the kinetic coefficients Γ can be determined (Arkhipov [29]). One finds, using the attenuation data of Chase [5], the following values:

$$a = 1.2\ 10^{43}\ \mathrm{s\ cm^{-5}\ deg^{-11}}, \quad b = 1.3\ 10^{50}\ \mathrm{cm^3\ s\ deg} \qquad (2,19)$$

We checked that the temperature variation of the roton parameters (Yarnell et al., ref. I, 18) does not influence the values of a and b appreciably.

Though there exists considerable uncertainty concerning the values of the derivatives of the roton paremeters on the pressure, there still is fairly good agreement between the theoretical values for the attenuation given by (2, 18) and the experimental values of Chase [5]. We note that the assumption is made that the total attenuation is simply the sum of viscous losses ($\alpha_\eta = (\omega^2/2\rho c^3)4\eta/3$) and relaxation losses. This is probably justified, since the viscous losses are relatively small.

Knowing the coefficients Γ, the attenuation of *second* sound due to the relaxation of the number densities can be calculated. It turns out that the loss due to the relaxing number density of the excitations is only a fraction of the total attenuation and that the loss $\alpha_{II\varkappa}$ due to thermal conductivity, is by far the most important one. Therefore, the value of $\varkappa$ is obtained rather accurately from measurements of the second sound attenuation.

In a formal way one can introduce into the hydrodynamic equations (1, 40) four coefficients of second viscosity, apart from the coefficient of shear viscosity and the coefficient of thermal conductivity [14, 20]. Khalatnikov showed that a certain combination of the second viscosity coefficients, ζ_I, is equal to the expression between square brackets in (2, 18). Thus the total attenuation of sound, at low frequencies (2, 17) and from about 1.0°K to 2.0°K, is

$$\alpha_I = \frac{\omega^2}{2\rho c^3}\left(\frac{4}{3}\eta + \zeta_I\right) \qquad (2,20)$$

3. *Remarks on the theory of Khalatnikov*

The equations for the time rate of change of N (2, 3) are macroscopic equations. We have seen already, in Chapter I, that at temperatures above 1.0°K the mean free path length of the excitations is much smaller than the wavelength of sound normally used. But we must make sure as well that the spatial extension of the excitations is not too large, otherwise we can not locate them well enough relative to the wavelength of sound.

A small extra energy is needed to localize an excitation. This extra energy should not be more than a fraction of the energy of a non-localized excitation.

We infer from the wavefunction for a localized excitation (1,15) together with the energy spectrum (1, 13) as well as from the uncertainty relation $\Delta x \, \Delta p_x > h/2\pi$, where we take Δx as the spatial extension, d, of an excitation, that d must be about ten times the central wavelength, h/p_0, of a wave packet in order to keep the excess energy within 1% of the energy of the non-localized excitation. Naturally, this gives only the order of magnitude of d. Thus rotons extend over about 10 times the atomic spacing and $d_r \simeq$

3×10^{-7} cm. Phonons have on the average an extension of about 10 times the wavelength of thermal sound. Thus at 1°K; d_{Ph} is approximately equal to 10^{-5} cm.

The conclusion is that we need not to be afraid of the excitations not being well enough localized, except for very high experimental sound frequencies or at extremely low temperatures. The values for d imply, however, that the excitations "overlap" each other since, for instance, at 1.0°K there are about 3×10^{19} of them per cm^3. However, the collision probability is small and on the average the quasi-particles run through 100 or 1000 others before "colliding".

From the theory of Khalatnikov we know that in helium II mechanical sound energy is transformed into heat since the production of excitations is a slow process. The integral over one period, $-\oint pdV$, which gives the loss of mechanical energy per period, must be positive. This reminds us of the picture Kneser [16] gave for the relaxation losses in a gas of polyatomic molecules.

When the gas is compressed the translational energy of the molecules is increased almost instantaneously, and all the compressional energy goes into the translational energy of the molecules. It takes some time or many collisions before the energy is redistributed over all the degrees of freedom of the molecules, i.e. over the vibrational and rotational modes as well. The result is that the pressure overshoots its equilibrium value and during each cycle a net work is done on the gas. Schematically this is indicated in figure 3.

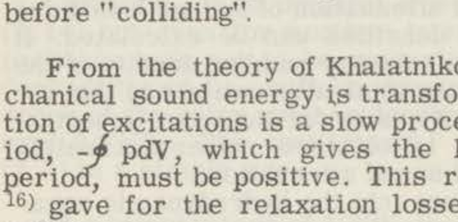

Fig. 3. Schematic representation of the energy loss in a sound wave in a gas of polyatomic molecules after Kneser [16]. p=pressure, V= volume, t=time. The representative point follows the trajectory in the p-V plane as indicated by the arrows.

We suspect that in helium II the situation is an analogous one. Let us take the case of rotons first. Initially, the change in roton number density is simply:

$$\delta N_{r,in} = - N_r \frac{\delta V}{V} \qquad (2,21)$$

The change in equilibrium number density is:

$$dN_{r,eq} = \left(\frac{\partial N_r}{\partial T}\right)_p dT + \left(\frac{\partial N_r}{\partial p}\right)_T dp \qquad (2,22)$$

The first term can be neglected since the propagation of ordinary sound in helium II is almost isothermal. From (1,7) the derivative $(\partial N_r/\partial p)_T$ is equal to

$$\left(\frac{\partial N_r}{\partial p}\right)_T = \chi_r N_T \left[2\,\frac{\rho}{p_o}\,\frac{\partial p_o}{\partial \rho} + \frac{1}{2}\,\frac{\rho}{\mu}\,\frac{\partial \mu}{\partial \rho} - \frac{\Delta}{T}\,\frac{\rho}{\Delta}\,\frac{\partial \Delta}{\partial \rho}\right] \qquad (2,23)$$

where χ_T is the isothermal compressibility. The expression in square brackets is positive and at 1.5°K equal to 8/3 when one uses the values for the derivatives given by Atkins [21] or equal to 3 when one uses the values given by Khalatnikov [14]. Thus

$$dN_{r,eq} \simeq \left(\frac{\partial N_r}{\partial p}\right)_T dp = -\chi_T N_r\,\frac{8}{3}\,c^2 \rho\,\frac{dV}{V} \qquad (2,24)$$

with $\chi_T = 10^{-8}\,\mathrm{cm}^2/\mathrm{dyne}$ we get

$$dN_{r,eq} \simeq -1.7\,N_r\,\frac{dV}{V} \qquad (2,25)$$

Comparing (2,21) and (2,25) we see that on compression initially N_r is less than N_r with $\mu_r = 0$. Thus, according to (2,2), the chemical potential per roton is negative.

From experimental values for the derivatives with respect to the pressure, of the roton parameters and of the velocity of ordinary sound, it follows that

$$(\partial p/\partial \mu_r)_{\rho,S} < 0 \quad \text{and} \quad (\partial p/\partial \mu_{ph})_{\rho,S} > 0 \qquad (2,26)$$

Thus, on compression, the excess pressure, $p - p_{eq}$ due to the relaxing of the roton number density is positive:

$$\delta p = \frac{\partial p}{\partial \mu_r}\,\mu_r > 0 \qquad (2,27)$$

For phonons an analogous calculation can be made. The phonon number density N_{ph} is, according to (1,6), proportional to $(T/c)^3$ where c is the velocity of sound. Neglecting the temperature changes in an ordinary sound wave, we see that the equilibrium number density decreases on compression. Since initially the number density is increased on compression, the phonon chemical potential, μ_{ph}, is positive. With $\partial p/\partial \mu_{ph} > 0$ this again leads to a too high non-equilibrium pressure on compression.

In this way we understand the relaxation attenuation of ordinary sound in helium II qualitatively.

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Chapter III

THE TECHNIQUE OF THE PULSE METHOD

In this chapter the pulse technique for measuring the attenuation of sound in liquids or solids is described. In section 1 we sketch the general aspect. In sections 2 and 3 some acoustic and in section 4 some electronic problems are considered in detail.

1. *The general aspect of the pulse method* [1,2,3]

A pulse generator, with a repetition frequency of about 1000 Hz, triggers a r.f. oscillator, and thus short radio frequency pulses of 1 to 50 μs duration are fed to a quartz crystal (see fig. 1).

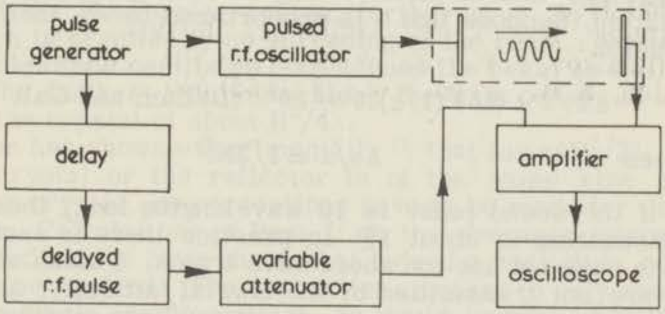

Fig. 1. Block diagram

The pulsed r.f. oscillator is tuned to the fundamental resonance frequency of the crystal or to an odd harmonic. In order to produce longitudinal sound, the quartz is cut perpendicular to the X-axis, and its thickness, t, is half a wavelength. Thus:

$$t = c_{\text{long}}/2f_{\text{res}} \qquad (3,1)$$

where c is the velocity of longitudinal sound in quartz. The crystal transforms part of the electrical energy into sound energy, and a short sound pulse travels through the liquid to the receiving crystal which in turn delivers to an amplifier a r.f. electrical signal. After amplification and detection the signals and some higher order reflections are displayed on an oscilloscope.

In order to determine the attenuation one varies the distance between the two crystals and observes the change in amplitude of the signals received.

Experimentally, the range of frequencies from 1 to about 200 MHz has been covered [3]. Generally, the accuracy in the attenuation which can be obtained is about 2% [3].

Instead of two separate crystals, a reflector and one crystal, as transmitter and receiver, can be used. Mostly one uses piezoelectric quartz crystals. We will see that it is sometimes advisable to use another piezoelectric or electrostrictive material, like barium titanate.

2. *Some acoustical details*

a) *The attenuation as a function of frequency.* The sound pulse has a finite extension, say N wavelengths, and thus is composed of a band of frequencies, the spread in the frequency being

$$\Delta\nu \simeq \nu/2N \qquad (3,2)$$

Because the attenuation is a function of frequency, the pulse must have greater than a definite length for a given accuracy in the attenuation. Suppose that α is proportional to ν^2, then the general expression

$$\Delta\alpha/\alpha = (\Delta\nu)^2(1/\alpha)\,\partial^2\alpha/\partial\nu^2 \quad \text{(Pellam and Galt [2])} \qquad (3,3)$$

becomes

$$\Delta\alpha/\alpha \simeq 1/2N^2 \qquad (3,4)$$

Thus if the sound pulse is 10 wavelengths long, the accuracy in the attenuation is about 1%. In practice there is another reason why one does not use too short wave trains. Pulses which are too short are not transmitted by the crystal faithfully, and they have no flat top even though the r.f. driving voltage may have a perfect square envelope and be delivered by a source with zero internal impedance. The reason for this is that the mechanical Q of the crystal prevents a rapid increase of the sound amplitude.

The Q of a quartz crystal, loaded at both sides by a medium of acoustic impedance Z_1, is, according to Huntington, Emslie and Hughes [4] equal to

$$Q = (n\pi/8Z_1)\,[2(2Z_o^2 - Z_1^2/4)]^{1/2} \qquad (3,5)$$

Here n is the order of the harmonic and Z_o is the acoustic impedance of quartz. Z is equal to the ratio of excess pressure to particle velocity in the sound wave and equals

$$Z = \rho c \qquad (3,6)$$

where ρ is the density and c the velocity of sound in the medium.

For water $Z_o \simeq Z_1$ and the Q for the first harmonic is about 8.

If at time $t=0$ an electrical voltage $\sin 2\pi\nu t$ is applied to the crystal, then the pressure amplitude in the liquid rises with time as $1-\exp(-\pi\nu t/Q_{mech})$. Thus in Q/π periods the final value of the pressure amplitude is reached within a fraction $1/e$. Since the height of the received signal on the scope can be measured with an accuracy of 1%, the duration of the pulse for a "flat top" must be at least 5 times Q/π periods. Thus in water the pulse must contain more than 10 periods. It is safe to use wave trains of 20 wavelengths. In liquid helium the mechanical Q is much higher and pulses of longer duration are to be preferred.

b) *Diffraction effects*. Because of the finite size of the transducer the sound beam spreads, and one has to investigate the influence of diffraction losses on the measured attenuation. The diffraction pattern is known from the theory of the diffraction of light. We suppose that the whole surface of a circular crystal vibrates in phase and with equal amplitude. Even if these conditions are not fulfilled, the diffraction pattern will be qualitatively the same as for a crystal vibrating like a piston. The sound field can be divided approximately into two regions: the Fresnel region extending the distance $z=R^2/\lambda$ from the transmitting crystal — where R is the radius of the crystal and λ the wavelength of sound in the liquid — and the Fraunhofer region, further away. In the Fresnel region there is practically no spreading of the beam, through the pressure amplitude oscillates both across the beam as well along the axis. The farthest minimum along the axis occurs at a distance from the crystal of about $R^2/4\lambda$.

Pinkerton has shown experimentally [1] that for $z<R^2/2\lambda$, if the receiving crystal or the reflector is of the same size as the transmitting crystal, no corrections have to be made for diffraction effects. Calculations of Bass [5] on the averaged intensity as a function of distance have shown that indeed a negligible correction should be applied. The correction amounts to only about 1.3 db over the whole length $z=R^2/2\lambda$. In liquid helium λ for 10 MHz sound is about 2.4×10^{-3}cm, and from a crystal with radius 1 cm the Fresnel region extends about 400 cm away. Thus $R^2/2\lambda$ equals 200 cm and a loss of (1.3/200) db/cm is negligible.

