Stringing together the quantum phases of matter

Lorentz Lectures, Leiden
May 7, 14, 21, June 4, 2012

Subir Sachdev

See also lecture at the 2011 Solvay conference,
Theory of the Quantum World, chair D.J. Gross.
100th anniversary of the first Solvay conference,
Radiation and the Quanta, chair H.A. Lorentz.
arXiv:1203.4565

Talk online at sachdev.physics.harvard.edu

PHYSICS
HARVARD

Sommerfeld-Bloch theory of
metals, insulators, and superconductors:
many-electron quantum states are adiabatically
connected to independent electron states

Modern phases of quantum matter

Not adiabatically connected
to independent electron states:

Modern phases of quantum matter

Not adiabatically connected to independent electron states:

many-particle, long-range quantum entanglement

States of quantum matter with long-range entanglement in *d* spatial dimensions

Useful classification is provided by
nature of excitations with vanishing energy:

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1. Gapped systems without zero energy excitations

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2. "Relativistic" systems with zero energy excitations at isolated points in momentum space

States of quantum matter with long-range entanglement in *d* spatial dimensions

Useful classification is provided by
nature of excitations with vanishing energy:

1. Gapped systems without zero energy excitations

2. "Relativistic" systems with zero energy excitations at isolated points in momentum space

3. "Compressible" systems with zero energy excitations on *d-1* dimensional surfaces in momentum space.

States of quantum matter with long-range entanglement in *d* spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

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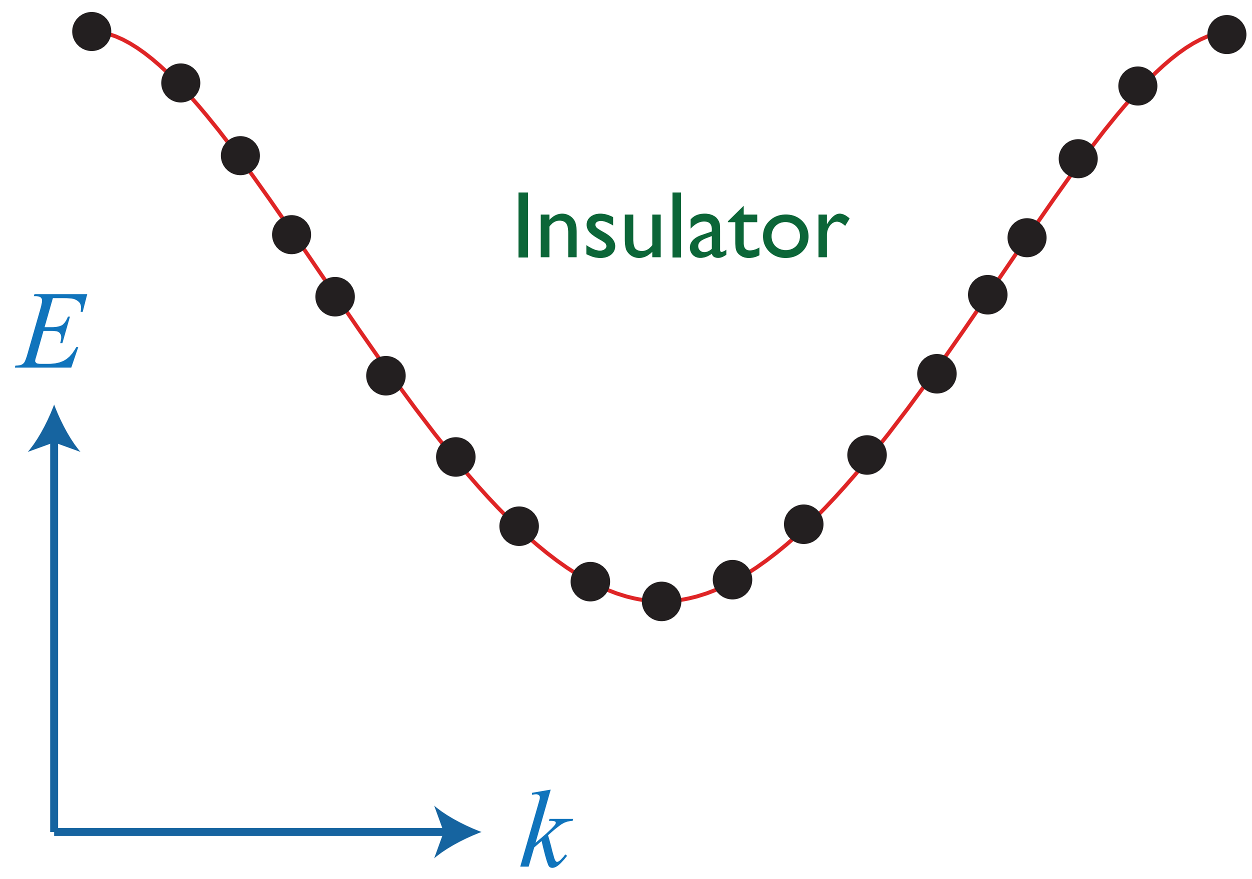
Graphene, strange metals in high temperature superconductors, spin liquids

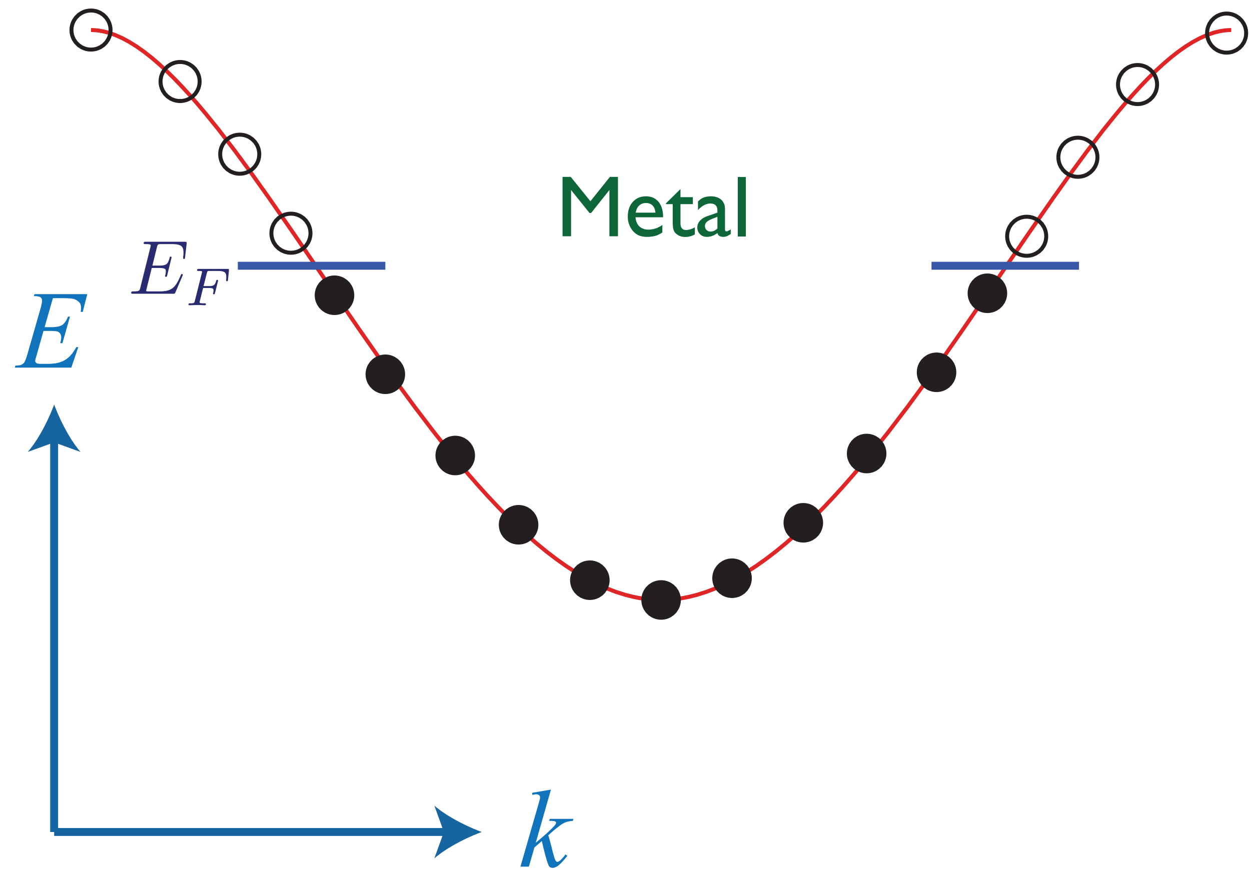
Gapped quantum matter

Band insulators



An even number of electrons per unit cell

Metals



An odd number of electrons per unit cell

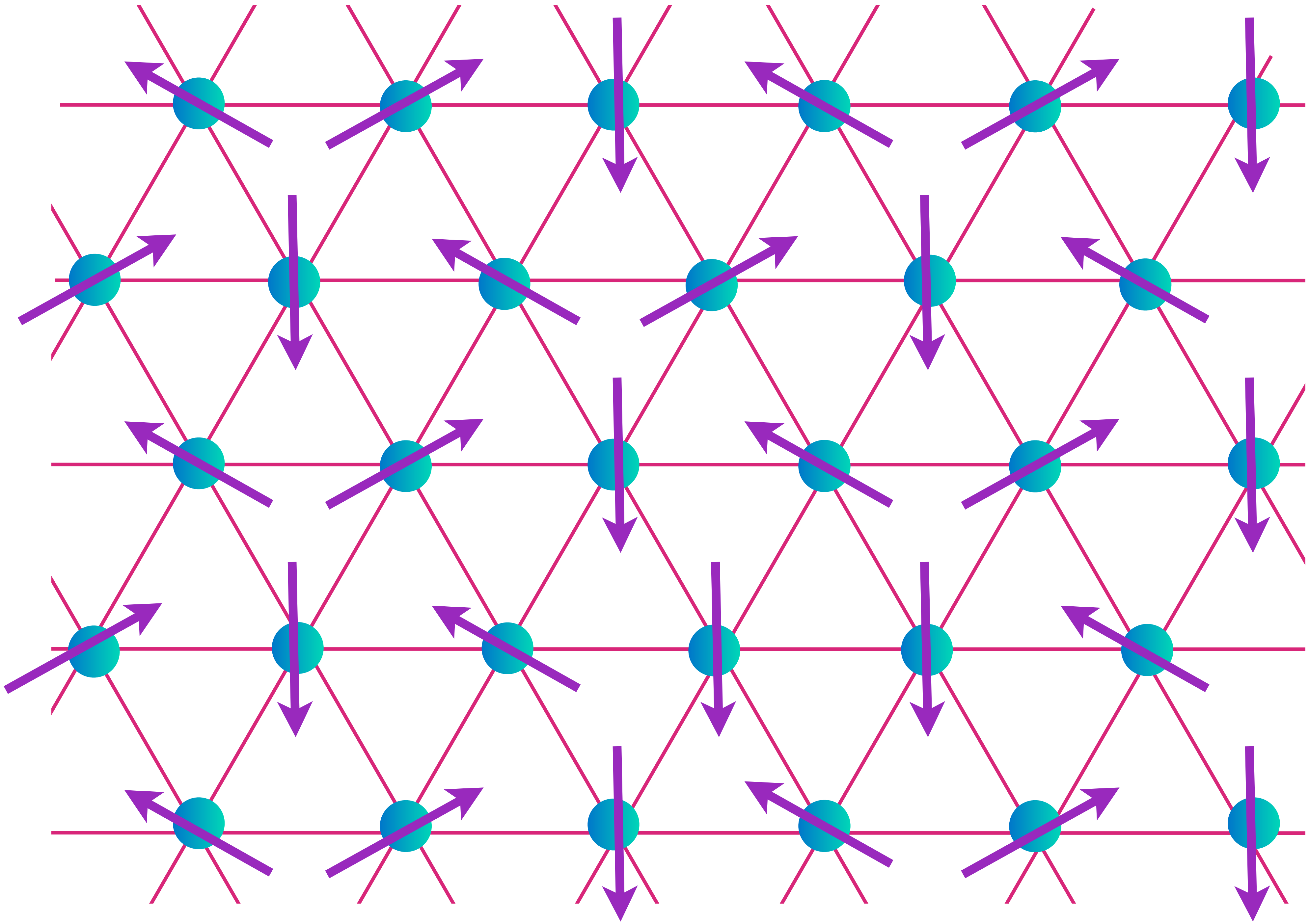
Mott insulator

Emergent excitations

An odd number of electrons per unit cell
but electrons are localized by Coulomb repulsion;
state has long-range entanglement

Mott insulator: Triangular lattice antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$



Nearest-neighbor model has non-collinear Neel order

Mott insulator: Triangular lattice antiferromagnet

$$H = J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

Imagine quantum fluctuations are so strong that the Neel order does not have long-range correlations.

Naive "classical" picture: we obtain a quantum *disordered* state in which all spin-spin correlations decay exponentially over a short length scale.