For large z, in the Fraunhofer region, the pressure amplitude along the axis falls like $1/z^2$. The size of the receiving crystal must be small, and a correction for beam spreading must be made [1]. In the Fraunhofer region a circular crystal, as transmitter or as receiver, has a directional pattern as shown in figure 2.

c. *Parallelism of transmitter and receiver*

It is clear that as near a perfect parallelism as possible between the transmitting and the receiving crystal must be achieved.

In the Fraunhofer region we may infer from figure 2 the magnitude of the angle , θ, for which the radiated energy has decreased by 3 db with respect to the energy which is radiated in the direction normal to the circular aperture. This angle equals

$$\Theta = 0.51\,\lambda/D \tag{3,7}$$

where D is the diameter of the aperture. When the crystal is used as a receiver in this direction too the power sensitivity has decreased by 3 db or the amplitude sensitivity by 30%. If the receiving crystal is not set perpendicular to the incident beam of sound an apparent loss results. The accuracy of setting needed depends on the value of the attenuation one wishes to measure and on the order of the reflection one receives. In addition to this condition of parallelism, in the Fraunhofer region, the receiving crystal must be placed accurately on the axis since there is beam spreading.

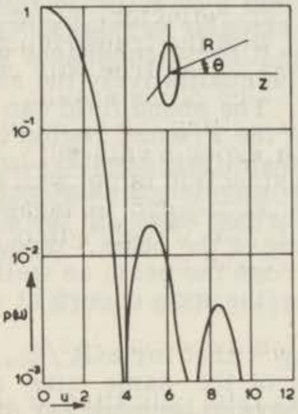

Fig. 2. Diffraction pattern of a circular transducer, of diameter D, in the Fraunhofer region. $u = \pi D \sin\theta/\lambda$; p(u) is the normalised power radiated in the direction designated by u.

Assuming in the Fresnel region a parallel beam, the criterion for parallelism is the same as in the Fraunhofer region.

Thus, in general, the wavelength of sound determines the accuracy of setting needed. The variation in distance between points on the circumference of the two crystals, or of the crystal and the reflector, should be a small fraction, say 1/50th, of the wavelength of sound. Thus, for 10 MHz sound in liquid helium, this variation must be within 0.5 μ. Chase (Chpt. II, ref. 5) achieved this by hanging the reflector from three wires attached to micrometer screws. Another possibility is to have as reflector a piston sliding in a brass cylinder. (See also ref. 3).

In connection with the difficulty of achieving almost perfect parallelism, we mention here a solution which always ensures "parallelism" between the reflector and the transmitting crystal. Figure 3 shows the special shape of the reflector.

The reflector is made of fused quartz in four parts: the body in the shape of a rectangular pyramide with a cylindrical hole along its axis, and the three side-pieces. The interfaces between the pieces were made optically flat and adhered without any kind of glue. A sound beam incident along the arrow reflects on the three side pieces and returns parallel to the direction of incidence.

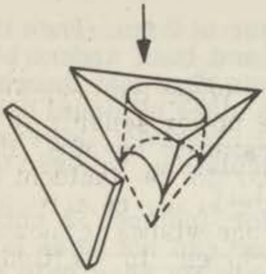

Fig. 3. The corner reflector. Only one of the sidepieces (before assembly) is drawn.

The device was meant to measure the attenuation of sound in liquid helium well above 10 MHz. However, the attenuation then is excessively high and one should have to decrease the path length in the liquid by projecting a rod of fused quartz – which material has a low acoustic loss – into the corner reflector. At one side the rod should be in acoustical contact with the transducing element. However, even if this could be done, mean free path effects (see Chpt. II, section 3) would limit the value of the results.

At room temperature the problem of setting the reflector can be solved rather more simply than in experiments with liquid helium where a sturdy mechanical construction is not well feasible. Thus Andreae, Lamb et al. have succeeded in measuring attenuations up to a frequency of 200 MHz; the wavelength of sound in a "normal" liquid at that frequency is about 6μ.

3. *The decay of a crystal vibrating in liquid helium*

The low acoustic impedance of a crystal vibrating in liquid helium results in a high mechanical Q and thus serious difficulties arise when one uses one and the same crystal for transmitting and for receiving the sound. After having been excited by the electrical signal from the pulsed r.f. oscillator the crystal vibrates acoustically for a very long time and one gets interference between the vibrations still present and the first and even the second reflection. In this section we will describe two different ways in which we tried to suppress the long tail of acoustical vibrations. We were led to use barium titanate, an electrostrictive material, as transducer in liquid helium (c.f. Andreae et al. [3]). The experiments, which are described in the next Chapter, were done with a cylinder of barium titanate which was first used for measurements in solid argon by Guptill et al. [9].

Our experience was that, with an applied voltage of a few volts, the received signals were of the order of millivolts if the attenuation was small. So the ringing amplitude of the crystal must have decreased to a fraction of a millivolt, that is to about 10^{-5} of the original amplitude, before the crystal is ready to receive a sound pulse. In Q/π periods the amplitude decreases to $1/e$. With a Q of 150 the recovery time of the crystal is then about 55 μs. (Numerical values are calculated for a frequency of 10 MHz). Suppose the distance between crystal and reflector is 2 cm. Then it takes 170 μs for a sound pulse to travel forth and back and no harm is done by the ringing of the crystal. We note here that theoretically the Q of a quartz crystal loaded at both sides by liquid helium is about 280. The clamping of the crystal, however, lowers the Q markedly. Therefore, in the above calculation, we used the experimental value for the Q of the crystal.

In the case of a high attenuation, for instance 2 cm^{-1}, the separation between crystal and reflector can not be as large as 2 cm, and interference results. The velocity need only depend very little on temperature to cause a drastic change in the shape and amplitude of the signal. For a phase change of π of the oncoming signal the velocity need only change by 0.1%, if the distance travelled is 1 cm. Chase was misled by this effect and thought that the varying signal strength as a function of temperature meant that there were two maxima in the attenuation instead of one. Whitney, with the electronic set-up of Chase, used two crystals instead of one to eliminate interference effects. Figure 2, Chapter II, for the attenuation of 12 MHz sound in liquid helium is drawn near 1^{o}K according to the data of Whitney. There is only one broad maximum.

It is worthwhile, at this point, asking whether it is possible to lower the mechanical Q of a crystal. There are, theoretically, two ways in which this can be done: mechanically or electrically. Of these electrical damping is the more alluring possibility. We must explain first why the obvious method of damping the crystal mechanically is cumbersome at liquid helium temperatures.

a) *Mechanical damping.* In order to lower the Q of a crystal mechanically the crystal must be coupled acoustically to a solid. The Q is lowered then to about 1. It is not possible to use a thin layer of liquid helium itself to achieve this coupling. The transmission coefficient 6) for sound from the crystal to the solid through liquid helium is too small since the contrast between the acoustical impedances of liquid helium and a solid is too large. A rough calculation shows that the layer of liquid helium should be much thinner than 230 Å, an impossible demand.

Several types of highly viscous liquids are in use to couple the transducer to a solid. We used Dow Corning Fluid with a viscosity of 25000 poise at room temperature. This fluid "freezes" at liquid air temperature, and the difference in contraction below that temperature is small. However, one has to cool down slowly exerting all the time a fairly high pressure to prevent the formation of too many cracks, which ruin the acoustical contact. The solid backing must be rather large compared with the dimensions of the cryostat.

b) *Electrical damping.* Technically it is not easy to damp a crystal mechanically at liquid helium temperatures. It might be possible to drain enough energy from the vibrating crystal in electrical form dissipating it simply in a resistance.

The electro-mechanical circuit, close to resonance, for an X-cut crystal, loaded on one side by a medium of acoustical impedance ρc per cm^2, is shown in figure 4a.

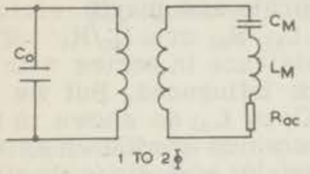

Fig.4a. Electromechanical circuit of a piezoelectric transducer, loaded on one side.

The values of the different elements are given by: 7)

$$
\begin{aligned}
C_o &= \varepsilon^{LC} A/4\pi t \\
C_M &= 2t/\pi^2 A Y_o (1-k^2) \\
L_M &= A t \rho_q /2 \\
R_{oc} &= \rho c A \quad \Phi = D\varepsilon^{LC} A/4\pi t = DC_o
\end{aligned}
\qquad (3, 8)\ *
$$

C_o is the electrostatical capacitance, measured at zero frequency and for the crystal clamped (LC means "longitudinally clamped"). D is the appropriate piezoelectric constant which relates surface charge density to stress. C_M is equal to $2/\pi^2$ times the static compliance of the crystal. L_M is half the mass of the crystal, and R_{ac} is the acoustic impedance of the load; k is the so-called electro-mechanical coupling coefficient. It can be shown that the square of k represents the percentage of the total electrical energy, applied to the crystal that is, at zero frequency, stored in mechanical form.

From figure 4b we read

$$k^2 = C_x/(C_x + C_o) \simeq C_x/C_o \qquad (3, 10)$$

since $C_x \ll C_o$ for all piezoelectric crystals.

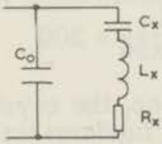

Fig. 4b. Equivalent electrical circuit.

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[*] Here A = area, t = thickness and ρ_q = density of quartz. Y_o is Young's modulus. The values of the electrical, mechanical and piezoelectric coefficients are, for quartz vibrating along the X- or thickness axis:

$$\varepsilon^{LC} = 4.55, \quad Y_o = 8.42\times10^{11}, \quad \rho_q = 2.65, \quad D = 14.3\times10^4 \qquad (3,9)$$

in c.g.s. electrostatical units. The coupling coefficient k = 0.0096. To get the value of L_x in henries one has to multiply by 9×10^{11}.

If we transform C_M, L_M and R_{ac} to the left side in the usual way (the transforming ratio is not dimensionless), we obtain the equivalent circuit of figure 4b which is more commonly known. We will need the representation of figure 4a, however, to illustrate the difference between barium titanate and quartz.

The mechanical Q of the crystal is $\omega L_M/R_{ac}$ or $\omega L_x/R_x$. Thus the damping can be increased by a resistance in series with R_x. The series branch, however, cannot be influenced. But we can tune out the electrostatic capacitance C_o by L_{el} as shown in figure 5. The tuning coil has its own losses which are shown as R_{el}. The quality factor of the coil, Q_{el}, must be *high* since then R_{el} has a large value, and the series acoustical branch is damped heavily.

Suppose we wish to bring down the Q_{mech} from 150 to the value of Q for a crystal vibrating in a "normal" liquid. In water $Q \simeq 8$.

We have to tune out with L_{el} the capacitance C_o plus the capacitance of the r.f. leads from the pulsed oscillator to the crystal: 50 pF is a conservative figure for the total capacitance. Cyrogenic demands make it difficult to make the capacitance smaller than this.

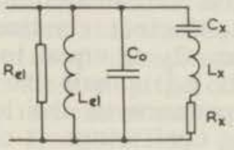

Fig. 5. The tuning of C_0 by L_{el}.

Let us take a crystal of 1 cm^2 surface and of thickness 0.028 cm (f_{res} = 10 MHz). We calculate from (3, 8):

$$L_x \simeq 7.5 \text{ mH/cm}^2, \quad C_x \simeq (1/32) \text{ pF/cm}^2 \qquad (3, 11)$$

L_{el} must resonate with 50 pF at the same frequency, thus $L_{el} = L_x/32 \times 50$ ($\simeq$ 0.45 μH). The Q of the circuit must be about 8: $\omega L_x/R_{el} \simeq 8$. Therefore the Q_{el} of the coil must be at least

$$Q_{el} = R_{el}/\omega L_{el} = \frac{(1/8)\,\omega L_x}{(\omega L_x)/32 \times 50} = 200 \qquad (3, 12)$$

This is an impractically high Q. In addition, the crystal is vibrating in spurious modes as well as in the fundamental. Especially since it is excited by a r.f. pulse which contains a narrow band of frequencies. The equivalent circuit for the crystal is not of the simple form of figure 4b. The conclusion is that it is extremely difficult to damp a quartz crystal electrically.

It might be possible, however, to succeed with electrical damping, if one uses as transducer another piezoelectric material. If L_x has a small value then, for the same $Q = \omega L_x/R_{el}$, R_{el} need not be so high. Perhaps even mistuning could be tolerated or in other words: the spurious modes might be damped as well by one single coil if the condition for Q_{el} is not so stringent. Let us look then for crystals with a much smaller L_x. The mechanical inductance, L_M, being half the mass, is about the same for all crystals. L_x, however, is equal to $L_M/4\Phi^2$ where $\Phi \propto D\varepsilon \propto k\sqrt{\varepsilon}$ since

$$k = D\,\sqrt{(\varepsilon/4\pi\,Y_o)} \qquad (3,13)$$

Most naturally-grown piezoelectric materials have a coupling coefficient between 10 and 20% and an ε of about 3. We therefore looked for a synthetic material with a high Φ. Barium titanate is such a material. It is electrostrictive, and equations can be set up for it which are analogous to the piezoelectric ones. Its electromechanical coupling-coefficient k is about 0.25 and at room temperature: $\varepsilon \simeq 1500$ [8]. At a temperature of 2°K the value of ε has decreased to about 90 [10]. Thus, roughly, we gain in Φ a factor of 10 and in L_x a factor of 100. Therefore the Q_{el} of the tuning coil need only be 2.

According to these ideas we tried a barium titanate disk as transducer and indeed got better results. There was much less interference and the shape of the signals, received in pure ^{4}He, was much better.