Mott insulator: Triangular lattice antiferromagnet

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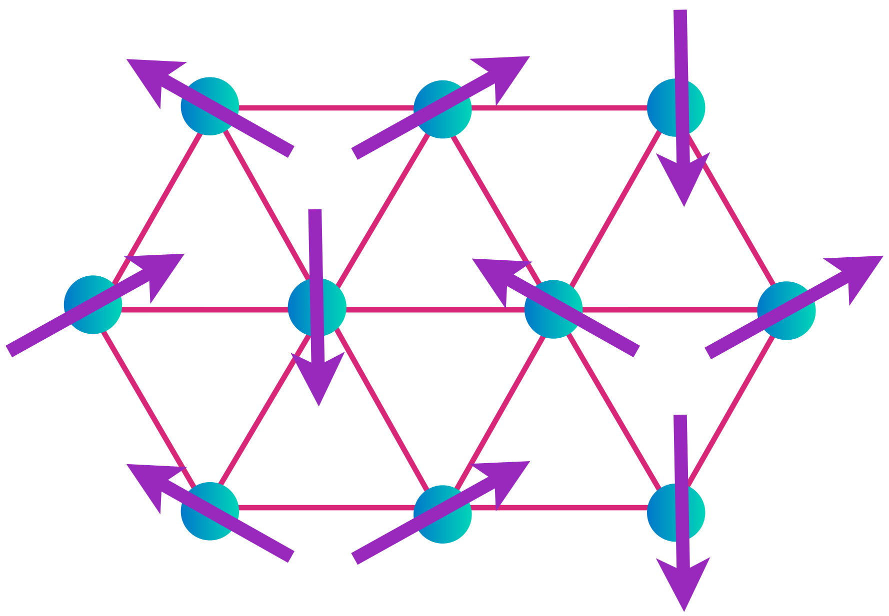
Modern "quantum" understanding: the discrete quantum degrees of freedom require a state with *long-range* entanglement.

Mott insulator: Triangular lattice antiferromagnet



Z_2 spin liquid
with neutral $S = 1/2$ spinons
and **vison** excitations

non-collinear Néel state

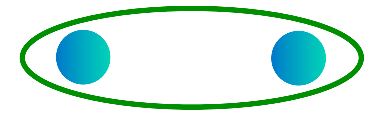
s_c

s

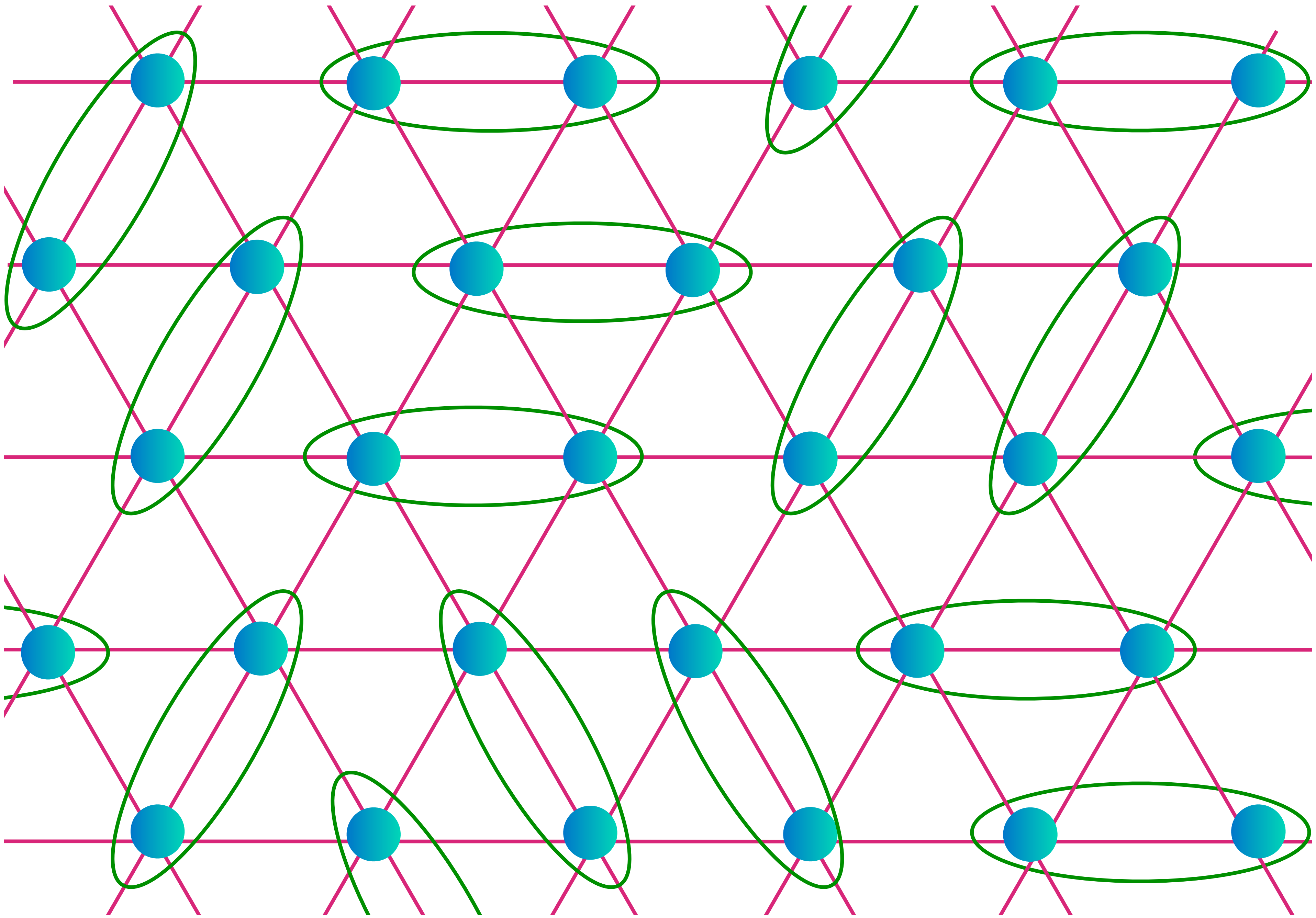
N. Read and S. Sachdev, *Phys. Rev. Lett*. **66**, 1773 (1991)
X.-G. Wen, *Phys. Rev. B* **44**, 2664 (1991)

Mott insulator: Triangular lattice antiferromagnet

Spin liquid obtained in a generalized spin model with $S=1/2$ per unit cell



$$= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$$



P. Fazekas and P. W. Anderson, *Philos. Mag.* **30**, 23 (1974).

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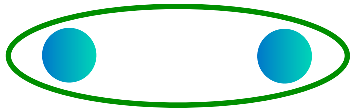

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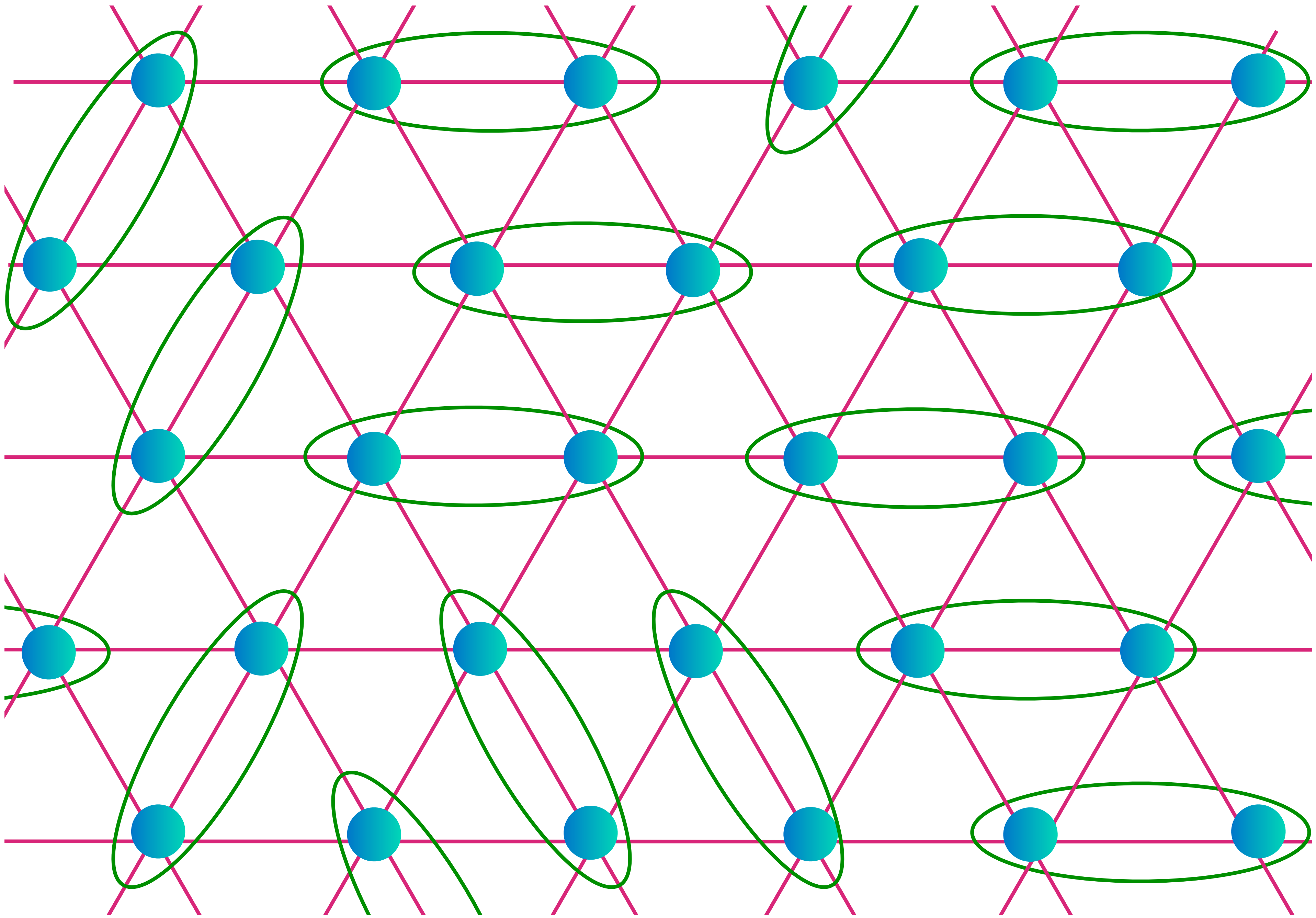

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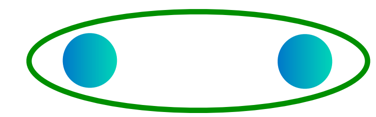

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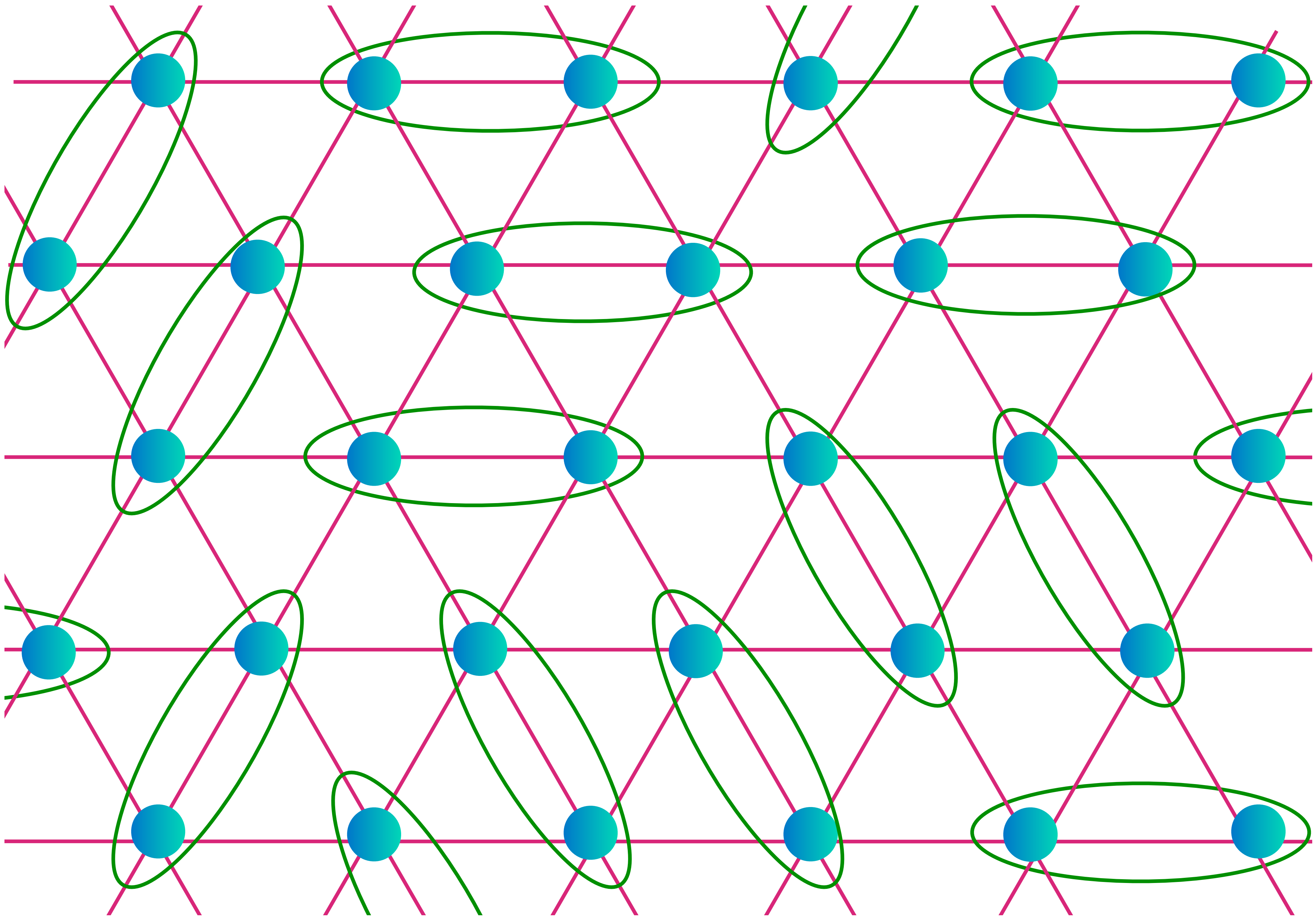