Nevertheless, we are not certain whether the better shape of the signals was due to electrical damping as described. The disk probably had more than one resonance frequency. Normal commercial barium titanate is a sintered material which has to be polarized electrically above the "Curie" temperature and then cooled down in an electric field in order to give it its useful properties as a transducer. The polarization in the sintered material must be far from homogeneous, and we estimate [8] that in fact only about 10% of the domains were oriented in the direction of the applied electric field. Hence there must result a whole band of "resonance" frequencies. It may be, however, that barium titanate has high internal losses at liquid helium temperatures (see also Andreae et al. [3]).

With the barium titanate cylinder described in Chapter IV we obtained a very good shape for the signals from the 5th reflection on.

4. *Electronics*

In the actual set-up a delayed calibration r.f. pulse was generated as well as the signal pulse. After having passed the variable attenuator the delayed pulse entered the amplifier at about the same signal strength as the sound signals received from the crystal. Both signals were amplified and in this way a possible non-linearity or variation with time of the amplification did not influence the accuracy of the attenuation measurements. By varying the strength of the r.f. calibration pulse with the help of the variable attenuator its height on.the oscilloscope was made equal, within ± 1 db, to the heighth of the received sound signal. In the range of 1 db we interpolated linearly. The error made in interpolating linearly is less than 1/50 db, which is negligible. The delay of the calibration pulse could be varied and the delayed r.f. pulse could be set alongside each of the successive sound reflections.

We will describe the different parts of the electronic set-up briefly (figure 1).

The pulse generator was a Hewlett Packard type 212A. The variable attenuator was a Rohde and Schwarz with a maximum attenuation of 100 db in steps of 1 db and a frequency range from DC to 300 MHz.

Below we will give a description of the pulsed r.f. oscillator, the delay and the amplifier. Throughout the remainder of this section we will refer frequently to volumes 18 and 19 of the Massachusetts Institute of Technology Radiation Laboratory Series: (MIT, Volume, Chpt-Section).

a) *Pulsed r.f. oscillator.* (MIT, 19, 4-13 and 14). (See figure 6).

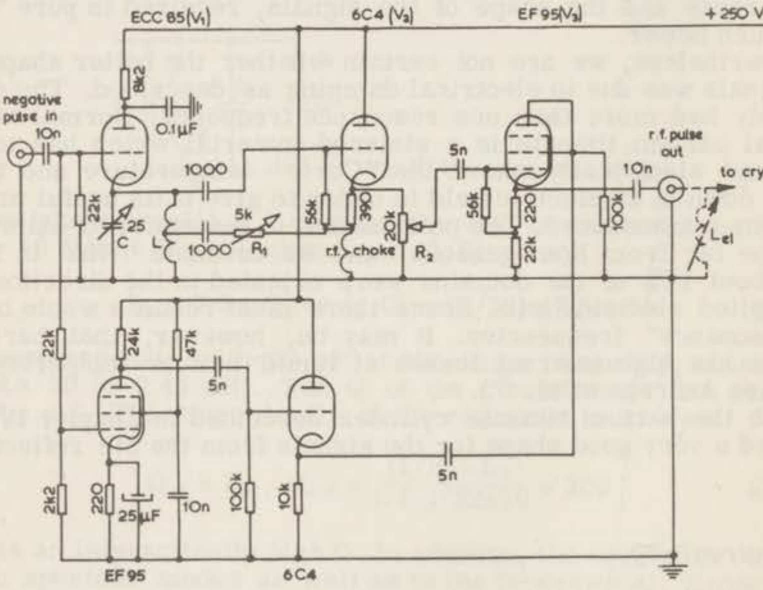

Fig. 6. Pulse r.f. oscillator.

The oscillator is of the "ringing circuit" type, and its frequency is determined by L and C. Most of the time tube V_1 conducts, and its low output impedance, 1/S, prevents oscillation. When a negative rectangular pulse is fed to the grid of V_1 the damping is removed and the current, I, flowing through L starts the oscillations immediately, the initial voltage amplitude being $I\omega L$. The amplitude of oscillation should decay with $\exp(-\omega t/2Q)$. However, just enough energy is fed to the LC circuit through R_1 by the oscillator tube, V_2, to keep the amplitude constant. As soon as V_1 starts to conduct again the oscillations are damped abruptly. R_2 is a simple amplitude control.

A positive pulse is applied to the anode of the output cathode follower during the time the r.f. pulse comes at its grid. This is done by inverting the negative pulse which is fed into the pulsed r.f. oscillator. The purpose of gating the output tube is to present to the crystal a high impedance immediately after having excited it. Should the tube V_3 stay conducting after having transmitted the r.f. signal to the crystal, then its low output impedance, 1/S, should shunt L_{el}. And as we have seen in section 3, to prevent a long acoustic recovery time of the crystal, the Q_{el} of the tuning inductance, L_{el}, should be high.

b) *Delay*. See figure 7. The same negative pulse which gates the pulsed r.f. oscillator triggers the time delay. Thus, the monostable multivibrator tube ECC91 (MIT, 19, 5) is triggered into its unstable position, and a positive gate of duration 4000 μs appears at the anode of the tube half which initially was on. This gate is fed to the suppressor grid of the Miller sweep generator tube, 6AS6. A negative going linear voltage is generated (MIT, 19,5-15), and the voltage at the anode of the 6AS6 drops about 250 V in 2500 μs. Thus the incoming pulse can be delayed to a maximum of 2500 μs. The negative going voltage is fed through a cathode follower, ECC83 a, and then compared, in the ECC83 b, with a reference voltage set by R. When the two voltages become equal, a negative step is produced at the anode of the ECC83 b. The EF80 is part of a regenerative loop to speed up the transition of the ECC83 b, (MIT, 19, 9-13).

Via a cathode follower, EC92, this step – which is delayed with respect to the incoming pulse, the delay being a function of the reference voltage – triggers a monostable multivibrator, ECC85. The stable position of the multivibrator is with the right half of the ECC85 on.

A positive pulse, of about 20 μs duration, is generated at the anode of the right half.

This pulse gates the 6AS6 time selector (MIT, 19, 10-3). Most of the time the current in the 6AS6 goes to the screen only, the suppressor voltage being about 10V negative with respect to the cathode. The continuous r.f. signal (from the General Radio type 805 C signal generator) does not appear at the anode. When the positive gate arrives at g_3, the current switches from screen to anode and a r.f. pulse appears at the output. This r.f. pulse is of the same frequency as the sound and is fed together with the received signals to the amplifier.

The delay offers a simple method to measure the frequency of the sound signal. One first tunes the pulsed r.f. oscillator (fig. 6), maximum height and a good shape of the received sound signals

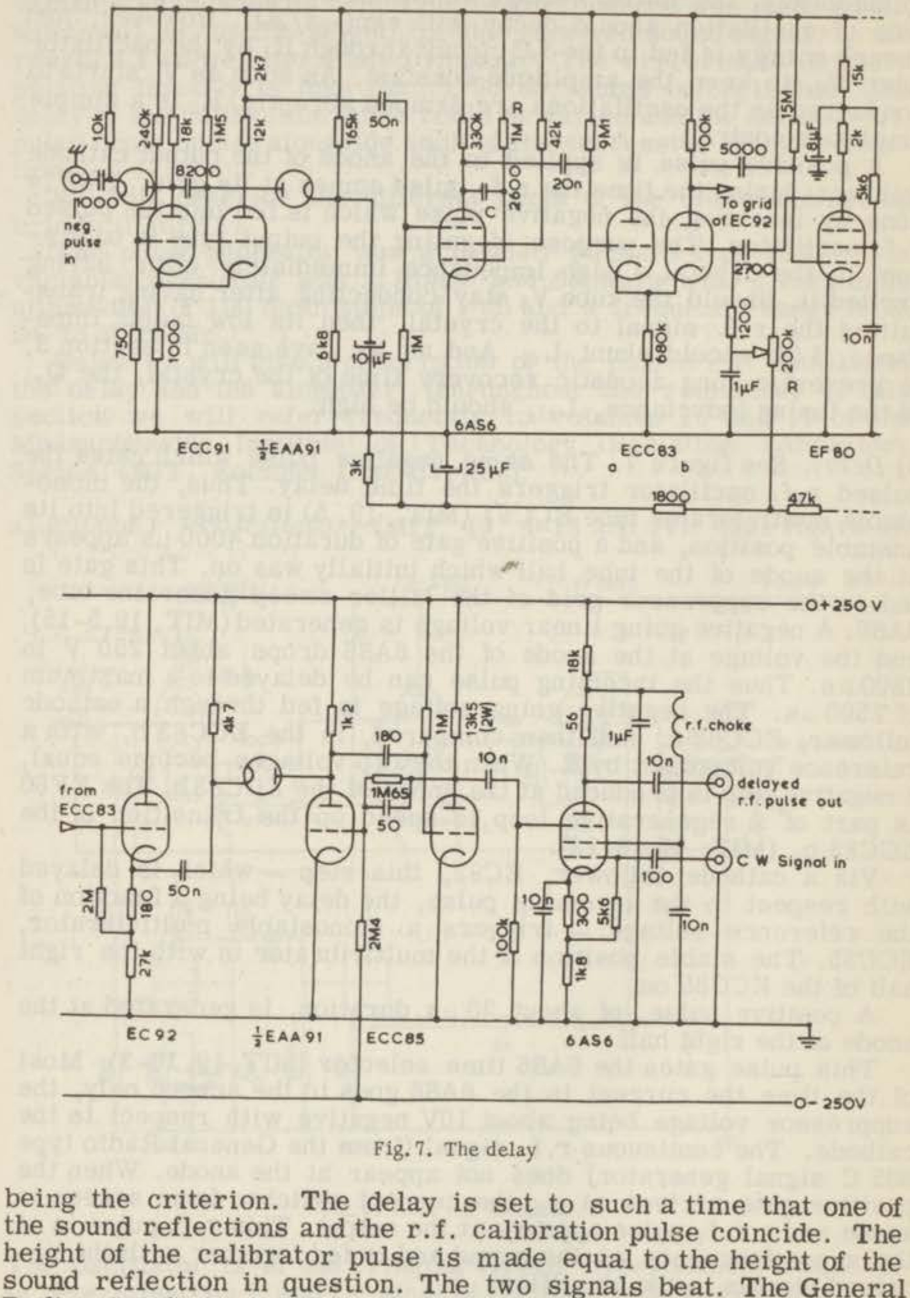

Fig. 7. The delay

being the criterion. The delay is set to such a time that one of the sound reflections and the r.f. calibration pulse coincide. The height of the calibrator pulse is made equal to the height of the sound reflection in question. The two signals beat. The General Radio signal generator, which determines the frequency of the calibrator pulse, is tuned to give zero beat. The amplitudes of the two signals add or subtract. There is no stable phase relationship and one sees the height of the interference signal change from zero to double the amplitude of a single signal. The frequency of the General Radio signal generator in this position gives the freqiency of the generated sound.

c) *The amplifier* (See figure 8). (MIT, 18, 4 and 5).

The amplifier is of the synchronous double-tuned type, consisting of a broad-band preamplifier, mixer, i.f. amplifier, detection and output stage.

The preamplifier, ECC88 and E180F, is centered at 7 MHz and has a voltage gain of about 100 at a bandwidth of 8 MHz. For calculating the values of the inductances and resistances in the double-tuned circuits the tables in section 5-5 of MIT, 18 are used.

The input is a so-called grounded-cathode grounded-grid cascode stage (MIT, 18, Chpt. 13, Minimal Noise Circuits) which has a low noise figure (MIT, 18, Chpt. 12: Amplifier Sensitivity) *if* there is the proper mismatch between the source conductance and the input conductance of the first stage (MIT, 18, 13-6). However, the source conductance, i.e. the conductance of the receiving crystal, is difficult to measure. Therefore we restricted ourselves to tuning out the capacitance of the source and associated leads by a variable inductance (not shown).

In the ECH81 the signal is mixed with the c.w. signal of the local oscillator, for which we used a Rohde and Schwartz type SMLM.

The intermediate frequency amplifier is centered at 27 MHz and has an over-all bandwidth of 0.6 MHz. Thus r.f. pulses of about 2 μs minimum duration are reproduced fairly well (MIT, 18, 13-1). The double tuned interstage circuits are calculated with the help of the theory given in MIT, 18, 5. The circuits are inductance-coupled to get an amplifier with a short recovery time. Should they be capacitance-coupled then, during a strong signal, the interstage coupling capacitances should be charged by grid conduction. These fairly large coupling capacitances should need a too long time to discharge themselves through the grid leak-resistances.

The coupling is slightly less than transitional (or critical which in the case of equal Q's is the same as transitional) to prevent a doubly-humped bandpass characteristic (MIT, 18, 5-3).

The ½EB91 detector and the output stage are conventional (as is the whole amplifier). We could vary the gain of the amplifier enough simply by varying the local oscillator voltage fed to the mixing tube.

The r.f. calibration pulse was fed into the amplifier at the second stage, the assumption having been made that the first stage amplifies linearly over a range of a few mV, the maximum strength of the signals.

The over-all voltage gain of the amplifier was about 20,000.

The resistances of 10 Ω in the grid circuits prevented any parasitic oscillations.

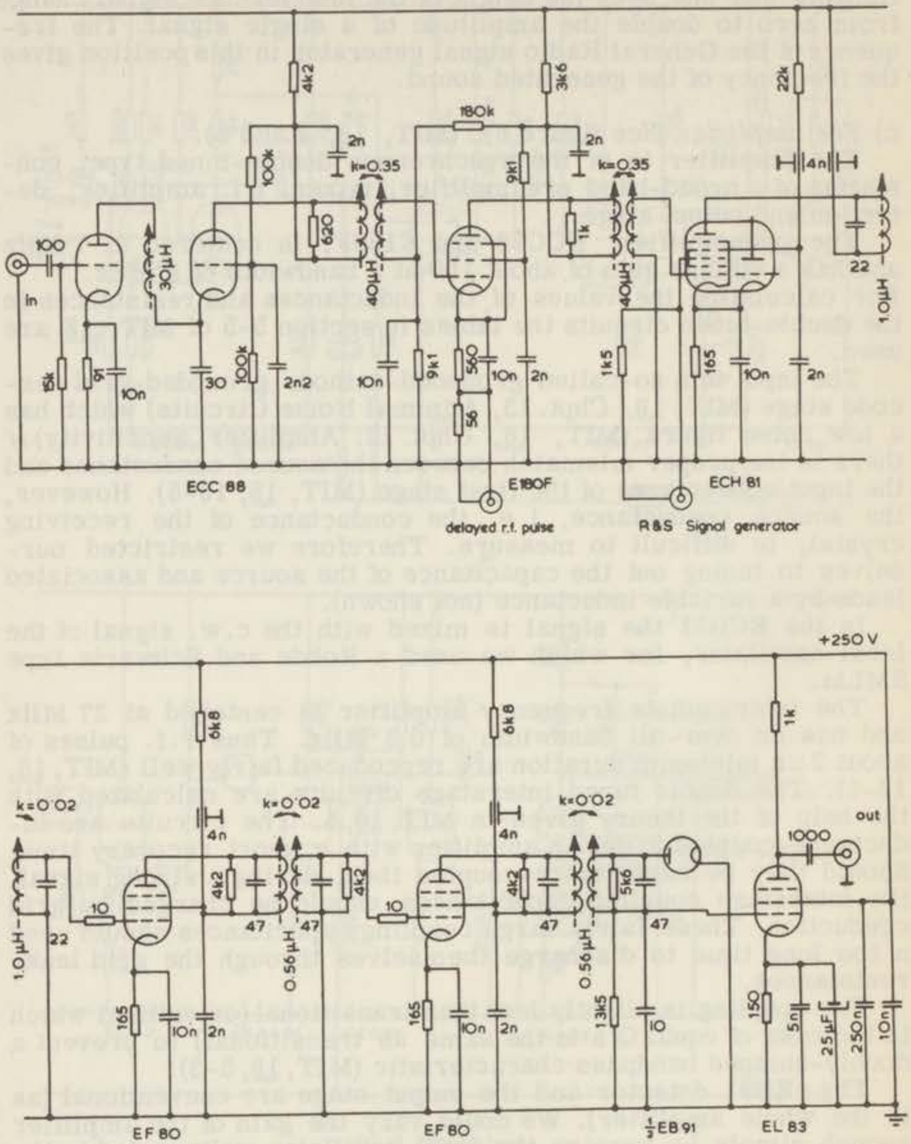

Fig. 8. The r.f. amplifier

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Chapter IV

THE EXPERIMENT

In the first section the barium titanate cylinder and its imperfections are described. Section 2 is devoted to the method of measuring the attenuation of sound with the cylinder and to the correction we had to apply to the apparent attenuation. It will become clear that the method is suitable only if the attenuation in another liquid, of about the same acoustic impedance as the liquid under test, is known. In section 3 the results are given for four ^{3}He-^{4}He mixtures, of ^{3}He concentration 0.5, 1, 3 and 11%.

1. *The barium titanate cylinder*

There is a change in structure for barium titanate at about 120°C. Above this temperature the structure is cubic (see fig. 1) the titanium atom on the average being in the center of the cell.

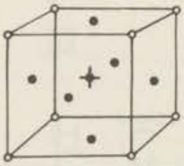

Fig. 1. The structure of barium titanate above 120°C.

Below 120°C the titanium atom is locked into one of six possible positions of minimum potential energy. Each of the positions is slightly away from the center of the oxygen atoms, and the structure changes from cubic to tetragonal. The field of the resulting electric dipole results in the adjacent cells in a preference for one of the six positions of minimum potential energy and domains of electrical polarization are formed. In the direction of polarization the cell-dimension changes from 4.0 Å to 4.026 Å and normal to it the dimensions shrink to 3.86 Å [1]. The dielectric constant of polycrystalline barium titanate, ε, has a value, at 120°C, of about 6000. Above 120°C it falls off following a Curie law. Below 120°C the value of ε drops to 1500 at room temperature and at the temperature of liquid helium ε amounts to about 90 [2].

In addition to the transition from a cubic to a tetragonal structure, there are two more changes in structure, at 10°C and at -80°C. Below -80°C the structure of barium titanate is trigonal, and the titanium nucleus spends equal times in three of the six positions of minimum energy which lie on mutually perpendicular axes, and very little time in the other three [1].

We used polycrystalline sintered barium titanate in the shape of a hollow cylinder. After silvering the inside and the outside, the cylinder was given a remanent polarization in the radial direction by heating it in an oil bath above 120°C and then cooling it in an electric field. We never applied a polarization field of more than 8 kV/cm for fear of breaking the 1 mm thick cylinder. When there are small defects, the internal stresses can easily become too high when the temperature is lowered through the Curie point.

When a small a. c. field is applied to the polarized cylinder, its thickness varies with the same frequency. Probably some domains grow at the expense of others, this being accompanied by a change in dimension.

The dielectric relaxation losses in barium titanate are rather high. Thus the transducer has a low internal mechanical Q which is exactly what we needed (Chpt. III, section 3).

Guptill and Hoyt [3] used a barium titanate cylinder, shown in figure 2, to measure the velocity of sound in solid argon.

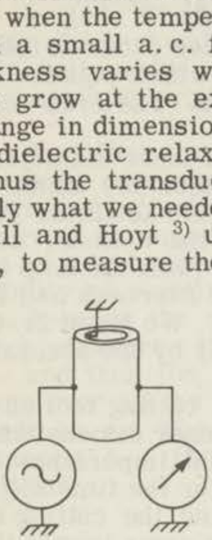

Fig. 2. The barium titanate cylinder for measuring the velocity. After Guptill et al. [3].

One half of the cylinder was used as transmitter; the other half detected the resonances set up in the liquid cylinder. The outer silvering was split in two electrodes by scratching the silver away along two diametrically opposed lines. In practice the receiving and transmitting halves were separated by narrow additional electrodes (four scratches in total) which were earthed. These served to prevent too large a direct acoustical and electrical pick-up.

The influence of the cylinder on the resonance frequencies of the liquid cylinder can be neglected since resonances in the liquid are set up far from a resonance frequency of the barium titanate cylinder itself. This separation of frequencies is possible since the electromechanical coupling coefficient for barium titanate is high and the internal Q is low. Moreover, in liquid ^{4}He – or in a ^{3}He-^{4}He mixture (Lim et al. [4]) – the acoustic impedance is much smaller than the acoustic impedance of barium titanate. Thus the radial velocity of the liquid at the boundary is zero, and the condition for the resonance frequencies, according to Lord Rayleigh [5], is:

$$J'_n(kr) = 0 \qquad (4,1)$$

assuming for the moment an infinitely long cylinder. The prime means differentiation with respect to the argument. k is the wave vector and r is the radius of the cylinder. The asymptotic form of the function $J'_n(kr)$ is a damped trigonometric function:

$$\begin{array}{lll} & J'_n(kr) \rightarrow \pm \sqrt{(2/\pi kr)}\,\sin(kr-\frac{\pi}{4}) & \text{n even} \\ \text{as } kr \rightarrow \infty & & \\ & J'_n(kr) \rightarrow \pm \sqrt{(2/\pi kr)}\,\cos(kr-\frac{\pi}{4}) & \text{n odd} \end{array} \qquad (4,2)$$

Thus, for n either odd or even, resonances occur at frequencies separated by a frequency difference, $\Delta\nu$, which equals $\Delta\nu = c/2r$, c being the velocity of sound in the liquid. The even resonances lie midway between the uneven ones.

We used, in liquid helium, a cylinder of inner diameter $\simeq$ 1 cm; c/2r was about 23 kHz, and the resonances lay about 11.5 kHz apart. Thus at 2 MHz we excited in the liquid about the 175th resonance. We determined the diameter of the cylinder accurately by measuring the resonance frequencies (with the help of a Berkeley counter) and using the data of van Itterbeek and Forrez 6) for the velocity of sound in liquid helium. We found 2r=0.987 cm ± 0.2%, the accuracy being limited mainly by the accuracy in the velocity data.

The cylinder was cut with a diamond cutting tool on a slowly turning lathe. Only the regularity of the inner dimensions and the finish of the inner surface are of paramount importance. We took off a few microns of material each cut. For the finishing cuts the cutting depth was not reset any more, and the cutting tool went back and forth across the rotating cylinder some twenty times, the disappearing of a high-pitched scratching noise being the sign that no more material was being taken off. The cylinder was silvered chemically. It was important to clean it very thoroughly (simply by scrubbing it with water and soap for a quarter of an hour).

Experimentally we found that the resonances split up into two or more peaks instead of one single resonance: the smaller the wavelength the more peaks we observed. In liquid air, around 2 MHz, we found a maximum difference in frequency between the peaks of one resonance of 1900 Hz. Secondly, we found that the even resonances did not lie midway between the uneven resonances.

A simple explanation for the splitting up of a resonance is that the cylinder is slightly conical or barrel-shaped, the broadest part coming into resonance at the lower frequency. From the difference in frequency between the peaks of a certain resonance we concluded that the effective radius varied by about 8 micron. As has been said we interpreted this as the maximum variation in diameter in going from top to bottom.

One might think, that a resonance is split because the column of liquid brought into vibration has a finite length whereas the condition (4,1) is for an infinite cylinder. One can easily calculate the splitting – following Lord Rayleigh (loc. cit. section 340) – assuming that the liquid is free to move in the radial direction at a distance from the bottom of the can (see fig. 6) equal to the penetration depth, λ. For a viscous wave: $\lambda = \sqrt{(\eta/\pi\nu\rho)}$. In liquid helium, at 1.5°K and for a frequency of 2 MHz, the penetration depth is about equal to 10^{-5} cm. We calculated that the relative shift in resonance frequency from the zeroth order harmonic in the axial direction to the second harmonic in the axial direction was only about 10^{-4}. Higher order axial harmonics were unlikely to be excited. The splitting, at 2 MHz, did not depend on the velocity of sound which should have been the case. Moreover, the geometrical conditions for resonance in the axial direction are as severe as in the radial direction and presumably were not fulfilled.

The fact that the even resonances did not lie midway between the uneven resonances taught us that the cross-section of the cylinder was not circular, but, presumably, slightly elliptical.

There is an important difference between the "J_o" and the "J_1" modes of resonance. We note here that there was only a slight probability of exciting other modes than the J_o and the J_1 ones. In the J_o-mode the velocity field does not depend on the angle Θ, – which measures the direction in a plane perpendicular to the axis – and thus the effective diameter of the cylinder is some average over all the directions Θ. Let the effective radius for the J_o-mode be r_o. The difference in frequency between successive resonances, $\Delta\nu_o$, then equals $c/2r_o$. For the J_o-mode the velocity potential Ψ:

$$\Psi = A \cos\Theta \, J_1(kr) \qquad (4,3)$$

depends on the angle Θ. Since there was a preferred direction for the nodal line in the J_1 mode, the effective radius, r_1, for the J_1-mode was not equal to the effective radius for the J_o-mode. The difference in frequency between the successive J_1 resonances, $\Delta\nu_1$, is $c/2r_1$.

It is clear that $\Delta\nu_1 = \Delta r_o(1+\delta/r)$, where $\delta = r_o - r_1$ is the difference in effective radius. We will see that δ/r was small, and, within the measuring accuracy, $\Delta\nu_1$ was equal to $\Delta\nu_o$. However, the effect of a slight difference in effective radius over many resonances is to shift the frequencies of one type of resonance (say the even ones) relative to those of the other type (the uneven ones). Let the difference between the frequency of an even resonance and the frequency midway between the two adjacent uneven resonances be ε. It is easy to show that:

$$\varepsilon/\Delta\nu = 2\,(r/\lambda)\,\delta/r \qquad (4,4)$$

Measuring around 2 MHz in liquid air we found: $\Delta\nu$ = 88.64 kHz and ε = 250 Hz. Thus δ/r was about 1.4×10^{-4} (the velocity of sound in liquid air is about 900 m/s).

We concluded that the cross-section of the cylinder was slightly elliptical, the difference between the axes being about 1.4 μ *.

2. *Measurement of the attenuation with the barium titanate cylinder*

If one remembers that the attenuation in liquid helium is rather small, it may be seen from the foregoing that it was impossible to measure the attenuation in pure ^{4}He directly by observing the half width of a resonance, $\Delta\nu$. The half width is related to the volume attenuation by the relationship

$$Q = \frac{\nu_{res}}{\Delta\nu} = \frac{\pi}{\alpha\lambda} \tag{4,5}$$

α being the amplitude attenuation coefficient. At 1.5°K, for instance, $\alpha/\omega^2 = 6 \times 10^{-17}$ cm^{-1} s^2 7). Thus $Q \simeq 6 \times 10^{10}/\nu$. The broadening of a resonance peak due to the non-perfect geometry must be much smaller than the real width. Thus for ν = 10 MHz, δ/r should be much smaller than $(5/3) \times 10^{-4}$, whereas in reality $\delta/r \simeq 1.4 \times 10^{-4}$ (see above).

Besides the condition of an almost perfect geometry, there are two more reasons why it is difficult to measure the attenuation directly from observing the Q of a resonance. First of all, the frequency of the generator must be stable. If one requires that during the measurement the drift in signal generator frequency should be less than 1/10 of the resonance width, then for ν =10 MHz the relative frequency drift should be less than $1/(6 \times 10^4)$. (All figures quoted are for a temperature of 1.5°K and a frequency of 10 MHz). Secondly, the bath temperature should be very constant, since the velocity, and thus the resonance frequency, are functions of temperature. Again, if one requires that the shift in resonance frequency should be less than 10% of the resonance width, the drift in temperature should be less than 5 millidegree.