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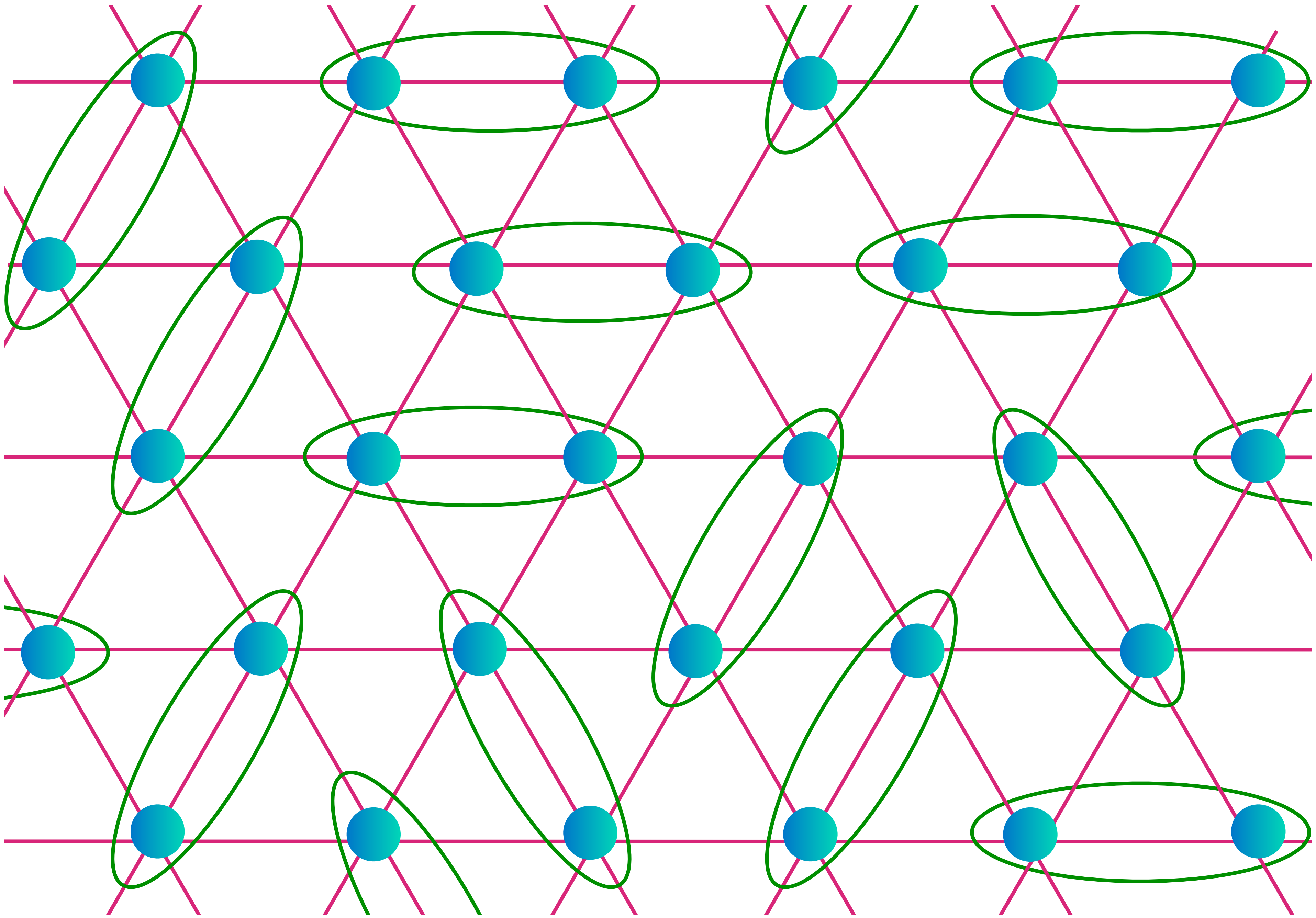

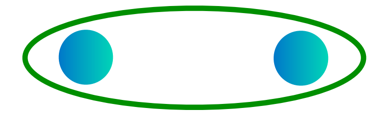

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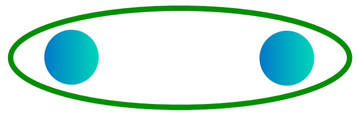

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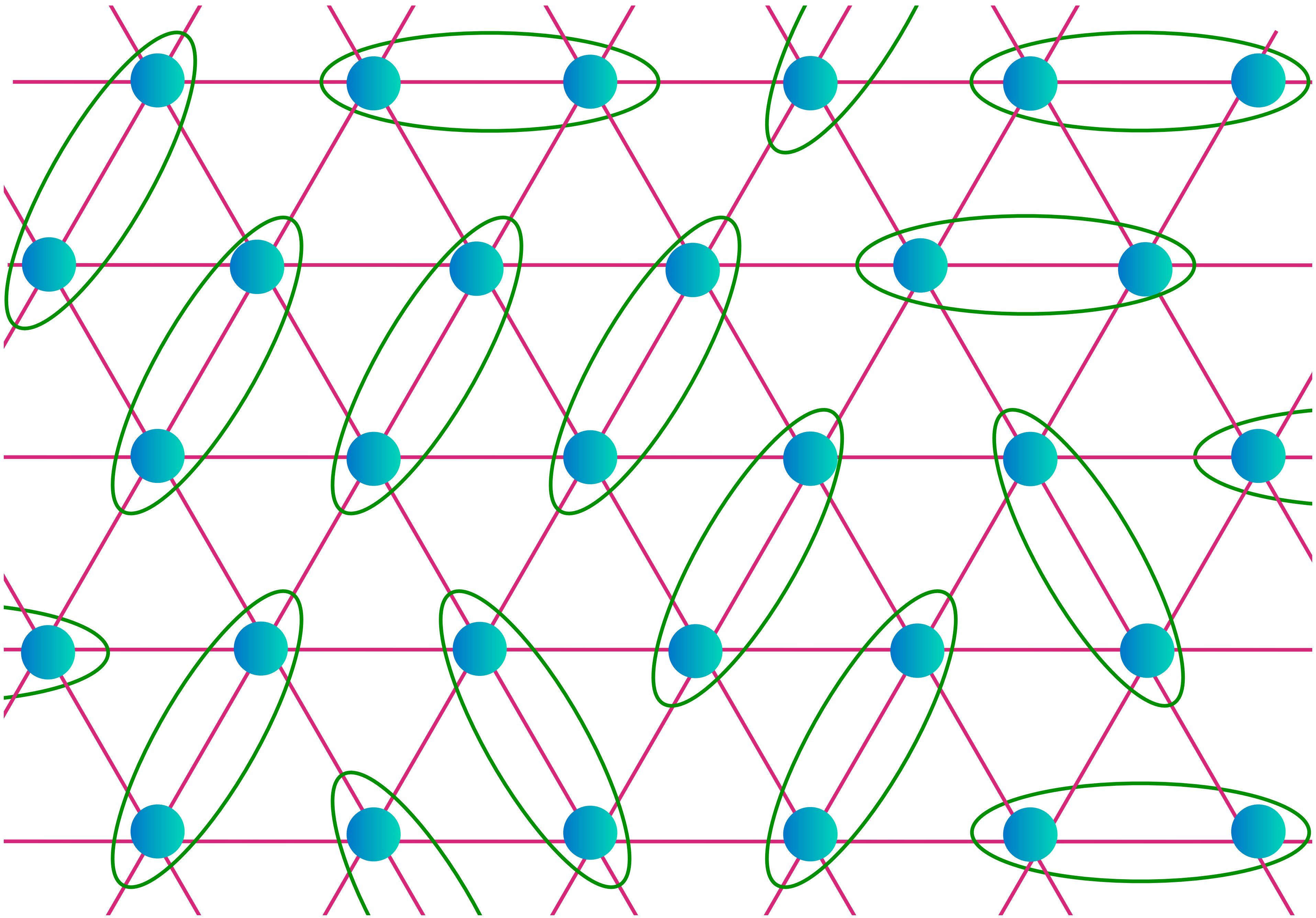

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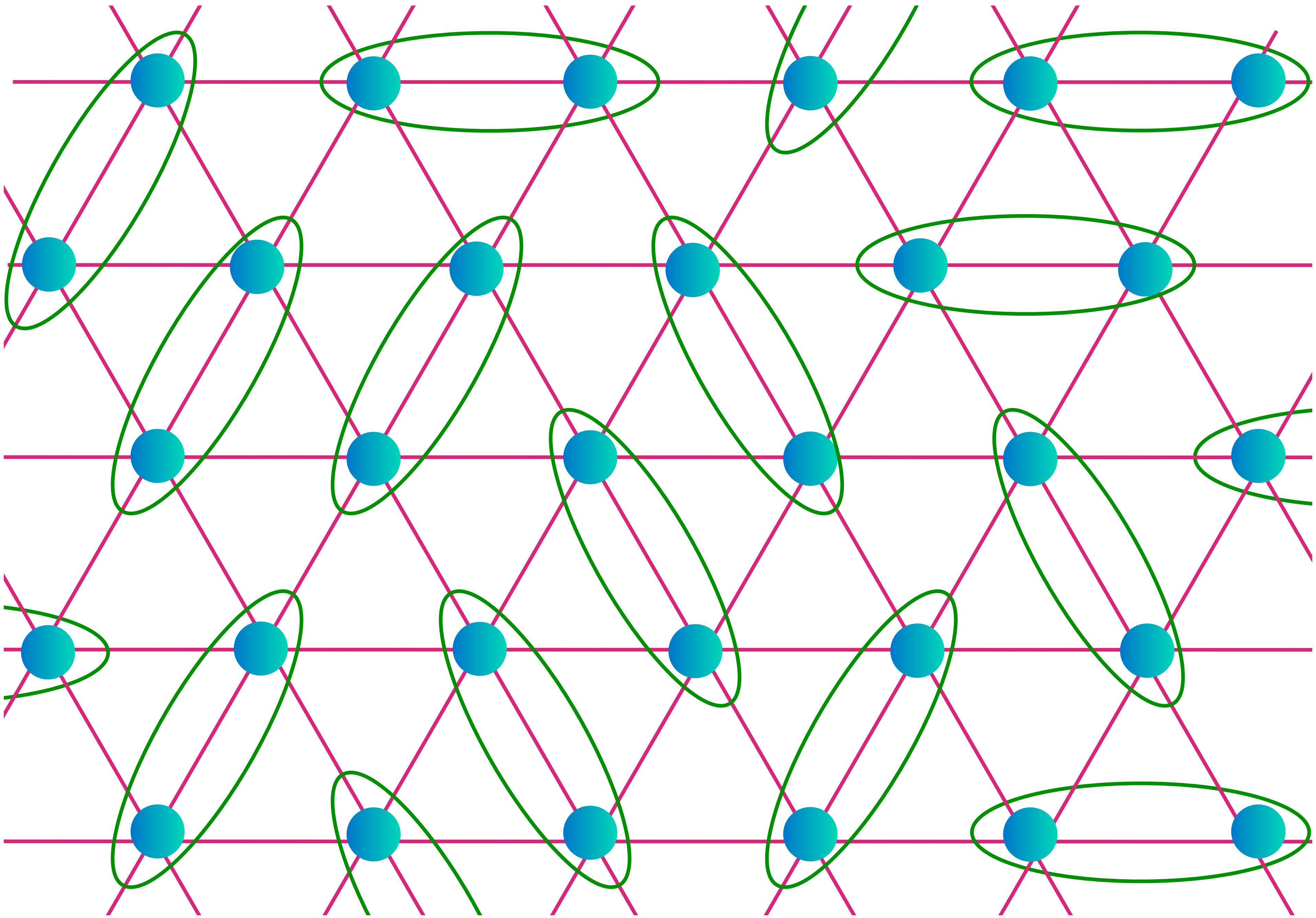

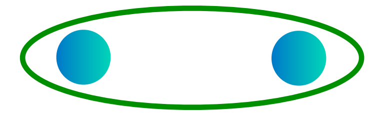

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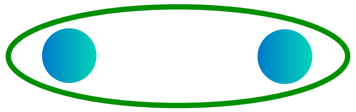
Excitations of the Z_2 Spin liquid