Both these stability conditions could be met, but if one works with pulsed rather than with continuous sound, the problems to be solved become much more simple. The condition for frequency stability is replaced by $\delta\nu/\nu = \frac{1}{2}\,\delta\alpha/\alpha < 1/20$, (supposing $\alpha \propto \nu^2$) and the second condition by $\delta T < 0.03$°K.

Naturally, if one wishes to determine the attenuation from the decay of successive sound reflections, the geometrical conditions are as severe as in the case of continuous waves. However, there is a simple way of camouflaging the non-perfect geometry of the barium titanate cylinder by using only a small section of the cylinder as receiver.

The effect of a receiver of smaller dimensions can be explained qualitatively by looking at the analogous case of plane circular

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* After completion of the measurements, in the Central Workshop of the "Technische Hogeschool" at Delft a cylinder was lapped which was out of round by not more than 0.1 μ and conical by 0.5 μ. After polarization this cylinder was 5 μ out of round'. Therefore it is almost certain that on polarization the cylinder changes of shape drastically.

receivers. From the diffraction pattern (fig. 2, Chapter III) one may infer that the angle of incidence for which the receiver sensitivity has decreased by, say, 30% is inversely proportional to the diameter of the receiver. It follows that for the same angle of incidence the apparent loss is smaller for a smaller receiver.

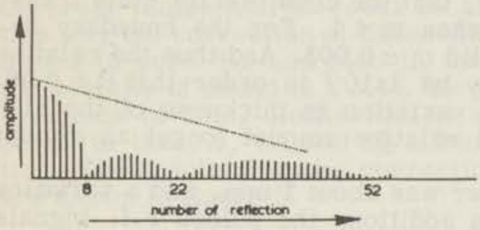

Fig. 3. Sketch of the reflections received in liquid air.

In liquid air, with a full half of the cylinder as receiver (fig. 2), we got, at a frequency of 7 MHz, a row of sound pulses as sketched in figure 3. From the minima, occurring at the 8th, 22th and 52th reflection, we estimated the variation in diameter of the cylinder at different points along its axis to be: 11, 6.5 and 4 μ respectively (compare with 8 μ in section 1). When we diminished the receiving area to a small curved square of 4x4 mm the envelope changed to the dotted line.

In liquid air we received a few hundred reflections the first 50 of which showed an exponential fall-off. This experimental fact meant that the relative loss at reflection did not depend on the number of the reflection for the first 50 ones. In liquid helium, at 7 MHz, the first 9, and at 12 MHz the first 5 reflections showed an exponential fall-off.

The decay of the successive reflections we observed was still much too fast. In order to determine the correction which had to be applied we compared our measurements in liquid ^{4}He with the data of Chase [7]. We found that our values for the apparent attenuation differed an amount, $\Delta\alpha$, from the values for the true attenuation. This correction, $\Delta\alpha$, depended slightly on temperature. There are two possibilities why the correction could change with temperature. The velocity of sound in the ^{4}He changes with temperature, and secondly the density or the acoustic impedance of liquid helium depends on temperature.

A change in velocity means a change in wavelength, and it is clear that for a smaller wavelength the angle of incidence is more critical. A change in acoustic impedance means that the fraction of the energy which at each reflection is lost in the barium titanate changes. We will argue that the variation of $\Delta\alpha$ with temperature was almost entirely due to the changing velocity.

When sound travelling in a medium of acoustic impedance $\rho_1 c_1$ falls at right angles on a slab of acoustic impedance $\rho_2 c_2$ and thickness t, the coefficient of reflection is given by a formula due to Lord Rayleigh [8]:

$$R = (m^2-1)^2/[4m^2 \operatorname{ctg}^2 \frac{2\pi t}{\lambda_2} + (m^2+1)^2] \qquad (4,6)$$

Here $m=\rho_1c_1/\rho_2c_2$. This means that the energy reflection coefficient, R, is zero when $t = (n + \frac{1}{2})\lambda_2$. And, indeed, we were exciting the barium titanate in its 3rd or 5th thickness harmonic. From (4, 6) it may be seen, however, that the condition for unity transmission is extremely sharp when $m \ll 1$. For the boundary between liquid helium and a solid $m \simeq 0.003$. And thus the relative change in frequency need only be 2×10^{-2} in order that $R \simeq 0.99$. Or for a given frequency the variation in thickness of the slab need only be the same small relative amount to get an almost total reflection.

The thickness of the cylinder was about 1 mm, and a variation of 10 μ was quite probable. In addition, the pulsed r.f. signals were frequency-modulated by at least 1%. We concluded that most of the energy, probably even more than 99%, was reflected back, and that even the relatively large change in acoustic impedance for the liquid helium (about 10% from 1.0°K to the lambda-temperature) did not effect the correction by more than an insignificant fraction of a percent.

Therefore, the correction that we had to apply to the apparent attenuation changed with temperature only because it was a function of the changing velocity. The wavelength of the sound decreased with increasing temperature, and the non-perfect geometry caused a larger apparent attenuation for a smaller wavelength or a higher temperature.

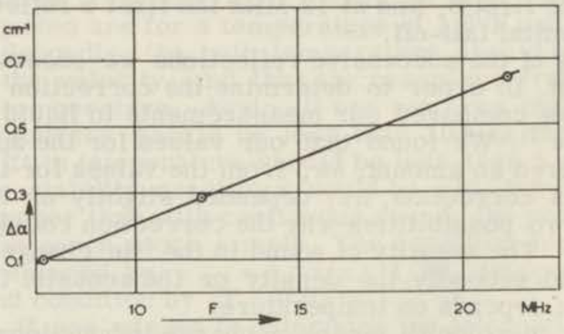

Fig. 4. The correction, Δα, as a function of frequency for a sound velocity of 237 m/s.

We determined the correction in pure ^{4}He at three different frequencies. The results are shown in fig. 4, for a velocity of 237 m/s. We note that for a frequency of 22 MHz the wavelength of sound is about 10 μ, comparable with the variation in diameter of the cylinder.

We reasoned that a change in velocity, at constant frequency, was equivalent, as far as the correction is concerned, to a change in frequency, at constant velocity, since the correction was determined only by the ratio of wavelength to variation in diameter of the cylinder. Thus we said that $(\Delta c/c)_{\nu}$ as a function of temperature had the same influence on the correction as $(\Delta\nu/\nu)_c$, where we put $\Delta c/c = \Delta \nu/\nu$. We calculated $\Delta c/c$ for pure ^{4}He from the data of van Itterbeek and Forrez [6], and then the correction for different temperatures could be read from the graph of fig. 4. In this way the correction at different temperatures could be determined in pure ^{4}He and also in the ^{3}He-^{4}He mixtures used since the velocity of sound in a 3.4% ^{3}He mixture was known from measurements of Lim, Hollis-Hallett and Guptill [4]. The velocity of sound in the 0.5, 1.1 and 3% mixtures was obtained by interpolation. The results for the correction as a function of temperature are shown in figure 5.

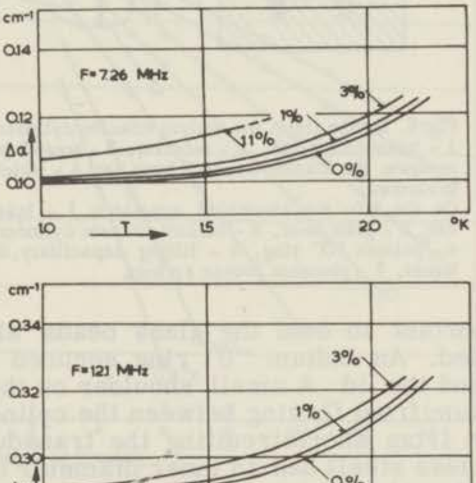

Fig. 5. The correction, Δα, as a function of temperature for the mixtures used.

Cryogenics. Figure 6 shows a drawing of the apparatus as it was mounted in the cryostat.

The barium titanate cylinder fitted in the brass can with a clearance of 0.1 mm. For insulating the electrodes thin paper was shoved between the cylinder and the container. Electrical contact to the electrodes was made by small phosphor bronze springs which were soldered to wires. These wires were pushed through hollow Philips glass beads and afterwards the glass beads were soldered tight. The electrical leads from the transmitter consisted simply of a pair of twisted wires. The lead to the amplifier went inside a german silver tube to prevent excessive electrical pick-up. Most measurements were done well above the noise level of the amplifier.

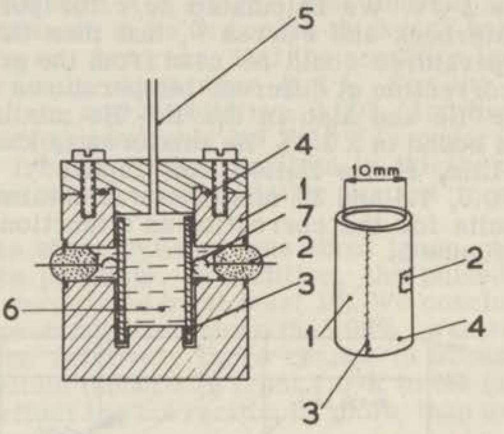

Fig. 6. On the right: the barium titanate cylinder; 1 - transmitting half, 2 - receiver, 3 - scratch to separate the transmitter half from part 4 - which is earthed.
On the left: the assembled apparatus; 1 - brass can, 2 - glass bead, 3 - barium titanate cylinder, 4 - indium "O" ring, 5 - filling capacillary, 6- liquid, 7 - phosphor bronze springs.

It was important to cool the glass beads slowly; otherwise cracks developed. An indium "0" ring secured a tight seal between the can and the lid. A small shoulder on the container prevented the indium from flowing between the cylinder and the container and thus from shortcircuiting the transducer. The filling capillary (stainless steel) had an inner diameter of 3/4 mm.

Condensing the mixture into the container was done in the normal way with the help of a Toepler pump. The apparent attenuation measured depended slightly on the height of the liquid level inside the cylinder; therefore the Toepler volume was calibrated and the same amount of gas was condensed for each run.

The cryostat itself was connected to an Edwards booster pump, type 9B3, which was backed by an Edwards prevacuum pump, type 1S450A. The pumping connection to the cryostat was as short as possible. A wide elbow, 30 cm. in diameter, on top of the booster, narrowed down into a horizontal stub, 8 cm in diameter, The booster could be closed off with a valve seating against the end of the elbow where it connected with the stub. The stub was connected by means of a rubber sleeve to another stub of the same diameter which projected from the cryostat head. Temperatures of the bath below 1.1°K could be regulated simply by positioning the valve.

The bath temperature was measured with the help of a carbon resistor soldered to the container. For calibrating the resistor we used at different temperature regions a mercury manometer, an oil manometer and a Mc Leod-gauge. The Mc Leod-gauge was connected to the cryostat head by means of glass tubing.

3. *Results*

A typical plot of the relative amplitude of successive sound reflections is shown in figure 7.

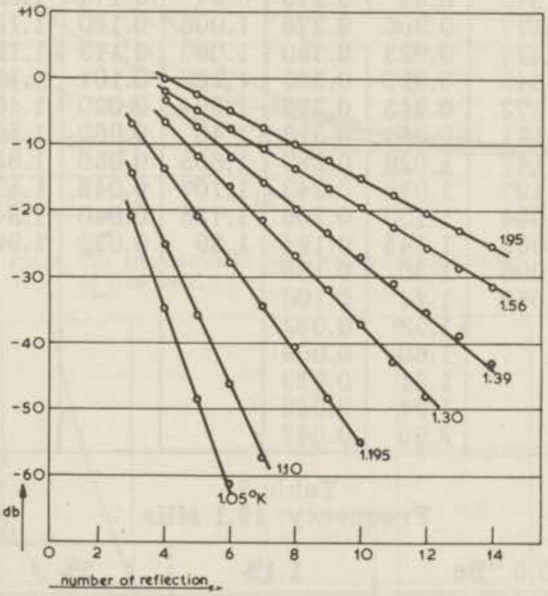

Fig. 7. Plot of the relative amplitude of the reflections at 7.26 MHz in ^{4}He.

In drawing the straight lines, the slope of which determined the apparent attenuation, we used at different temperatures the same sequence of reflections, that is to say at 7.26 MHz we used the 4th to 9th reflection and at 12.1 MHz we used the 2nd to 5th reflection.

The concentration of the mixtures followed from their λ-temperature together with the value for the shift in λ-temperature of 1.49°K per mole ^{3}He as given by Wansink [10]. We assumed that the λ-temperature coincided with the temperature for which the attenuation was largest (c.f. Sydoriak and Roberts [9]). In this way we obtained for the concentration of the mixtures used: 11 ± 0,3%, 3.0 ± 0,1% and 1.1 ± 0.1%. The 0.5% mixture was obtained by diluting the 1.1% mixture.

The results for the true attenuation are given in tables 1 and 2. They are shown in figures 8 and 9.

Table 1
Frequency: 7.26 MHz

0.5% ^{3}He		1.1%		3%		11%	
T(^{O}K)	α(cm^{-1})	T	α	T	α	T	α
0.900	0.543	0.87	0.399	0.88	0.178	0.89	0.058
0.906	0.540	0.88	0.379	0.94	0.174	1.02	0.053
0.944	0.477	0.905	0.378	1.006	0.150	1.105	0.053
0.973	0.434	0.923	0.350	1.095	0.113	1.124	0.052
1.040	0.345	0.930	0.339	1.205	0.107	1.185	0.049
1.104	0.273	0.943	0.325	1.304	0.090	1.40	0.046
1.214	0.191	0.984	0.313	1.40	0.060	1.50	0.039
1.295	0.147	1.026	0.283	1.605	0.050	1.61	0.038
1.424	0.107	1.075	0.249	1.707	0.048	1.85	0.032
1.52	0.084	1.130	0.206	1.785	0.040	1.945	0.045
1.60	0.069	1.145	0.194	1.89	0.032	1.990	0.123
1.71	0.056	1.30	0.195				
1.80	0.053	1.44	0.106				
		1.52	0.082				
		1.60	0.069				
		1.91	0.039				
		1.94	0.038				
		2.00	0.042				

Table 2
Frequency: 12.1 MHz

0.5 ^{3}He		1.1%		3%	
T(^{O}K)	α(cm^{-1})	T	α	T	α
1.013	0.967	0.88	0.915	0.875	0.466
1.06	0.833	0.90	0.873	0.90	0.446
1.106	0.709	0.95	0.780	0.95	0.394
1.153	0.563	1.075	0.588	1.00	0.378
1.214	0.468	1.145	0.495	1.10	0.317
1.25	0.407	1.25	0.379	1.20	0.277
1.262	0.381	1.50	0.173	1.255	0.229
1.30	0.334	1.70	0.108	1.31	0.231
1.35	0.274	1.99	0.111	1.397	0.157
1.40	0.221			1.50	0.157
1.50	0.173			1.60	0.140
1.60	0.119			1.62	0.112
1.70	0.092			1.705	0.104
1.80	0.080			1.852	0.094
				1.95	0.103

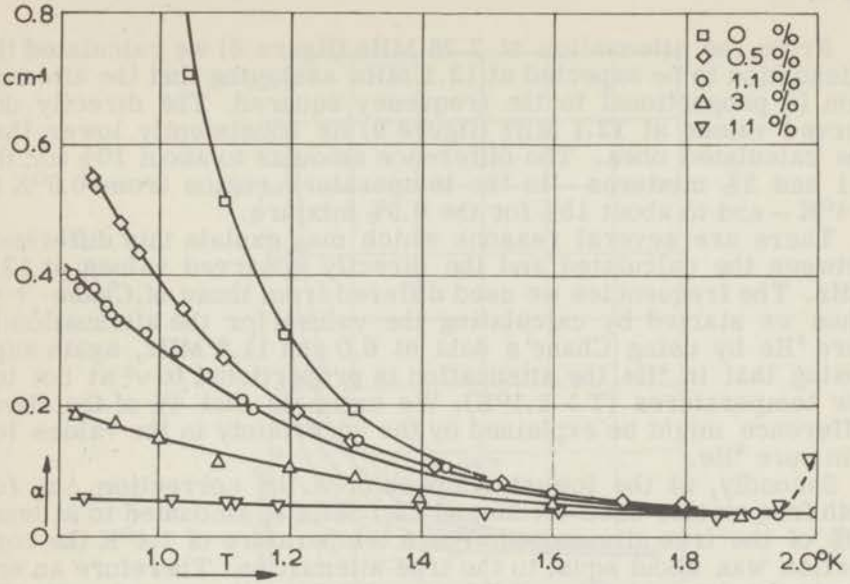

Fig. 8. The attenuation in the mixtures for a frequency of 7.26 MHz.

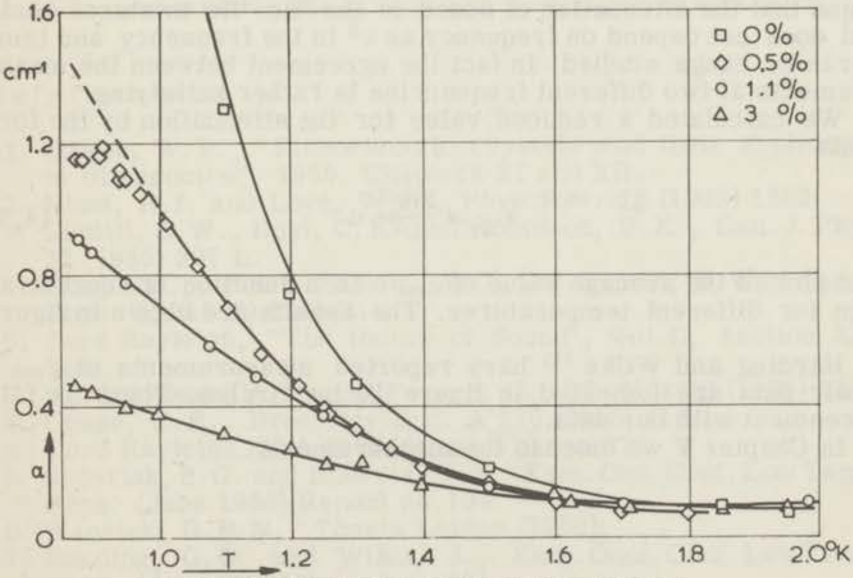

Fig. 9. The attenuation in the mixtures for a frequency of 12.1 MHz.

In extrapolating the curve for the attenuation at 12.1 MHz in the 0.5% mixture, the measurements at the lowest temperatures – around 0.9°K – have been neglected.
Since there the attenuation was large we could observe only the first reflection and severe interference made the results doubtful.

From the attenuation at 7.26 MHz (figure 8) we calculated the attenuation to be expected at 12.1 MHz assuming that the attenuation is proportional to the frequency squared. The directly observed values at 12.1 MHz (figure 9) lie consistently lower than the calculated ones. The difference amounts to about 10% for the 1.1 and 3% mixtures – in the temperature region from 0.9°K to 1.4°K – and to about 15% for the 0.5% mixture.

There are several reasons which may explain this difference between the calculated and the directly observed values at 12.1 MHz. The frequencies we used differed from those of Chase [7] *. Thus we started by calculating the values for the attenuation in pure ^{4}He by using Chase's data at 6.0 and 11.8 MHz, again supposing that in ^{4}He the attenuation is proportional to ν^2 at not too low temperatures ($T > 1.1^\circ K$). We estimate that 4% of the above difference might be explained by the uncertainty in the values for α in pure ^{4}He.

Secondly, at the lowest temperatures the correction, $\Delta\alpha$, for both frequencies used – 7.26 and 12.1 MHz –, amounted to at least 20% of the true attenuation. For a temperature of 1.4°K the correction was about equal to the true attenuation. Therefore an error in the determination of the correction easily caused a significant error in the true attenuation.

In view of what has been said we conclude that there are no signs that the attenuation of sound in the ^{3}He-^{4}He mixtures studied does not depend on frequency as ν^2 in the frequency and temperature range studied. In fact the agreement between the measurements at two different frequencies is rather satisfying.

We calculated a reduced value for the attenuation by the formula

$$\alpha_{red} = \frac{2\rho c^3}{\omega^2}\,\alpha \qquad (4,7)$$

and plotted the average value of α_{red} as a function of concentration for different temperatures. The results are shown in figure 10.

Harding and Wilks [11] have reported measurements of α_{red}. Their data are indicated in figure 10 by circles. There is fair agreement with our data.

In Chapter V we discuss the measurements.

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* Chase [7] gave 12.1 MHz as the value of his highest frequency. Whitney, who used the same crystal, later stated (ref. 8,- Chpt. III) that the resonance frequency of the crystal in reality was 11.8 MHz.

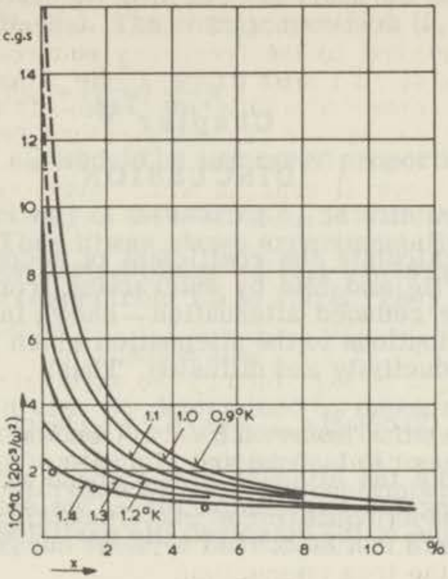

Fig. 10. Plot of the reduced attenuation, $\alpha_{red} = (2\rho c^3/\omega^2)\alpha$ as a function of the ^{3}He concentration for different temperatures. x denotes the molar concentration.

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Chapter V

DISCUSSION

We shall calculate the coefficient of second viscosity, ζ, in mixtures of ^{3}He and ^{4}He by subtracting from the experimental values for the reduced attenuation – shown in figure 10, Chapter IV – the contributions to the attenuation which result from viscosity, heat conductivity and diffusion. Thus:

$$\zeta = (2\rho c^3/\omega^2)\,\alpha_{\mathrm{exp}} - 4\eta/3 - (\gamma-1)\varkappa/c_p - \alpha_{D,\mathrm{red}} \qquad (5,1)$$

By α_D we denote the attenuation in a sound wave due to diffusion. Before deriving the expression for α_D we shall speak about the energy spectrum of the dissolved ^{3}He particles.

1. *Energy spectrum of the dissolved ^{3}He particles*

Apart from phonons and rotons in a dilute ^{3}He-^{4}He mixture there are excitations which are due to the presence of the ^{3}He atoms. For the energy spectrum of the translational motion of the dissolved particles, Pomeranchuk [1)] has proposed two logical possibilities:

$$E = E_o + p^2/2m_3^* \qquad (5,2a)$$

or

$$E = E_o + (p-p_o)^2/2m_3^* \qquad (5,2b)$$

where $\underline{p}$ is the momentum connected with the motion and m_3^* is an effective mass. The ^{3}He particles do not take part in the motion of the superfluid (Landau and Pomeranchuk [2)], Taconis et al. [3)]) but rather contribute to the normal fluid density ρ_n. For not too high concentrations [1)]

$$\rho_n = \rho_{no} + \rho_{ni} \qquad (5,3)$$

where ρ_{no} is the normal density due to the inertia of phonons and rotons, and ρ_{ni} is the contribution of the ^{3}He excitations.

In dilute solutions and for temperatures above 0.2$^{\mathrm{o}}$K the dissolved ^{3}He particles obey classical statistics (Pomeranchuk [1)]). The energy spectrum (5, 2a) gives for ρ_{ni}:

$$\rho_{ni} = \frac{\rho}{m_4}\, m_3^*\, x \qquad (5,4a)$$

where x is the molar ^{3}He concentration and m_4 is the mass of a ^{4}He atom; $x = N_3/(N_3+N_4)$. The energy spectrum (5, 2b) leads to

$$\rho_{ni} = \left(\frac{p_o^2}{3kT}\right)\frac{\rho}{m_4}x \tag{5, 4b}$$

which means that ρ_{ni} should be inversely proportional to temperature.

The most direct way of measuring ρ_{ni} is with the torsion pendulum technique. Thus it was shown experimentally by Berezniak and Esel'son [4] (see also Pellam [5]) that ρ_{ni} is independent of temperature in the region from 1.5 to 1.9°K. They found that

$$m_3^* \simeq 3m_3 \tag{5, 5}$$

The value of ρ_{ni} can also be determined by measuring the second sound velocity in mixtures [6,7,8]. The value of the effective mass obtained in this way is not quite independent of temperature but in any case does not decrease with increasing temperature. Therefore the energy spectrum (5, 2b) is certainly not the correct one. We shall use for m_3^* the value of Berezniak and Esel'son.

2. *Diffusion, viscosity and heat conductivity losses*

a) *Diffusion losses.* We shall derive the expression for diffusion losses by proceeding from the hydrodynamic equations for ^{3}He - ^{4}He mixtures. We follow Khalatnikov [9] who has derived these equations taking into account dissipative effects. In their exact form the hydrodynamic equations are very unwieldy. For sound of small amplitude we can neglect all terms quadratic in the velocities, $\underline{v}_n$ and $\underline{v}_s$, and in the gradients of pressure, p, temperature, T, and the mass concentration, c. c is defined by

$$c = N_3 m_3/(N_3 m_3 + N_4 m_4) \tag{5, 6}$$

In deriving α_D we shall neglect the terms in the hydrodynamic equations due to first and second viscosity and to heat conductivity. We make a few other introductory remarks.

The thermodynamic identity for the energy of a ^{3}He - ^{4}He mixture per cm^3, E, in a system of reference moving with the velocity of the superfluid, $\underline{v}_s$, can be written as:

$$dE = TdS + \Phi d\rho + Zdc + (\underline{v}_n - \underline{v}_s)d\underline{p} \tag{5, 7}$$

Here S is the entropy per cm^3, ρ the density and Φ the thermodynamic potential per gram of the mixture. Z is a potential for changing the concentration c of the mixture and can be expressed in terms of the chemical potentials per gram of the added particles, μ_3, and of the solvent, μ_4. Khalatnikov showed that

$$Z/\rho = \mu_3 - \mu_4 \tag{5, 8}$$

The vector $\underline{p}$ is the momentum per cm^3 of the normal fluid, again relative to the superfluid velocity $\underline{v}_s$:

$$\underline{p} = \rho_n(\underline{v}_n - \underline{v}_s) \tag{5,9}$$

The total momentum per cm^3 in a fixed coordinate system, $\underline{j}$, equals:

$$\underline{j} = \rho \underline{v}_s + \underline{p} \tag{5,10}$$

The two components do not mix perfectly – in the sense of the mixing of perfect gases – (c.f. Wansink 10)). We therefore proceed from the general expression for the chemical potentials in weak solutions (see Landau and Lifshitz 11)) which are

$$\begin{aligned} \mu_4 &= \mu_{4o} - \frac{kT}{m_4} x \\ \mu_3 &= \Psi(p, T) + \frac{kT}{m_3} \ln x \end{aligned} \tag{5,11}$$

For perfect mixing $\Psi(p, T)$ equals μ_{3o}, the chemical potential in pure ^{3}He *.