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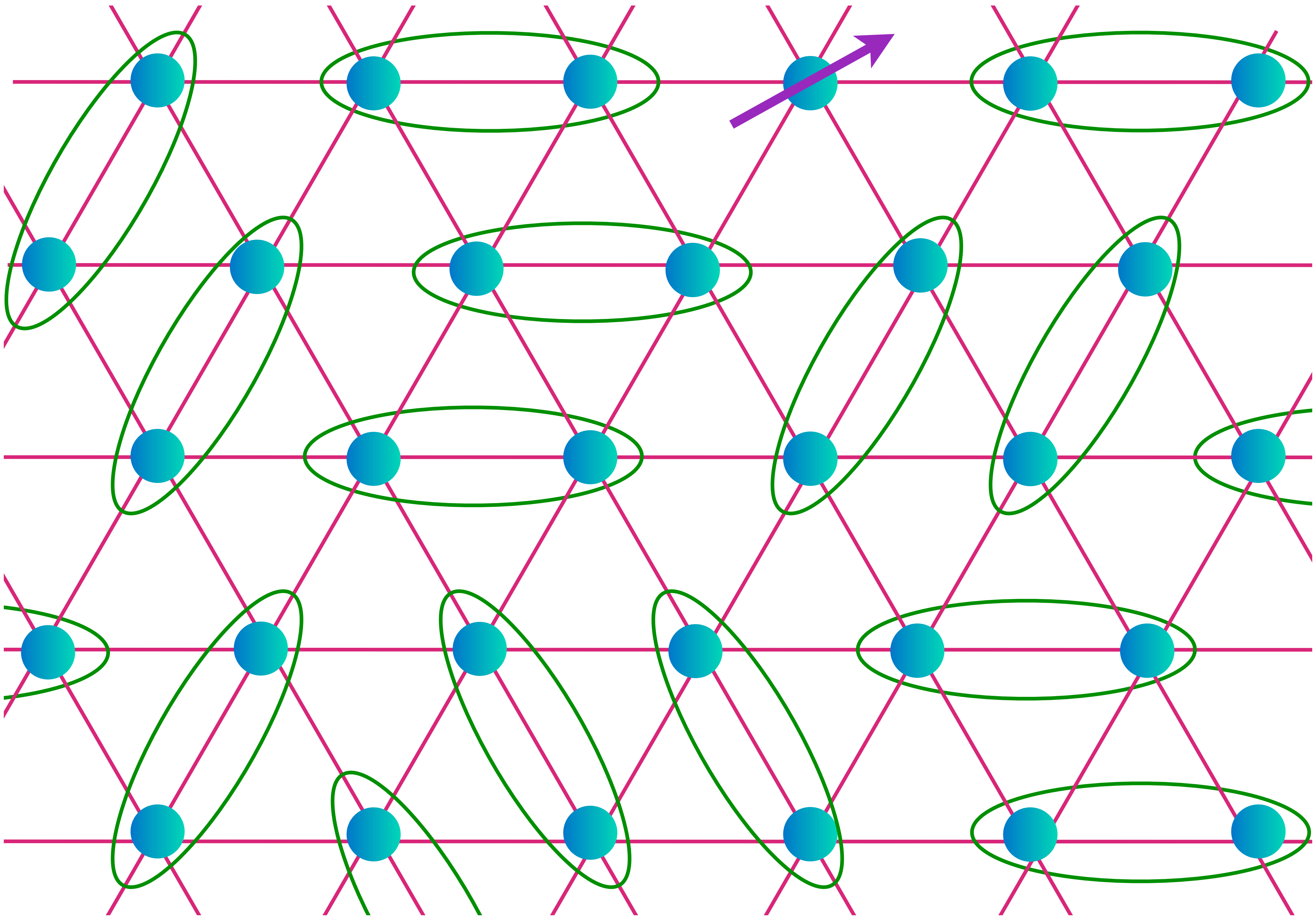
Spinon: S=1/2, charge 0



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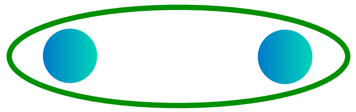

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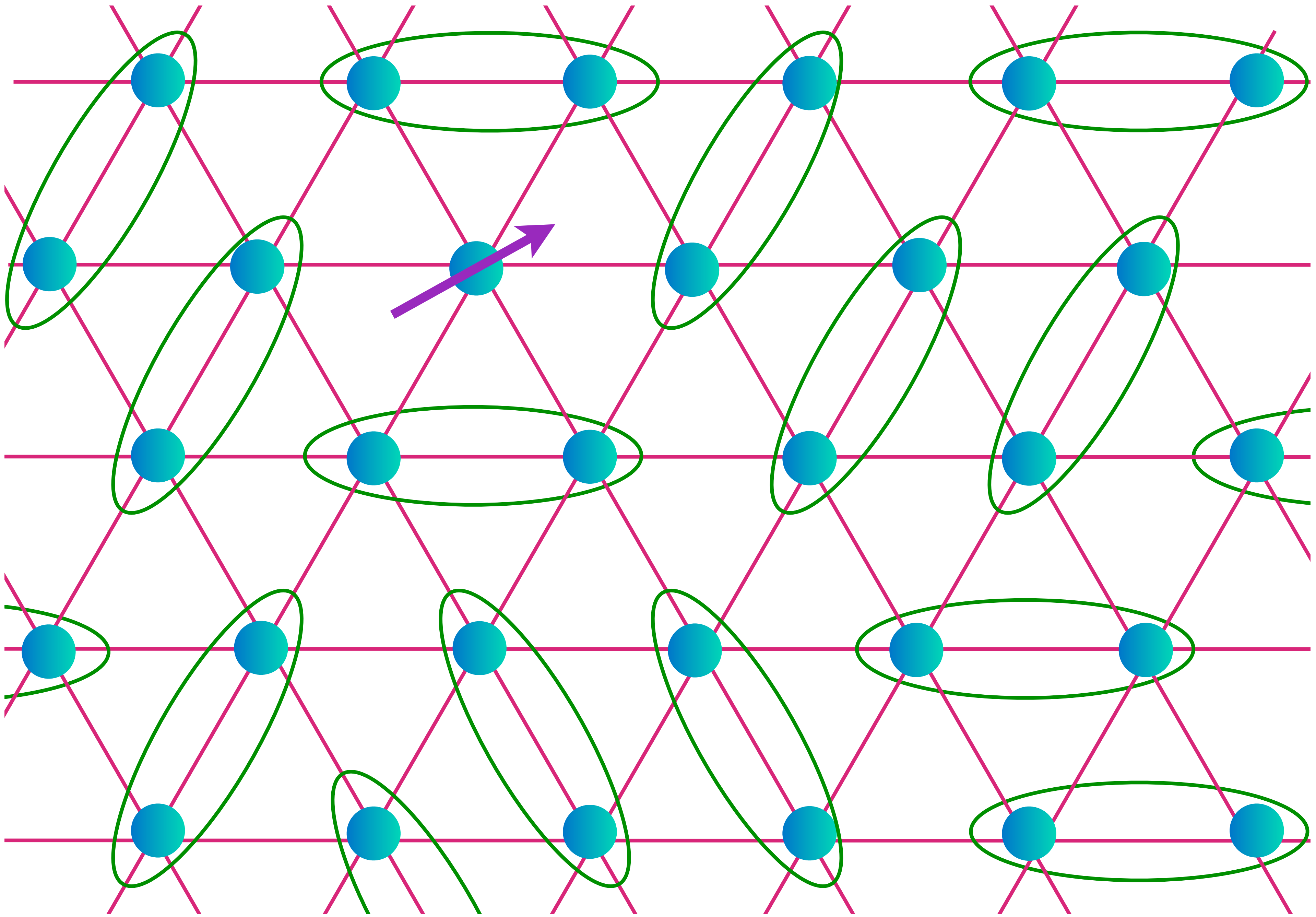

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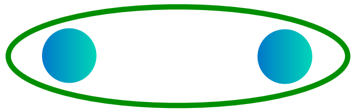

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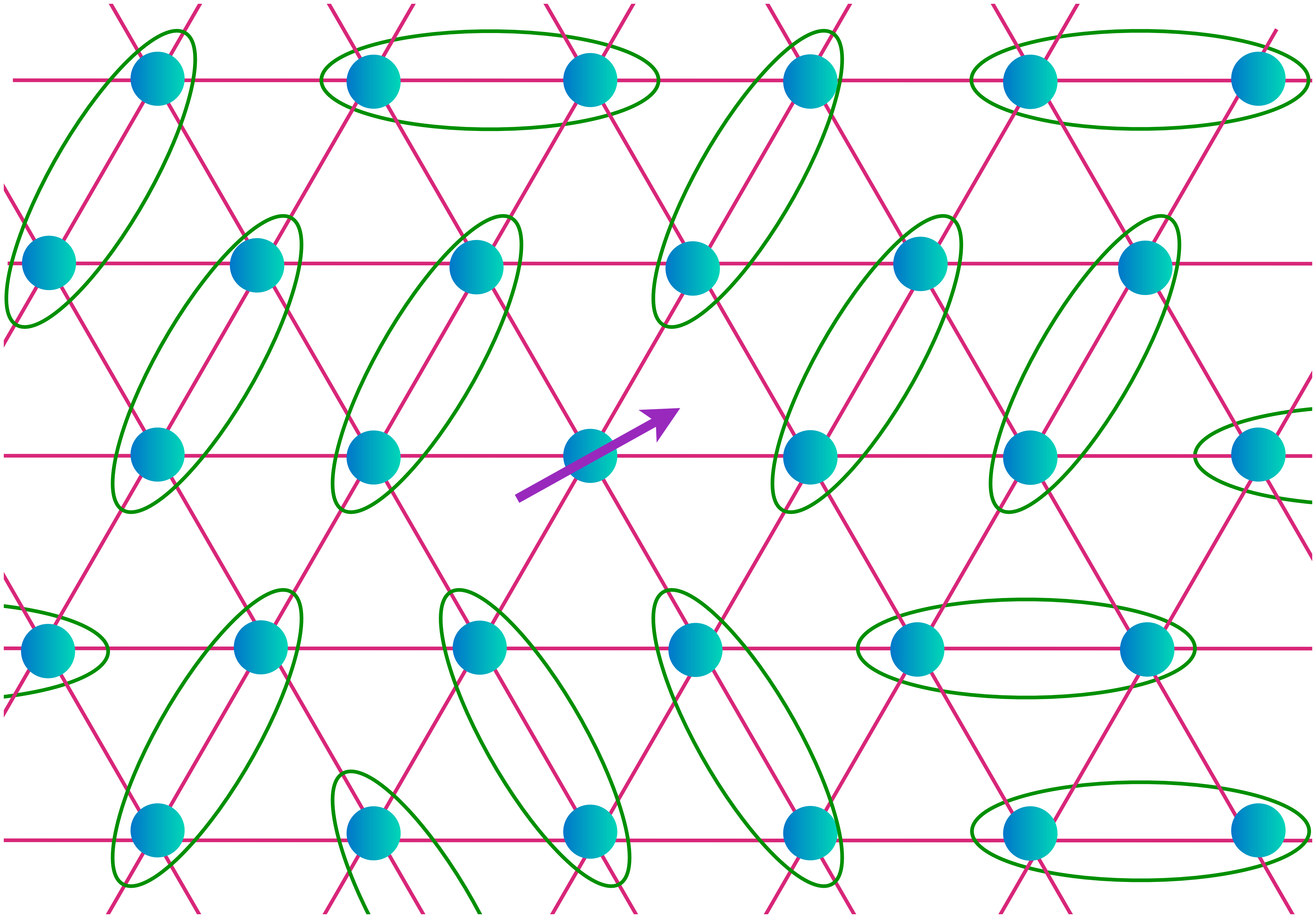

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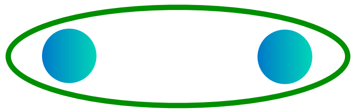

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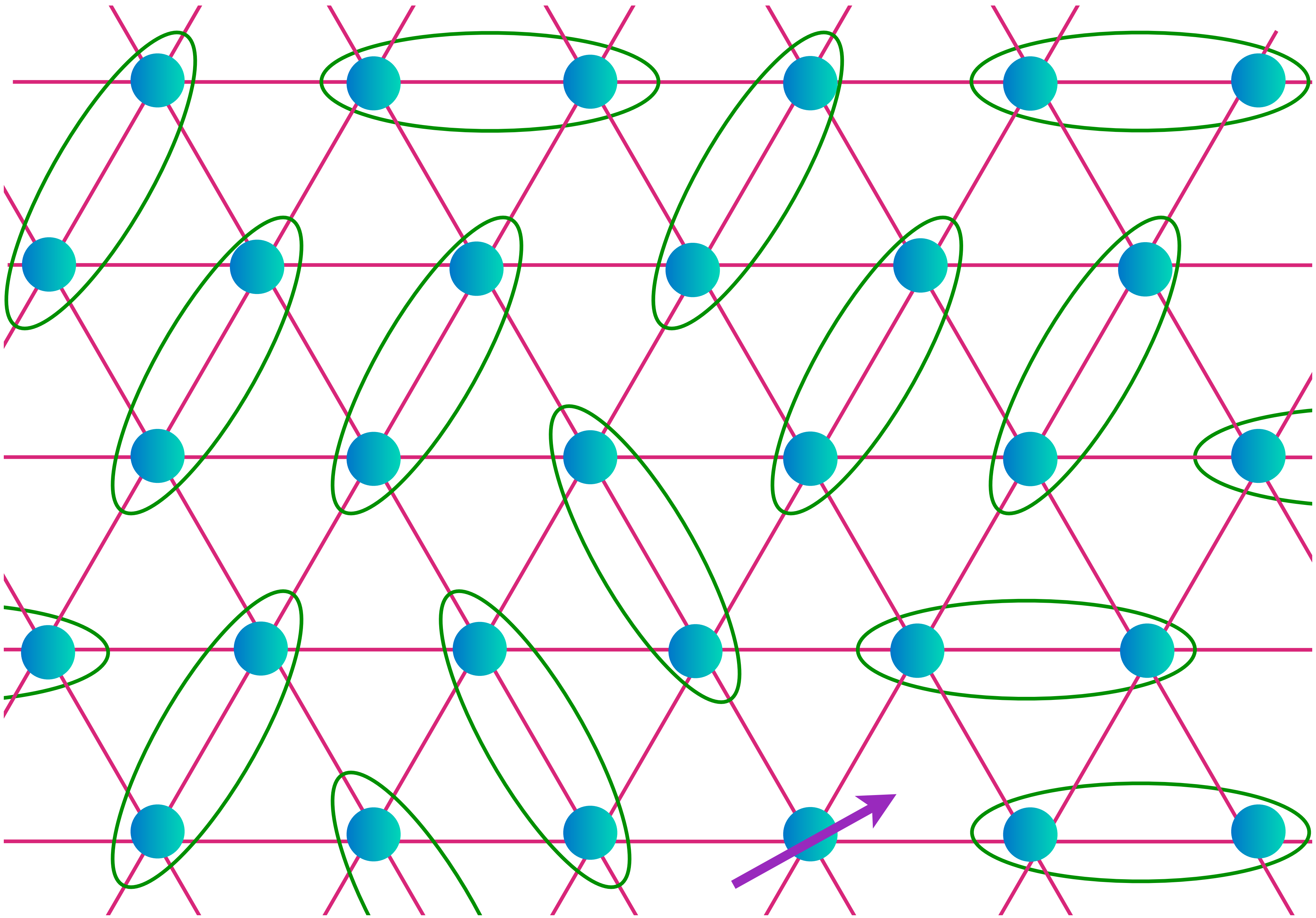