For future use we shall need the derivatives of Z/ρ. From (5,7) follows:

$$d\Phi = \frac{1}{\rho} dp - \sigma dT + \frac{Z}{\rho} dc + (\underline{v}_n - \underline{v}_s) d\underline{p} \tag{5,12}$$

where σ is the entropy per gram. Thus:

$$\frac{\partial(Z/\rho)}{\partial T} = - \frac{\partial\sigma}{\partial c}, \quad \frac{\partial(Z/\rho)}{\partial p} = \frac{\partial(1/\rho)}{\partial c} \tag{5,13}$$

The derivative $\partial(Z/\rho)/\partial c$ follows from (5, 8) and (5, 11):

$$\frac{\partial(Z/\rho)}{\partial c} = \frac{kT}{m_3 c} \tag{5,14}$$

With the simplifications mentioned above the hydrodynamic equations take the form:

---

* Khalatnikov assumes perfect mixing and makes a mistake by writing c instead of x. For weak solutions the formula of Pomeranchuk 1) for the velocity of second sound and the van 't Hoff law for the osmotic pressure do not contain the function $\Psi(p, T)$ since $\Psi(p, T)$ can be neglected with respect to ln x. With the specific heat it is otherwise, however, since there (5, 11) is differentiated twice with respect to temperature and the term with ln x drops out leaving $\partial^2\Psi/\partial T^2$.

$$\begin{gathered}\dot{\rho} + \operatorname{div} \underline{j} = 0 \\ \frac{\partial \underline{j}}{\partial t} + \nabla p = 0 \\ \frac{\partial}{\partial t}(\rho c) + \rho c \operatorname{div} \underline{v}_n = - \operatorname{div} \underline{g} \\ \frac{\partial \underline{v}_s}{\partial t} + \nabla(\Phi - \frac{Z}{\rho} c) = 0 \\ \dot{S} + S \operatorname{div} \underline{v}_n = [\frac{\partial}{\partial T} (\frac{Z}{\rho})_{p,c} - \frac{k_T}{T} \frac{\partial}{\partial c} (\frac{Z}{\rho})_{p,T}] \operatorname{div} \underline{g}\end{gathered} \qquad (5,15)$$

The vector $\underline{g}$ denotes the diffusion flow of the added particles. Thus:

$$\underline{g} = -\rho D(\nabla c + \frac{k_T}{T} \nabla T + \frac{k_p}{p} \nabla p) \qquad (5,16)$$

where D is the coefficient of diffusion and $k_T D$ is the coefficient of thermal diffusion. The quantity $k_p D$ is called the coefficient of barodiffusion.

The thermal diffusion ratio, k_T, has been calculated by Khalatnikov and Zharkov [12]. The ratio k_T is always smaller than the concentration c and it can be shown that the terms in (5, 15) containing k_T can be neglected.

For the barodiffusion ratio, k_p, Khalatnikov [9] derived the expression:

$$k_p = p \frac{\frac{\partial}{\partial p} (Z/\rho T)}{\frac{\partial}{\partial c} (Z/\rho T)} \qquad (5,17)$$

In ordinary sound the barodiffusion term in $\underline{g}$ cannot be neglected. It can be shown that in an ordinary sound wave

$$\frac{k_p}{p} \nabla p \simeq \frac{\rho_n}{\rho_s} \frac{u_I^2}{c^2 \frac{\partial}{\partial c} (Z/\rho)} \nabla c \qquad (5,18)$$

where u_I is the velocity of ordinary sound. In deriving (5, 18) we neglected temperature fluctuations * and dissipative effects.

For temperatures below 1.2°K and for concentrations between about 1 and 10%: $\rho_n/\rho_s \simeq \rho_{ni}/\rho$ and thus from (5, 4a) and (5, 5): $\rho_n/\rho_s \simeq 3c$. From (5, 14) and after insertion of numerical values (5, 18) becomes

$$\frac{k_p}{p} \nabla p \simeq \frac{60}{T} \nabla c \qquad (5,19)$$

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* The expansion coefficient in mixtures below T_λ is small (Ptukha [13]). Thus the ratio of the specific heats is almost 1 and ordinary sound is propagated almost isothermally.

Thus in ordinary sound the barodiffusion term in the diffusion flow $\underline{g}$ is overwhelmingly large and we neglect the term ∇c. The hydrodynamic equations, after elimination of $\underline{v}_n$ and $\underline{v}_s$, become

$$\ddot{\rho} = \Delta p$$

$$\frac{\rho_n}{\rho_s}\frac{\ddot{\sigma}}{\sigma} = \sigma \Delta T + c\Delta\left(\frac{Z}{\rho}\right) - \frac{\rho_n}{\rho_s \sigma} D \frac{k_p}{p} \frac{\partial}{\partial T}\left(\frac{Z}{\rho}\right) \Delta \dot{p} \qquad (5,20)$$

$$\frac{\ddot{c}}{c} = \frac{\ddot{\sigma}}{\sigma} + D \frac{k_p}{p}\left(\frac{1}{c} + \frac{1}{\sigma}\frac{\partial}{\partial T}\frac{Z}{\rho}\right)\Delta \dot{p}$$

where the dash means differentiation with respect to time. Using the relation $\partial(Z/\rho)/\partial T = \partial\sigma/\partial c$ from (5, 13) and denoting

$$\sigma - c\frac{\partial\sigma}{\partial c} \quad \text{by} \quad \bar{\sigma}$$

$$-D\frac{k_p}{p}\frac{\partial\sigma}{\partial c} \quad \text{by} \quad a \qquad (5,21)$$

$$\frac{D}{c\sigma}\frac{k_p}{p}\bar{\sigma} \quad \text{by} \quad b,$$

the equations (5, 20) can be rewritten

$$\ddot{\rho} = \Delta p$$

$$\frac{\rho_n}{\rho_s}\frac{\ddot{\sigma}}{\sigma} = \sigma \Delta T + c\Delta\left(\frac{Z}{\rho}\right) + \frac{\rho_n}{\rho_s c} a \Delta \dot{p} \qquad (5,22)$$

$$\frac{\ddot{c}}{c} = \frac{\ddot{\sigma}}{\sigma} + b\Delta \dot{p}$$

We choose as independent variables p, T and c and put the deviations from their average values proportional to $\exp[i(\omega t - kx)]$. In the usual way there results a determinantal equation for the velocity, u, of a sound wave in which now dissipative effects due to diffusion have been taken into account. In that equation we neglect terms with $\partial\rho/\partial T$ and we split the determinant in two terms. There results:

$$u^2 \begin{vmatrix} \frac{\partial\rho}{\partial p}u^2 - 1 & 0 & \frac{\partial\rho}{\partial c}u^2 \\ 0 & c\frac{\partial\sigma}{\partial T} & -\bar{\sigma} \\ \frac{c}{\rho^2}\frac{\partial c}{\partial\rho} & \frac{\rho_n}{\rho_s\sigma}\frac{\partial\sigma}{\partial T}u^2 - \bar{\sigma} & \frac{\rho_n}{\rho_s\sigma}\frac{\partial\sigma}{\partial c}u^2 - \frac{kT}{m_3} \end{vmatrix} +$$

$$+ i\omega \frac{\partial\rho}{\partial c} u^2 \begin{vmatrix} 0 & 0 & 1 \\ c\sigma b & c\frac{\partial\sigma}{\partial T} u^2 & -\bar{\sigma} u^2 \\ -\frac{\rho_n}{\rho_s\sigma} a & \frac{\rho_n}{\rho_s\sigma}\frac{\partial\sigma}{\partial T} u^2 - \bar{c} & \frac{\rho_n}{\rho_s\sigma}\frac{\partial\sigma}{\partial c} u^2 - \frac{kT}{m_3} \end{vmatrix} = 0 \qquad (5,23)$$

The first determinant, when put equal to zero, gives the velocities of first and second sound, u_{Io} and u_{IIo}, when no dissipative effects are taken into account. Since $u^2_{IIo} \ll u^2_{Io}$ the solutions of that determinantal equation:

$$\frac{\partial\rho}{\partial p}\frac{\partial\sigma}{\partial T}\frac{\rho_n}{\rho_s}\Big\{u_o^4 - u_o^2\,[\frac{\rho_s}{\rho_n}(c^2(\partial T/\partial \bar{c}) + \frac{kTc}{m_3}) + \frac{\partial p}{\partial\rho}(1 + \frac{\rho_s}{\rho_n}(\frac{c}{\rho}\frac{\partial\rho}{\partial c})^2)] +$$

$$+ \frac{\rho_s}{\rho_n}[\bar{\sigma}^2(\partial T/\partial\sigma) + \frac{kTc}{m_3}]\frac{\partial p}{\partial\rho}\Big\} = 0 \qquad (5,24)$$

are, to a good approximation, equal to [9]:

$$u^2_{Io} = (\frac{\partial p}{\partial\rho})_{c,T}\;[1 + \frac{\rho_s}{\rho_n}(\frac{c}{\rho}\frac{\partial\rho}{\partial c})^2]$$

$$u^2_{IIo} = \frac{\rho_s}{\rho_n}\,[\sigma^2(\frac{\partial T}{\partial\sigma})_{c,p} + \frac{kTc}{m_3}]/[1 + \frac{\rho_s}{\rho_n}(\frac{c}{\rho}\frac{\partial\rho}{\partial c})^2] \qquad (5,25)$$

The velocity of second sound, u_{IIo}, in mixtures was first derived by Pomeranchuk [1].

To solve equation (5, 23) for the velocity of ordinary sound, but now including diffusion losses, we put:

$$u_I^2 = u^2_{Io} + i\frac{\omega}{\rho}\alpha_{red,D} \qquad (5,26)$$

where $\alpha_{red,D}$ is the diffusion reduced attenuation. We neglect terms quadratic in α^2 and make use of the equation (5, 24) for μ_{Io}. After some calculation we get the equation for $\alpha_{red,D}$

$$\frac{\partial\sigma}{\partial T}\frac{\rho_n}{\rho_s}\alpha_{red,D} + \rho\frac{\partial\rho}{\partial c}\,[\frac{\rho_n}{\rho_s}\frac{\partial\sigma}{\partial T} u^2_{Io}\; c(\frac{a}{\sigma} + b) - c\,b\,\sigma\,\bar{\sigma}] = 0 \qquad (5,27)$$

From (5, 21):

$$c(\frac{a}{\sigma} + b) = D\frac{k_p}{p} \quad \text{and} \quad c\,b\,\sigma\,\bar{\sigma} = D\frac{k_p}{p}\bar{\sigma}^2 \qquad (5,28)$$

Thus:

$$\frac{\partial\sigma}{\partial T}\frac{\rho_n}{\rho_s}\alpha_{red,D} + \rho\frac{\partial\rho}{\partial c}D\frac{k_p}{p}[\frac{\rho_n}{\rho_s}\frac{\partial\sigma}{\partial T}u^2_{Io} - \bar{\sigma}^2] = 0 \qquad (5,29)$$

The term $(\rho_n/\rho_s)\partial\sigma/\partial T\ u_{Io}^2$ is an order of magnitude larger than $\bar{\sigma}^2$. Therefore:

$$\alpha_{red,D} = -\rho \frac{\partial\rho}{\partial c} D \frac{k_p}{p} u_{Io}^2 \qquad (5,30)$$

We drop the subscript o in u_{Io}. From (5,17), (5,13) and (5,14) finally:

$$\alpha_{red,D} = \frac{cD}{\rho} \left(\frac{\partial\rho}{\partial c}\right)^2 \frac{m_3}{kT} u_I^2 \qquad (5,31)$$

The derivative $\partial\rho/\partial c$ can be calculated from the measurements of Ptukha [13]: $\partial\rho/\partial c \simeq 0.4\,\rho$.

The coefficient of diffusion, D, was measured by Beenakker et al. [14] in mixtures of molar concentration 0.6 to 2.8 10^{-4}. The diffusion coefficient has been calculated theoretically by Zharkov and Khalatnikov [15] and Khalatnikov and Zharkov [12]. They have shown that for concentrations below 10^{-2}, and temperatures above 1.2°K, the diffusion coefficient equals

$$D = t_{ir} \frac{kT}{m_3} \qquad (5,32)$$

where t_{ir} is of the order of the average collision time between rotons and impurity atoms. From D_{exp} (Beenakker et al.) Khalatnikov and Zharkov calculated a value for t_{ir} and from this a value for the interaction energy between a roton and a ^{3}He atom. However, they did not use the correct value of D_{exp} (see reference 14). Therefore their final expression which they obtained for D, valid for all concentrations and temperatures above 0.6°K, is changed slightly in the numerical factor. D_{theor} then becomes:

$$D_{theor} = 4.8\ 10^{-6} \exp(\Delta/kT)\ (\rho_{no}/\rho_n)^2 \text{ c.g.s.} \qquad (5,33)$$

where Δ is the roton minimum energy.

With decreasing temperature D rises exponentially as long as $\rho_{ni} \ll \rho_{no}$ or as long as the ^{3}He atoms are scattered mainly by rotons. When ρ_{ni} becomes comparable with ρ_{no} the value of D goes through a maximum.

With the spin echo technique the diffusion coefficient was measured in a 2% mixture by Garwin and Reich [16]. Their values are a factor 4 to 8 smaller than the theoretical values (5,33) in the temperature region from 1.3 to 1.6°K. They have reported, however, that there are indications for clustering of the ^{3}He atoms.

There are not enough measurements of D for an accurate calculation of the diffusion losses. We used the theoretical values of D to estimate the losses due to diffusion:

$$\alpha_{red,D} \simeq 2\ 10^{-6}\, c\, \frac{\exp(\Delta/kT)}{T} \left(\frac{\rho_{no}}{\rho_n}\right)^2 \qquad (5,34)$$

The estimated values of $\alpha_{\text{red},D}$ at two temperatures, 0.9° and 1.4°K, are given in table I, from which it may be seen that the diffusion losses are small.