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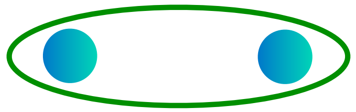

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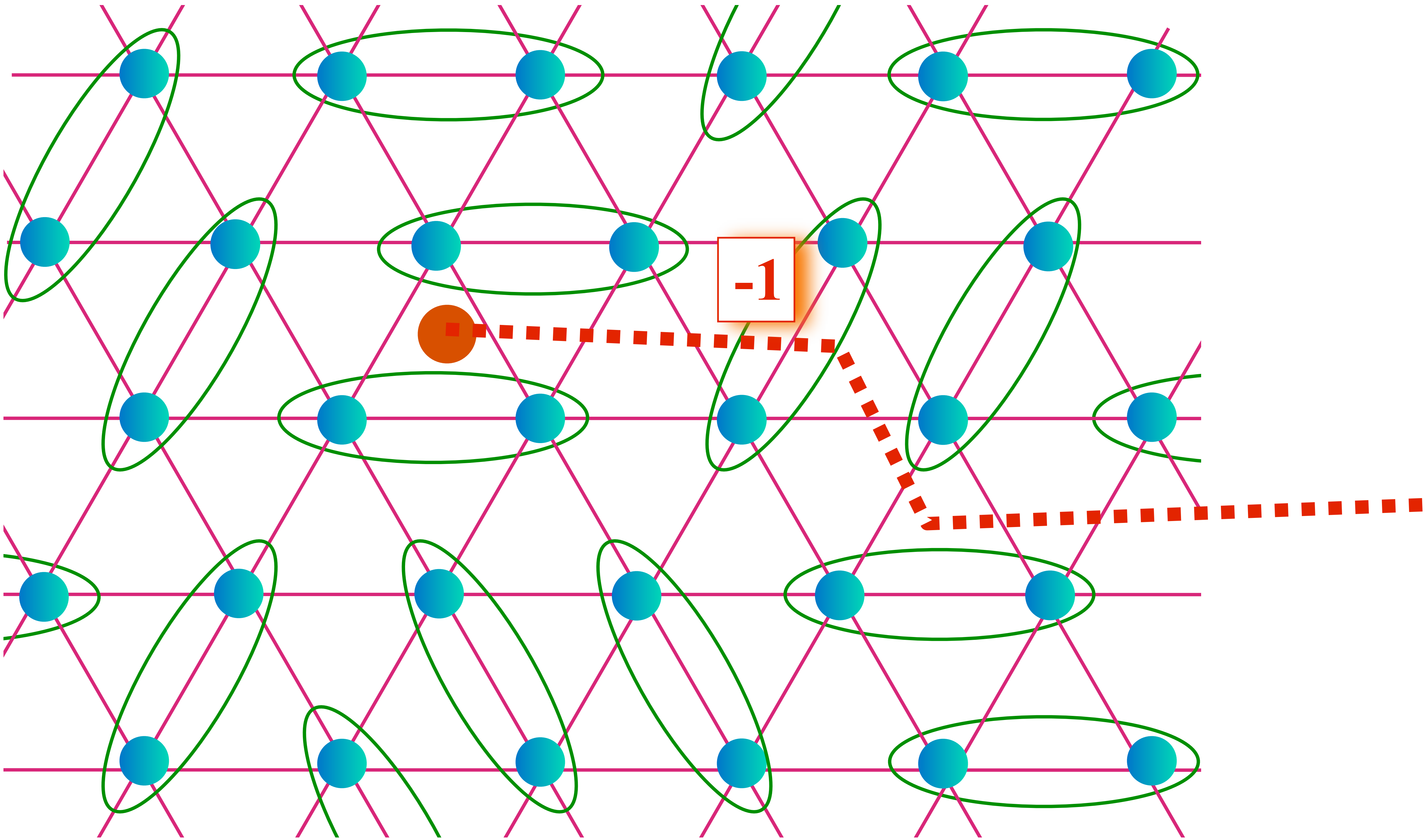
A vison



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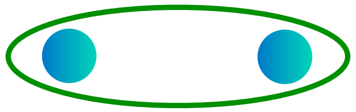

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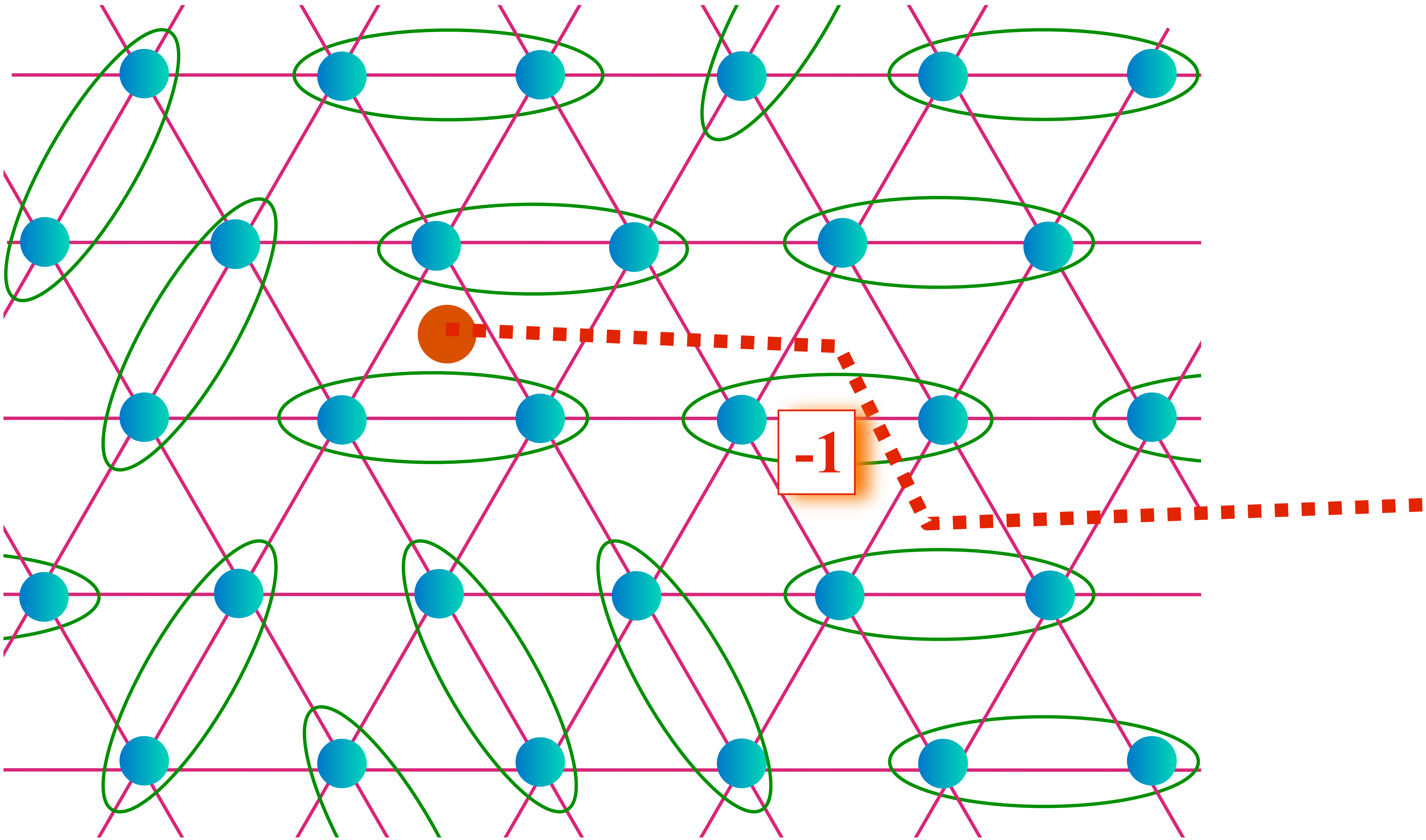

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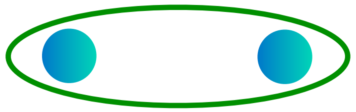

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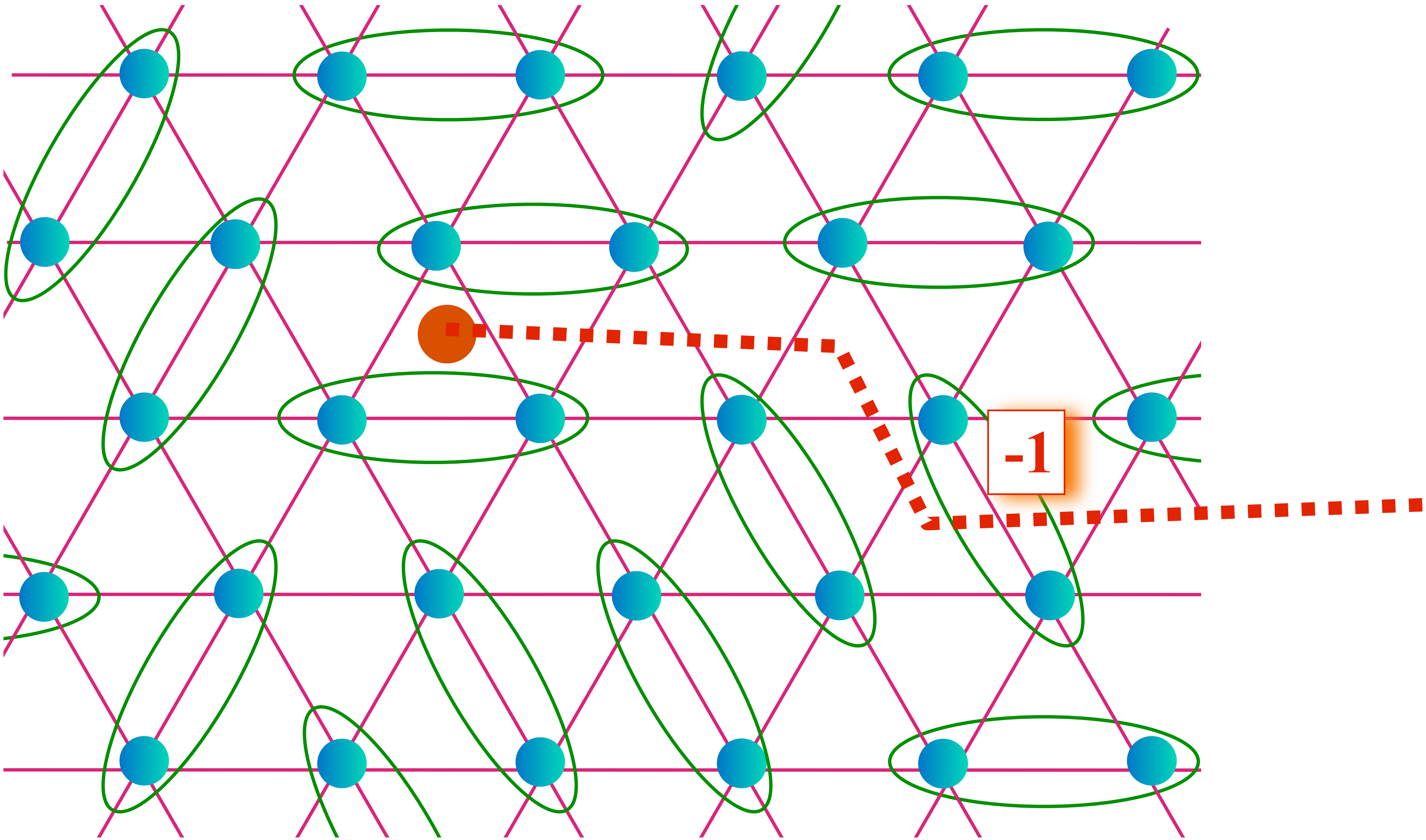