Table I

Estimated values for $\alpha_{\text{red},D} \times 10^6$

^{3}He concentration (x)	0.9°K	1.4°K
0.5%	11	3
1.1	7	5
3	2.5	5
11	0.7	3

b) *Viscosity losses.* The reduced attenuation equals $4\eta/3$. There do not seem to be reliable measurements of the viscosity coefficient of ^{3}He - ^{4}He mixtures. Wansink [10)] determined η in a flow experiment using an annular slit of 0.33 μ width. His method depends on the existence of an osmotic pressure and hence cannot be used for pure ^{4}He, which makes it difficult to judge the reliability of his results. In pure ^{4}He in narrow slits a too small value for η is found. Dash and Taylor [17)] obtained values for η using the torsion pendulum technique in mixtures of about 4.7 and 11%. Their value for η in pure ^{4}He at 1.3°K, however, lies a factor of 2 higher than the values obtained from the surface damping of second sound or with the help of the rotating viscometer.

Zharkov [18)] has given theoretical values for η in mixtures at temperatures above 0.6°K and mass concentrations, c, below 1%.

We estimate that in a mixture of 1.1% molar concentration from 0.9 to 1.4°K the reduced viscosity attenuation amounts to about 7% of the total reduced attenuation. In a 11% mixture this relative contribution may be 10%.

We note here that by adding a small amount of ^{3}He to He II the mean free path length of the ^{4}He excitations is decreased strongly. We assume that the mean free path length of the phonons in our mixtures even at 0.9°K was much smaller than the wavelength of 10 MHz sound (see Chapter II). This meant that the average collision time was much smaller than the period of the experimentally generated sound and that the viscous attenuation in the mixtures was equal to $(\omega^2/2\rho c^3)4\eta/3$ over the whole temperature region studied.

c) *Heat conductivity losses.* The reduced attenuation due to heat conductivity equals $(\gamma-1)\varkappa/c_p$ which can be written as

$$(\gamma-1)\,\varkappa/c_p = \frac{T\,u_I^2\,\alpha_p^2}{c_p^2}\,\varkappa \qquad (5,35)$$

where α_p denotes the expansion coefficient at constant pressure.

Theoretical values for $\varkappa$ have been given by Khalarnikov and Zharkov [12]. We shall estimate the contribution of heat conductivity to α_{red} in two extreme cases:

1) 11% mixture, $T = 1.4^{\circ}K$.

We put $\alpha_p \simeq \alpha_{sat}$. From Ptukha s measurements [13] we calculate $\alpha_p \simeq -4.5\ 10^{-3}$ c.g.s. Kapadnis [19] has measured the specific heat of a 7% mixture. Extrapolating his data we find $c_p \simeq c_{sat} \simeq 1.1\ 10^7$ erg/gm °K. From reference 12, the thermal conductivity $\varkappa$ is less than 10^4 c.g.s. Thus we find:

$(\gamma-1)\varkappa/c_p < 10^{-7}$ c.g.s. which is negligible compared with the total reduced attenuation of about 10^{-4} (figure 10, Chapter IV).

2) 0.5% mixture, $T = 0.9^{\circ}K$.

Here the values, of α_p and c_p are not known, the coefficient $\varkappa$ is of the order 3×10^4. A rough estimate shows that a considerable fraction, 10% or more, of the total attenuation might be due to heat conductivity losses.

3. *Conclusion*

We estimate that in the mixtures studied the added contributions of viscosity, heat conductivity and diffusion amount to not more than 20% of the total observed attenuation at all temperatures from 0.9 to 1.4°K. The remaining 80% must be considered as relaxation losses. From our measurements, however, it is obvious that these relaxation losses are strongly diminished by the addition of ^3He to He II. It is natural to suppose that the ^3He atoms strongly influence the relaxation times connected with the creation or annihilation of phonons and rotons and that the relaxation times in mixtures are much shorter than in pure ^4He.

In Khalatnikov's theory the number of ^4He excitations can be changed in inelastic collisions between phonons (5 phonon process) or between phonons and rotons (phonon-roton process). We computed the relative contribution to the attenuation of ordinary sound in pure ^4He due either to the slowness of the 5-phonon process or to the slowness of the phonon-roton process. The results are tabulated in table II.

Table II

Contribution to the reduced attenuation (pure ^4He)

T(°K)	PhPh term (μP)	PhR term (μP)
1.2	470	80
1.4	150	50
1.6	50	30

From table II it may be seen that in pure ^4He the PhPh term increases rapidly with decreasing temperature and that it becomes the most important term. We therefore suppose that in a mixture, through the presence of ^{3}He atoms, the probability of creating or annihilating phonons is enlarged greatly.

In a mixture there are several new processes possible which cannot occur in pure ^{4}He and in which the number of phonons is changed. And if we knew how the probability of occurrence of those inelastic processes depends on temperature and ^{3}He concentration, then it might be possible to give a quantitative explanation of the observed influence of ^{3}He on the attenuation.

But first we must be sure that the energy spectrum from which we start is the correct one.

As is known, the Pomeranchuk type ^{3}He excitations:

$$E = E_o + (p^2)/2m_3^* \qquad (5,2a)$$

give a contribution to the specific heat of (3/2) N_3k, and this is too small a value to account for the observed increase in the specific heat of a mixture relative to pure ^{4}He (Kapadnis 19), Dokoupil 21)).

Khalatnikov and Zharkov 12) have calculated the interaction energy between rotons and ^{3}He excitations. The number of rotons per cm^3 depends on temperature, and thus there is a contribution to the specific heat of a mixture. From their calculations it follows that the interaction energy per cm^3 between rotons and ^{3}He excitations equals:

$$V_{int} = N_3 N_r V_{ir} \qquad (5,36)$$

where N_3 and N_r are the numbers of ^{3}He atoms and rotons per cm^3, respectively. The interaction energy between a roton and a ^{3}He excitation follows from the experimental data on the coefficient of diffusion (Beenakker et al. 14)). Khalatnikov and Zharkov give for the value of V_{ir} :

$$V_{ir} \simeq 8 \mathrm{x} 10^{-38} \text{ erg cm}^3 \qquad (5,37)$$

The contribution to the specific heat, due to the temperature-dependent interaction energy (5, 36) is equal to:

$$N_3 V_{ir} \frac{\partial}{\partial T} N_r \simeq N \frac{\Delta}{T^2} N_r V_{ir} \qquad (5,38)$$

which amounts, at 1.1^{O}K in a 1% mixture, to only about 0.7x10^4 erg/g ^{O}K whereas the observed increase in specific heat equals about 1.8x10^5 erg/g ^{O}K (Kapadnis 19)).

It does not seem probable that the ^{3}He excitations have such a relatively large interaction with phonons that in this way the difference in specific heat between a mixture and pure ^{4}He could be explained. Khalatnikov and Zharkov 12) make rather vague assumptions for the interaction between a phonon and a ^{3}He excitation. We will not relate them here.

Thus the energy spectrum of the ^{3}He excitations together with the ^{4}He phonons and rotons does not seem to describe the thermal properties of a mixture correctly, even not if one takes into account possible interactions between the excitations. In fact, when the interaction becomes too strong, it becomes impossible to describe the liquid as an aggregate of elementary excitations.

Qualitatively, the concept of a phonon being a pure ^{4}He excitation seems to be rather hazardous for mixtures of not very low concentration. We have seen in Chapter II that phonons, being collective excitations, extend over a region in space of many atomic dimensions. If the ^{3}He atoms are evenly distributed in the mixture – which might not be the case – then there are many ^{3}He atoms included in the volume "occupied" by a phonon.

It is not difficult to imagine processes in which the number of phonons changes by one. For instance a 3 phonon process becomes possible in which a phonon collides with a ^{3}He excitation giving rise, after the collision, to two phonons. Or when two ^{3}He excitations collide, a phonon may be emitted (Khalatnikov and Zharkov [12]).

However, as we have said, a calculation of the temperature-dependence of the scattering cross-sections for one of these processes is extremely uncertain as long as one proceeds from an energy spectrum which does not agree with thermal data.

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SAMENVATTING

In dit proefschrift worden de resultaten beschreven van metingen over de absorptie van geluid in mengsels van ^{4}He en maximaal 11% ^{3}He. Het temperatuurgebied, waarin de absorptie werd gemeten, strekte zich uit van 0.9 tot 2.0°K. In zuiver ^{4}He neemt in dat temperatuurgebied de absorptie van geluid sterk toe wanneer de temperatuur lager wordt en rond 1°K treedt een maximum in de absorptie op.

Een verklaring voor dit verschijnsel werd gegeven door Khalatnikov. Hij toonde aan dat grote verliezen optreden ten gevolge van een relaxatieverschijnsel, hetwelk hierin bestaat dat de omzetting van normaal in superfluide helium – wat in de geluidsgolf geschiedt omdat de hoeveelheid normaal helium van de dichtheid afhangt – traag verloopt met betrekking tot de periode van het geluid. Khalatnikov slaagde erin de afhankelijkheid van de temperatuur van de met deze omzetting verbonden relaxatietijden uit te rekenen. Daarbij ging hij uit van de theorie van Landau voor He II, die zegt dat de thermische energie van He II opgevat dient te worden als een som van energieën van elementaire excitaties, phononen en rotonen genaamd. Khalatnikov rekende de waarschijnlijkheid uit voor inelastische botsingsprocessen tussen phononen en rotonen waarbij hun aantal met één verandert.

Het was te verwachten dat de toevoeging van een weinig ^{3}He aan He II de genoemde relaxatietijden sterk zou verkorten. Wij bepaalden de afhankelijkheid van de absorptie van de concentratie van het ^{3}He in het mengsel. In het algemeen neemt de absorptie af bij toevoeging van ^{3}He.

Het lijkt ons toe dat het moeilijk zal zijn de theorie van Khalatnikov uit te breiden tot mengsels van ^{3}He en ^{4}He. De thermische energie van een mengsel is misschien niet op zulk een eenvoudige wijze te beschrijven als die van zuiver ^{4}He in de theorie van Landau. Pomeranchuk heeft weliswaar een voorstel gedaan voor de afhankelijkheid van de translatorische energie van een ^{3}He atoom, in een mengsel, van de impuls, maar deze excitaties leveren een te kleine bijdrage tot de soortelijke warmte om de experimenteel gevonden soortelijke warmte te kunnen verklaren. De interacties in een mengsel tussen de phononen en rotonen enerzijds, en de ^{3}He excitaties anderzijds, spelen waarschijnlijk een grote rol.

STELLINGEN

1. De interpretatie die Dransfeld, Newell en Wilks geven van de theorie van Khalatnikov voor de relaxatieabsorptie van geluid in He II is onjuist. In het bijzonder vatten zij de in de theorie optredende relaxatietijden verkeerd op.
   Dransfeld, K., Newell, J.A. en Wilks, J., Proc.Roy.Soc. A 243 (1958) 500.

2. Uit de resultaten van zijn experimenten met "second sound" heeft Kramers de gevolgtrekking gemaakt dat, beneden 0.5°K, de gemiddelde vrije weglengte van phononen kleiner is dan die welke de viscositeit karakteriseert. Dit resultaat is waarschijnlijk in overeenstemming met berekeningen van Landau en Khalatnikov voor de gemiddelde vrije weglengte van phononen die onder kleine hoeken botsen.
   Kramers, H.C., Proefschrift, Leiden, 1955.
   Landau, L.D. en Khalatnikov, I.M., Zh.eksper.teor.Fiz. 19 (1949) 637, 709.

3. De formule voor de snelheid van "second sound" in mengsels van ^{3}He en ^{4}He kan worden afgeleid uitgaande van de veronderstelling dat het ^{3}He met het normale fluidum meebeweegt, maar zonder gebruik te maken van het excitatiespectrum van Pomeranchuk.
   Pomeranchuk, I.Ia., Zh.eksper.teor.Fiz. 19 (1949) 42.

4. Ondanks het feit dat de menging van ^{3}He en ^{4}He verre van ideaal is, is het niet verwonderlijk dat de wet van van 't Hoff, voor de osmotische druk van ^{3}He-^{4}He mengsels, met zo grote nauwkeurigheid op blijft gaan tot vrij hoge ^{3}He concentraties.
   Wansink, D.H.N., Proefschrift, Leiden, 1957, Hoofdstuk II.
   Landaul, L.D. en Lifshitz, E.M., "Statistical Physics", Hoofdstuk IX.

5. Bij het maken van dunne laagjes dienen meer voorzorgen in acht te worden genomen dan tot dusverre is geschied.

6. De numerieke uitwerking door Bolton en Wild van de door hen voorgestelde methode, om een galactisch magneetveld te meten met behulp van de Zeemansplitsing van de 21 cm lijn, is onjuist
   Bolton, J.G. en Wild, J.P., Ap.J. 125 (1957) 296.

7. De mening van Rowlinson dat vloeibare zuurstof gerekend zou moeten worden tot de niet-samengestelde vloeistoffen is onjuist.
Rowlinson, J. S., Disc. Far. Soc. 15 (1955) 52.
Perrier, A. en Kamerlingh Onnes, H., Leiden, Comm. 139d.
Wiersma, E. C. en Gorter, C. J., Physica 12 (1932) 316, Comm. Suppl. 73c.

8. De verwaarlozing, door Smith en Symons, van het effect van "ion-pairing" op de overgangsenergie E_{max} behorend bij de eerste electronenabsorptieband van het J^--ion in oplossing, is onvoldoende gemotiveerd en waarschijnlijk niet toelaatbaar.
Smyth, M. en Symons, M. C. R., Trans. Far. Soc. 54 (1958) 338.

9. Vinen heeft het ontstaan, de groei en de afbraak van wervels in het superfluide helium bestudeerd. Uit zijn experimenten volgt niet dat het ontstaan van wervels een gevolg zou zijn van een relatieve snelheid tussen de twee fluida.
Vinen, W. F., Proc. Roy. Soc., A 240 (1957) 114, 128; A 242 (1957) 493; A 243 (1957) 400.

10. Om een gunstige signaal-ruis verhouding te bereiken is het soms beter in de ingangskring van een middenfrequentversterker de spoel af te stemmen met een schroevendraaier dan om gebruik te maken van een ferrox-cube kerntje.