Excitations of the Z_2 Spin liquid

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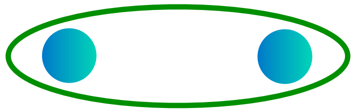

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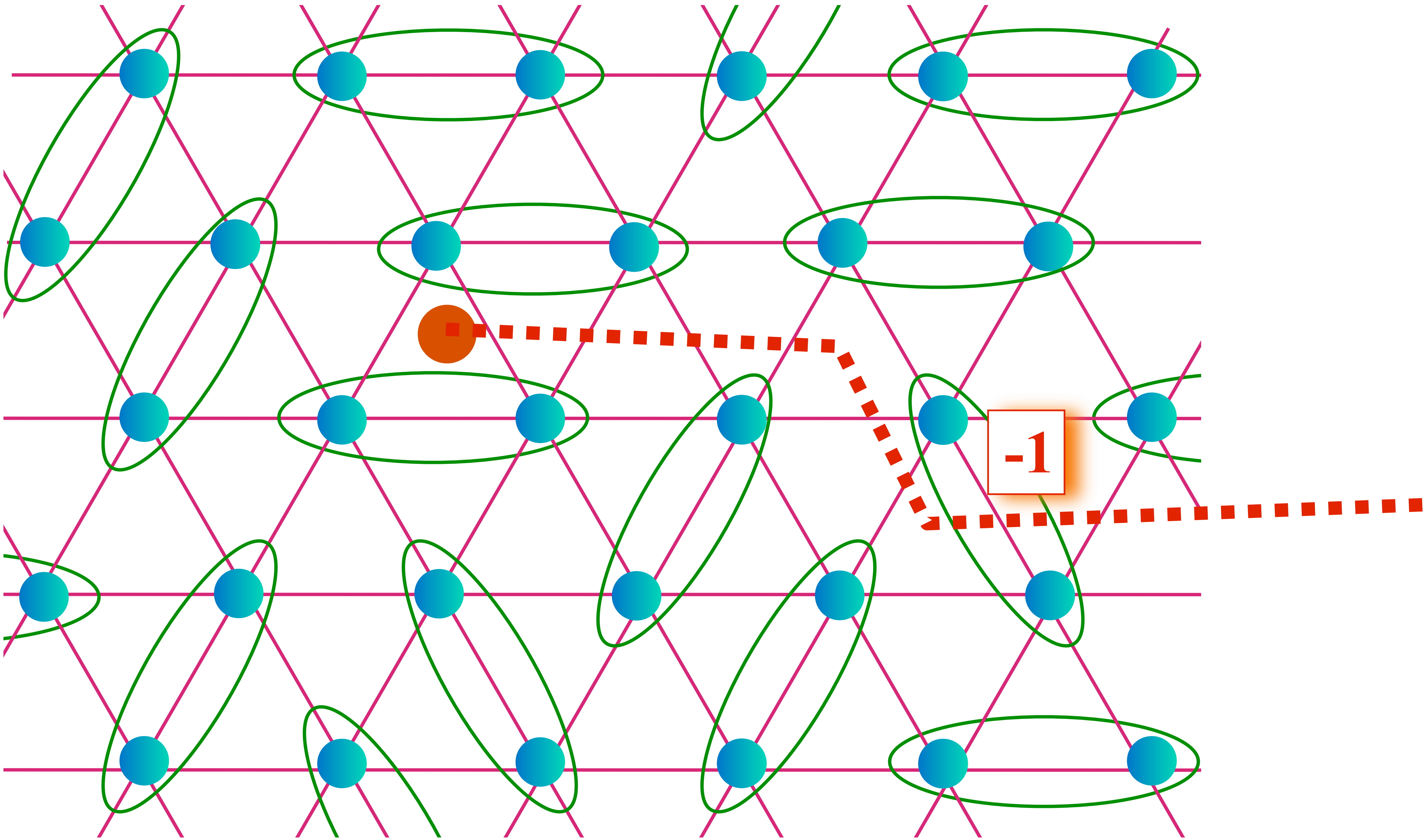

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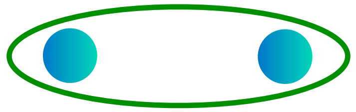

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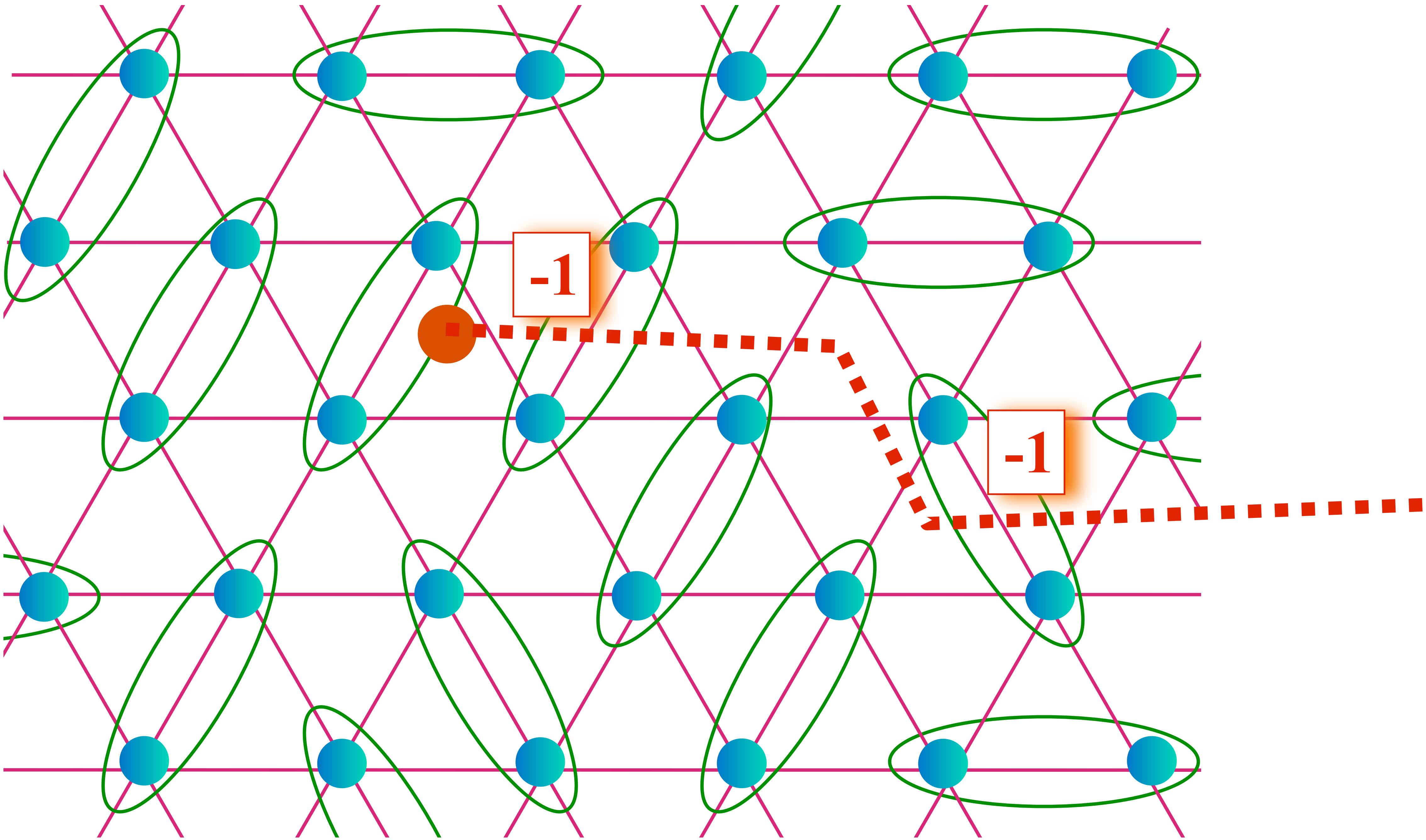

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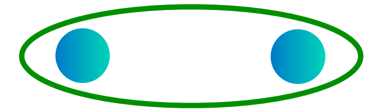

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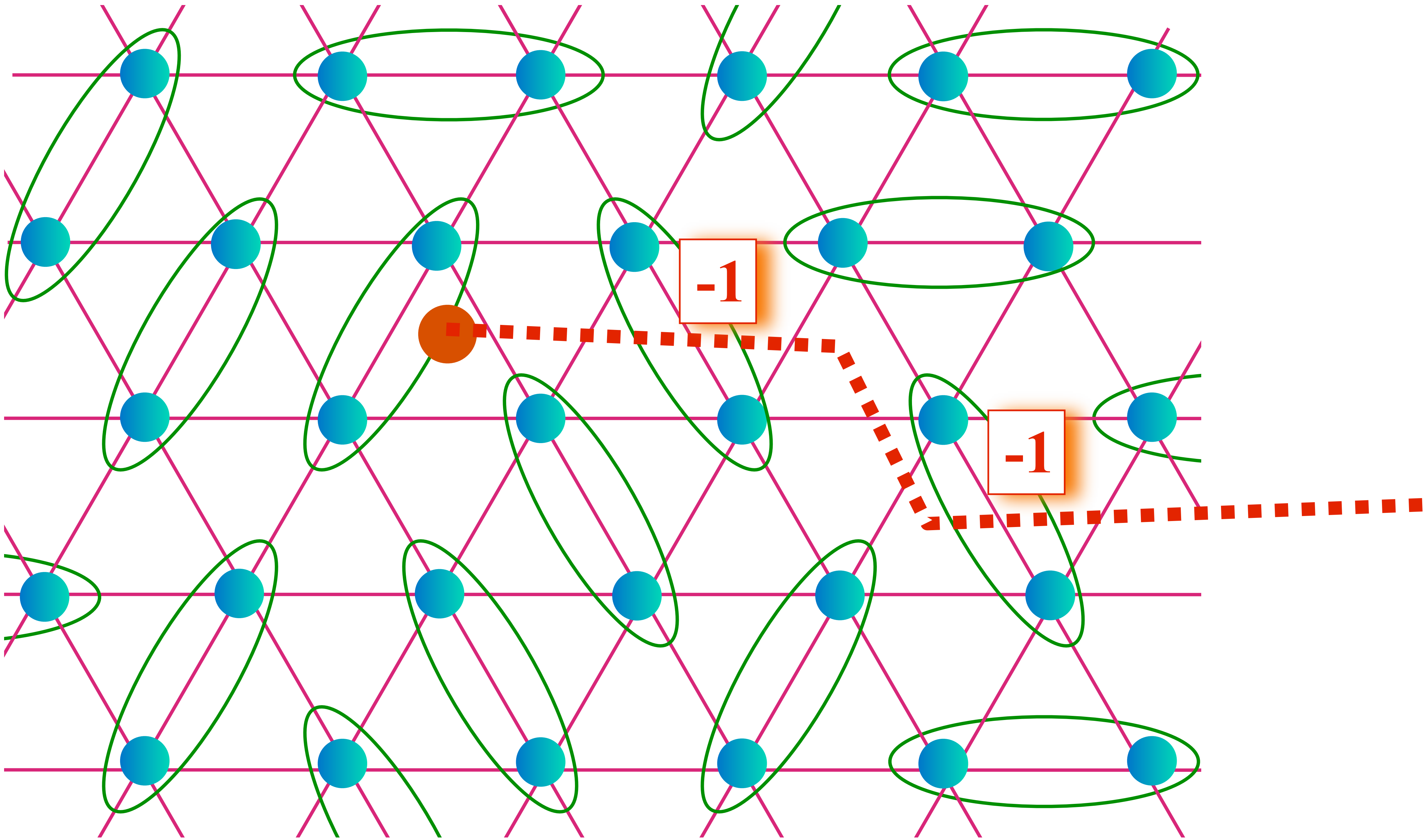

Excitations of the Z_2 Spin liquid

A vison

- Visons are are the **dark matter** of spin liquids: they likely carry most of the energy, but are very hard to detect because they do not carry charge or spin.

N. Read and B. Chakraborty, *Phys. Rev. B* **40,** 7133 (1989)
N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
T. Senthil and M.P.A. Fisher, *Phys. Rev. B* **63**, 134521 (2001)

Topological order in the Z_2 spin liquid ground state

4-fold degeneracy on the torus

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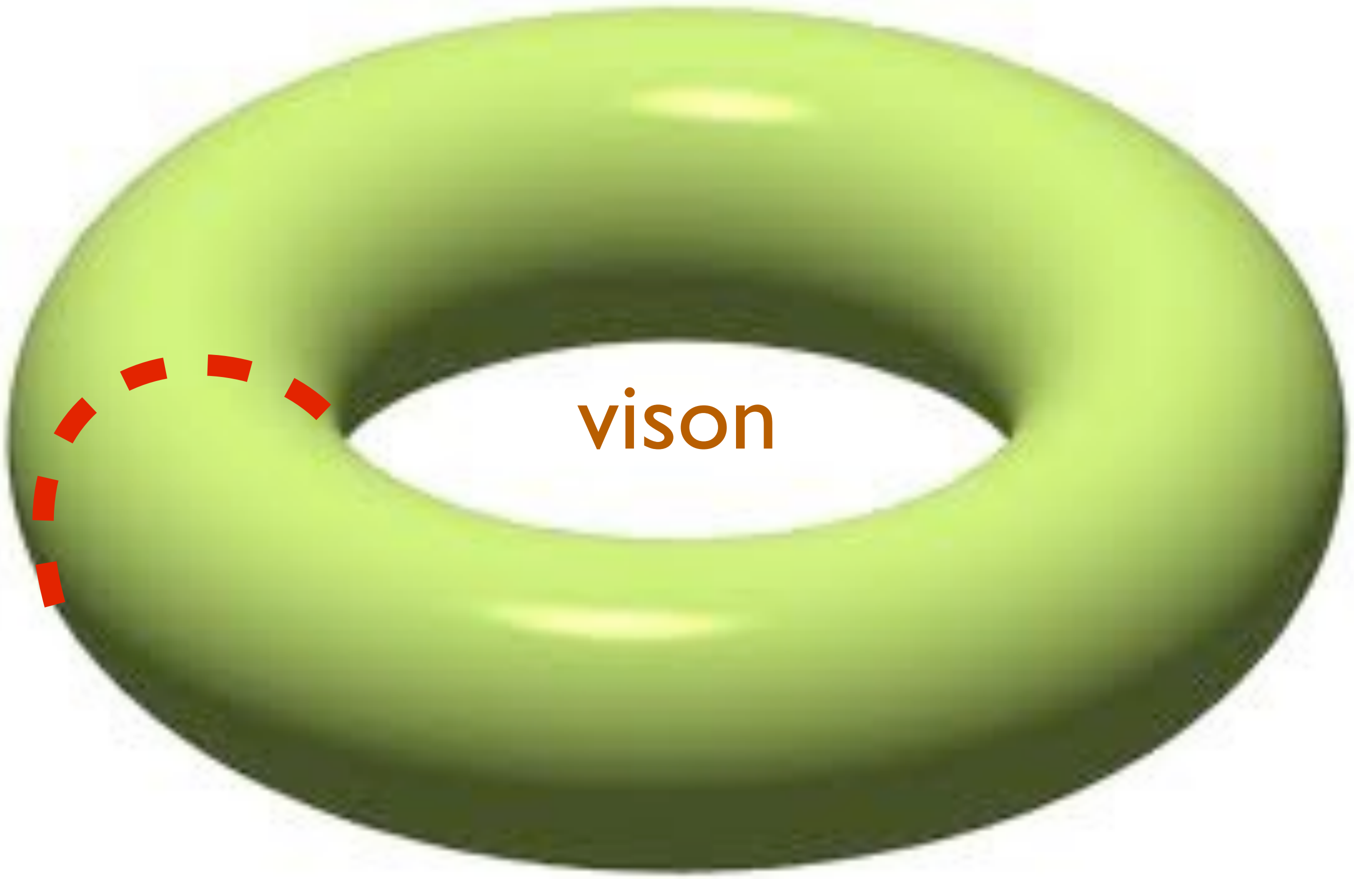

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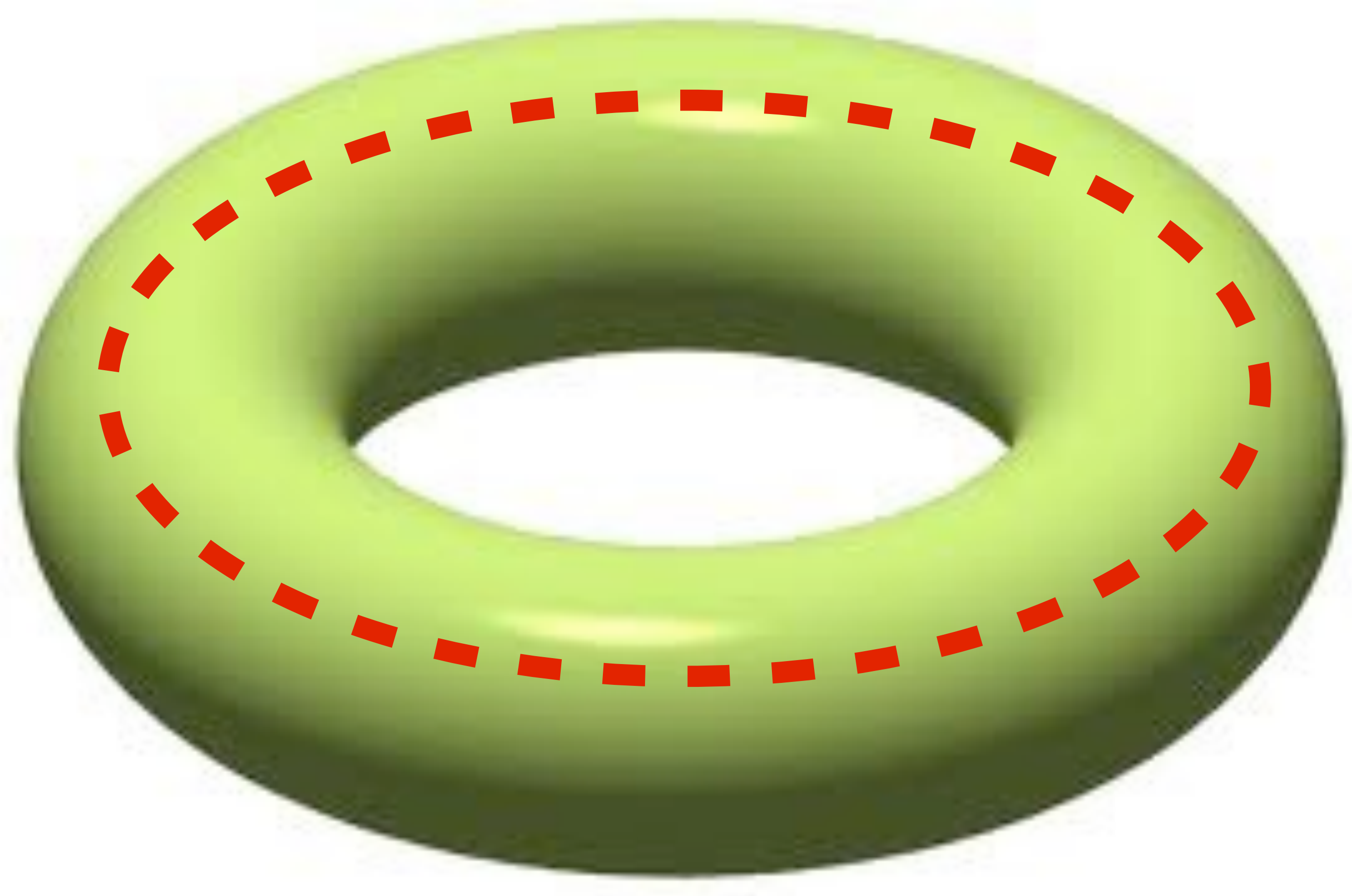

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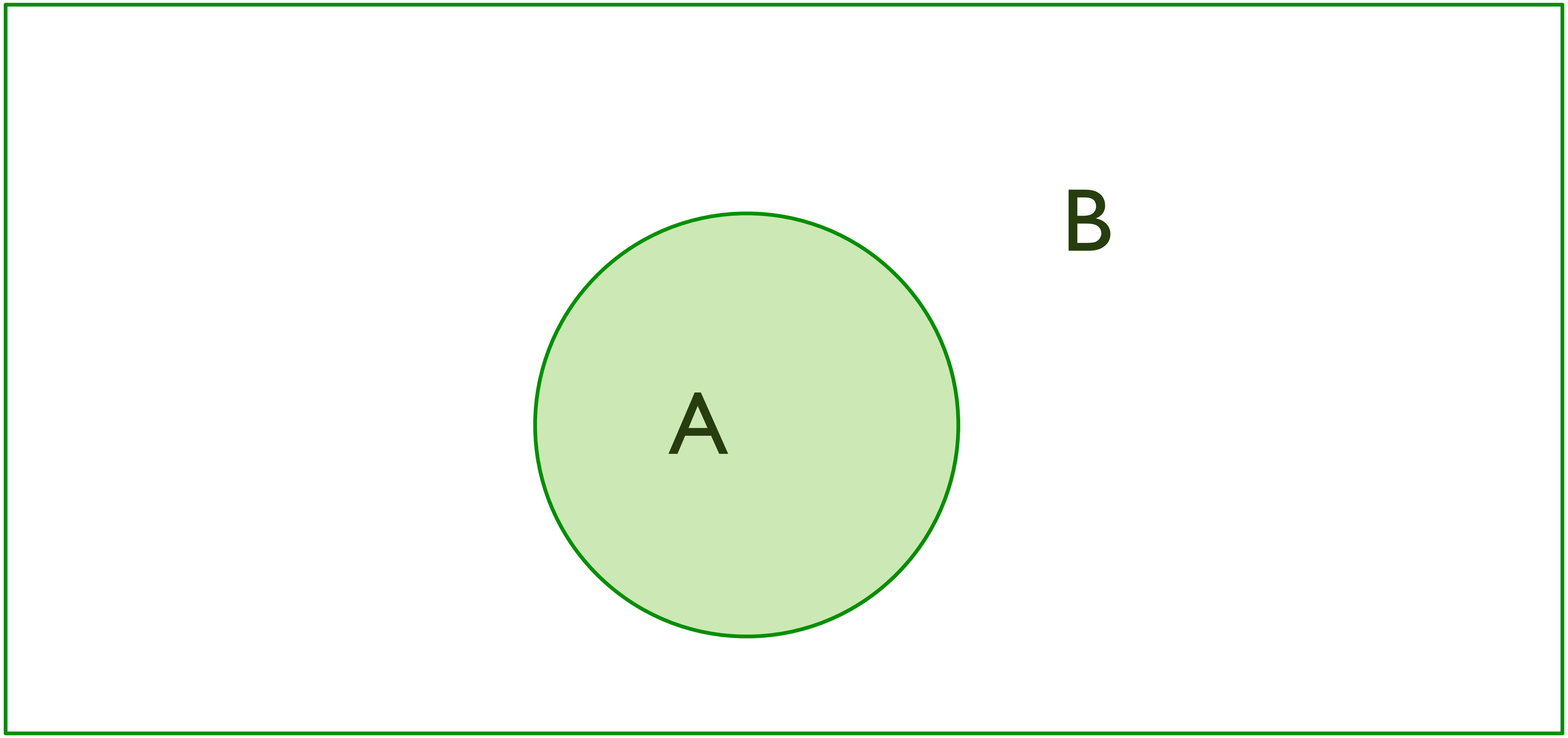
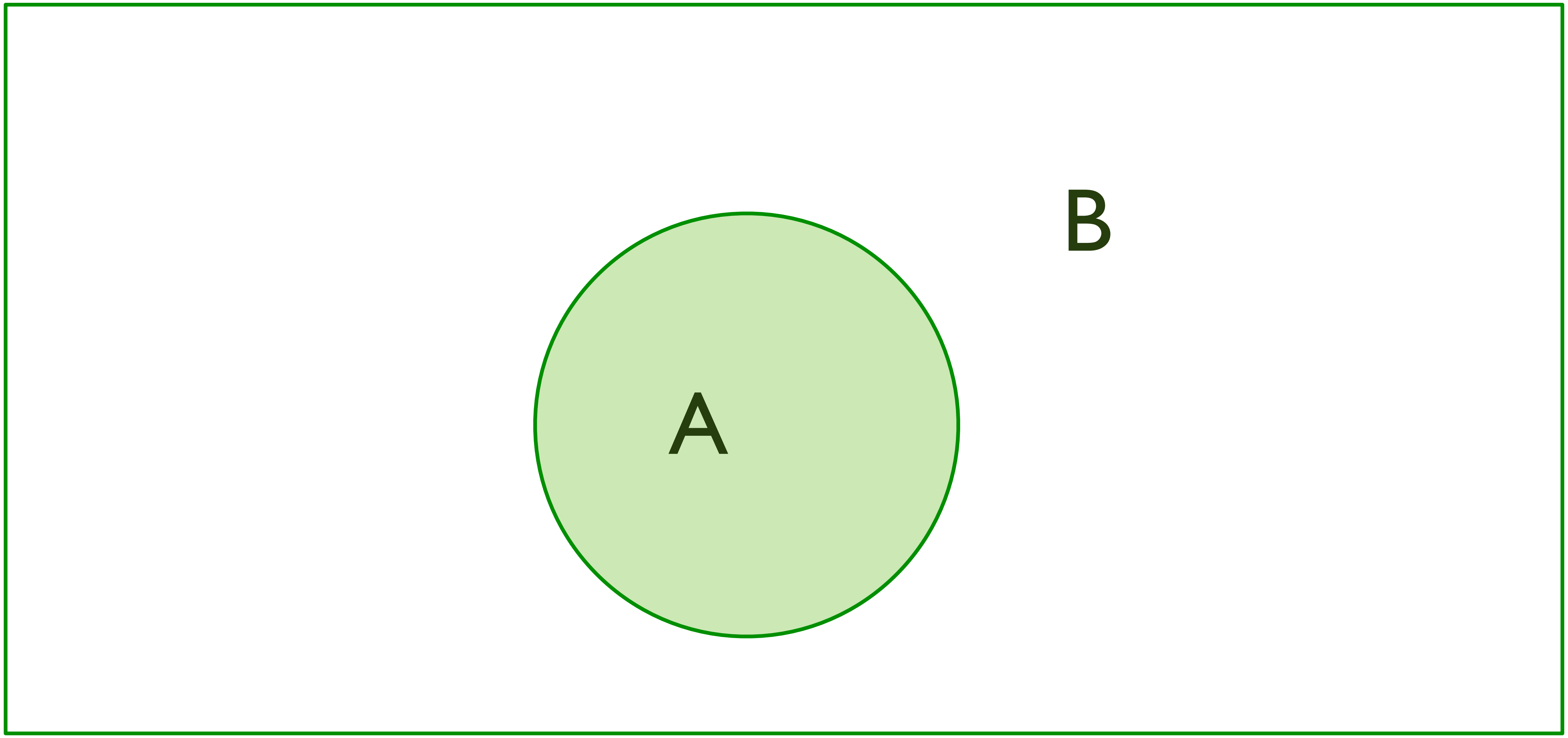


$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Topological order in the Z_2 spin liquid ground state

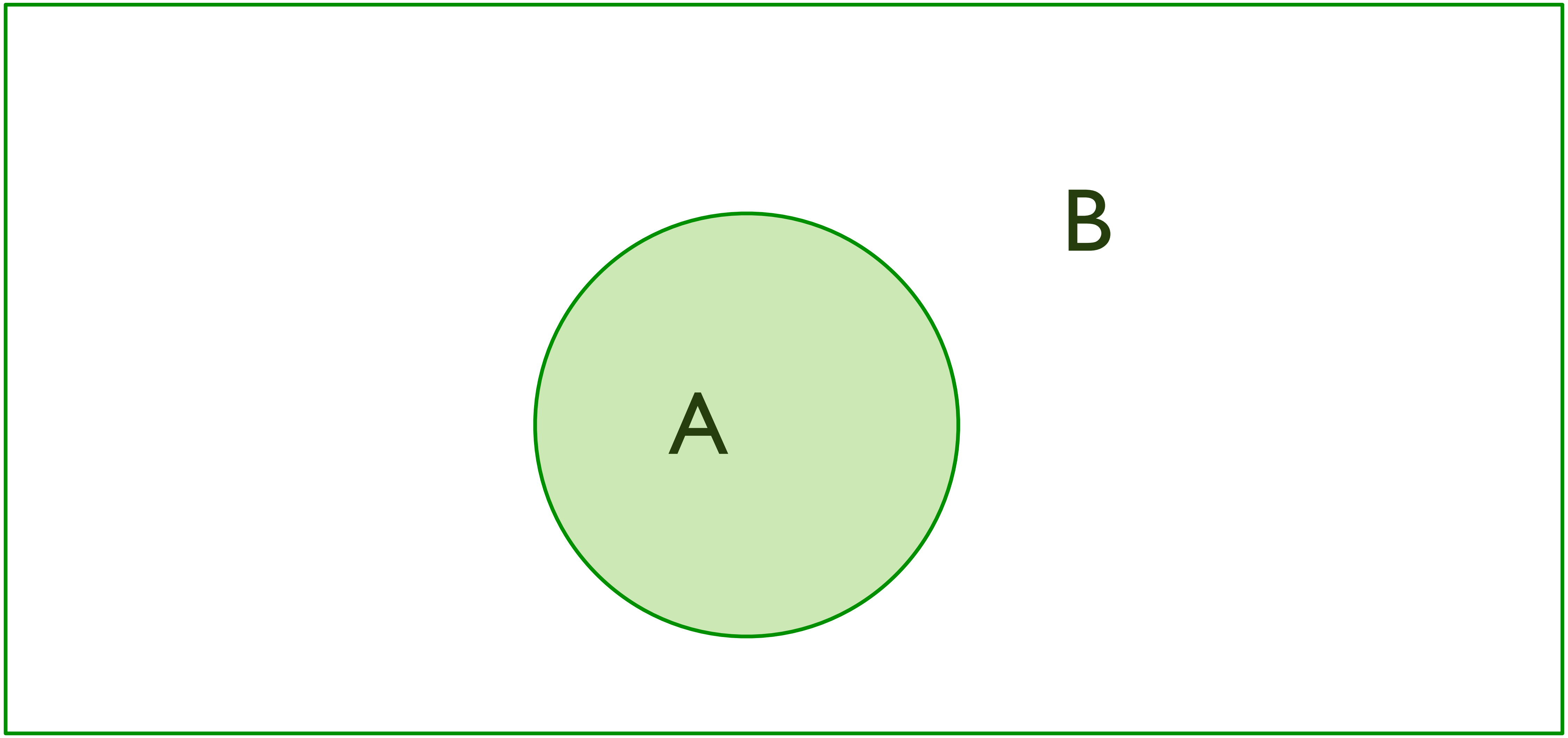


Entanglement entropy of a band insulator:

$$S_{EE} = aL - exp(-bL)$$

where L is the perimeter of the boundary between A and B.

Topological order in the Z_2 spin liquid ground state



Entanglement entropy of a Z_2 spin liquid:

$$S_{EE} = aL - \ln(2)$$

where L is the perimeter of the boundary between A and B.
The $\ln(2)$ is a universal characteristic of the Z_2 spin liquid, and implies *long-range* quantum entanglement.

M. Levin and X.-G. Wen, *Phys. Rev. Lett.* **96**, 110405 (2006); A. Kitaev and J. Preskill, *Phys. Rev. Lett.* **96**, 110404 (2006);
Y. Zhang, T. Grover, and A. Vishwanath, *Phys. Rev. B* **84**, 075128 (2011).

Topological order in the Z_2 spin liquid ground state

These properties of the ground state can be described by effective theories:

- deconfined phase of a Z_2 gauge theory

  N. Read and S. Sachdev, *Phys. Rev. Lett.* **66**, 1773 (1991)
  F. A. Bais, P. van Driel, and M. de Wild Propitius, `Phys. Lett. B **280**, 63 (1992).
  T. Senthil and M.P.A. Fisher, *Phys. Rev. B* **63**, 134521 (2001)

- topological doubled Chern-Simons gauge theory

  J. Maldacena, G. Moore, and N. Seiberg, JHEP 0110:005 (2001).
  M. Freedman, C. Nayak, K. Shtengel, K. Walker, and Z. Wang, *Annals of Physics* **310**, 428 (2004).

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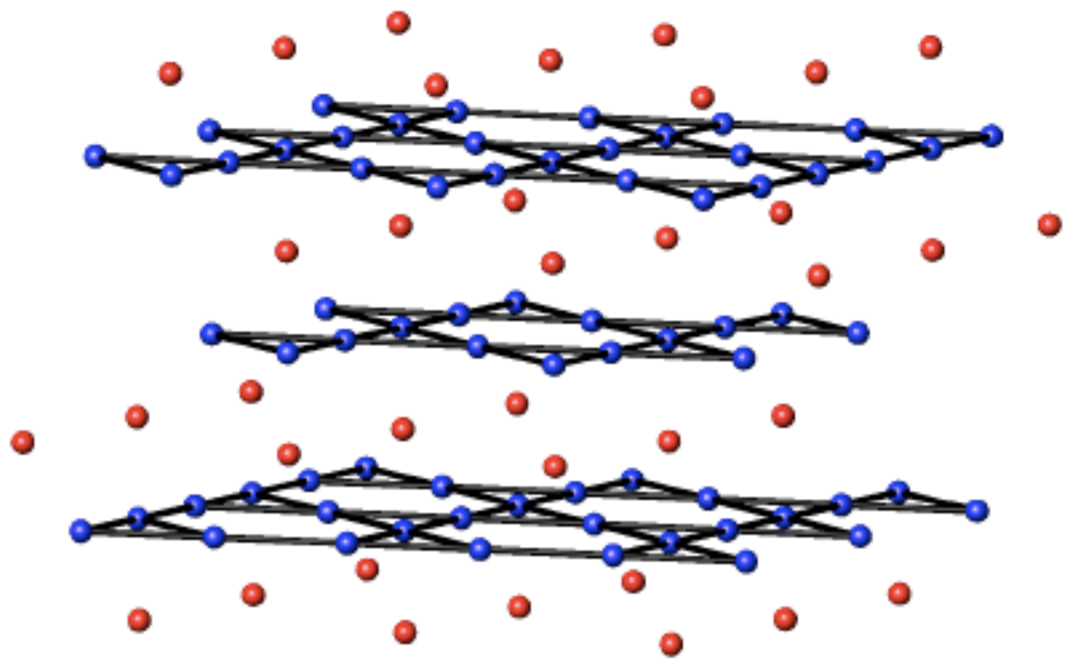
- Good recent numerical evidence of Z_2 spin liquid on kagome and square lattices

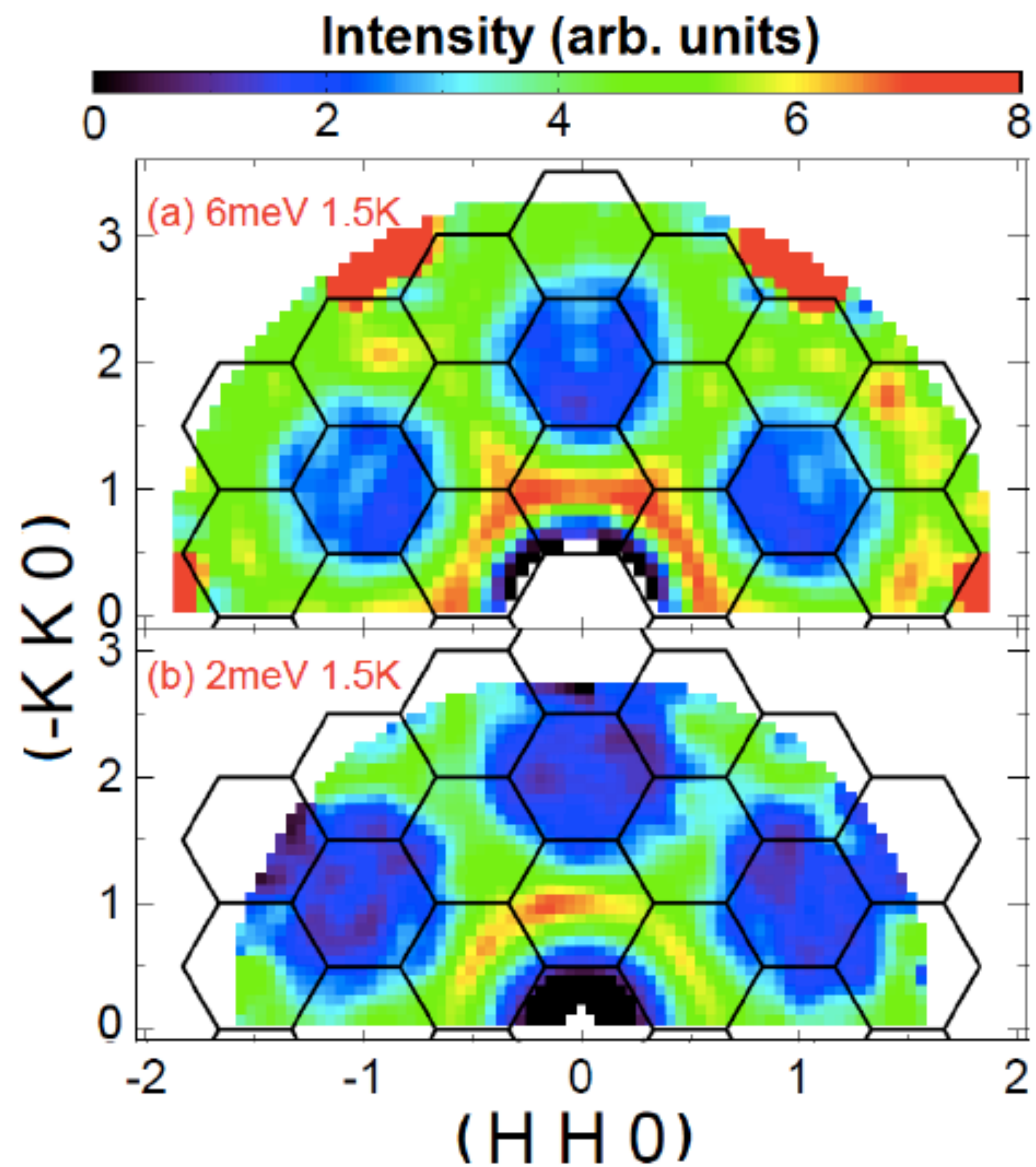
Simeng Yan, D. A. Huse, and S. R. White, *Science* **332**, 1173 (2011).
J. Hong-Chen Jiang, Hong Yao, and L. Balents, arXiv:1112.2241.
Ling Wang, Zheng-Cheng Gu, Xiao-Gang Wen, and F. Verstraete, arXiv:1112.3331

Promising experimental candidate: the kagome antiferromagnet

Young Lee, APS meeting, March 2012

$ZnCu_3(OH)_6Cl_2$ (also called Herbertsmithite)





Quantum Hall states

Similar topological properties,
but no time-reversal symmetry:

- ground state degeneracy on a torus
- universal entanglement entropy
- gapless edge states on spaces with boundaries (can also happen for some spin liquids)
- topological Chern-Simons gauge theories

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Conformal quantum matter

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Conformal quantum matter

A. Field theory:graphene

B. Field theory: superfluid-insulator transition

C. Field theory: antiferromagnets

D. Gauge-gravity duality

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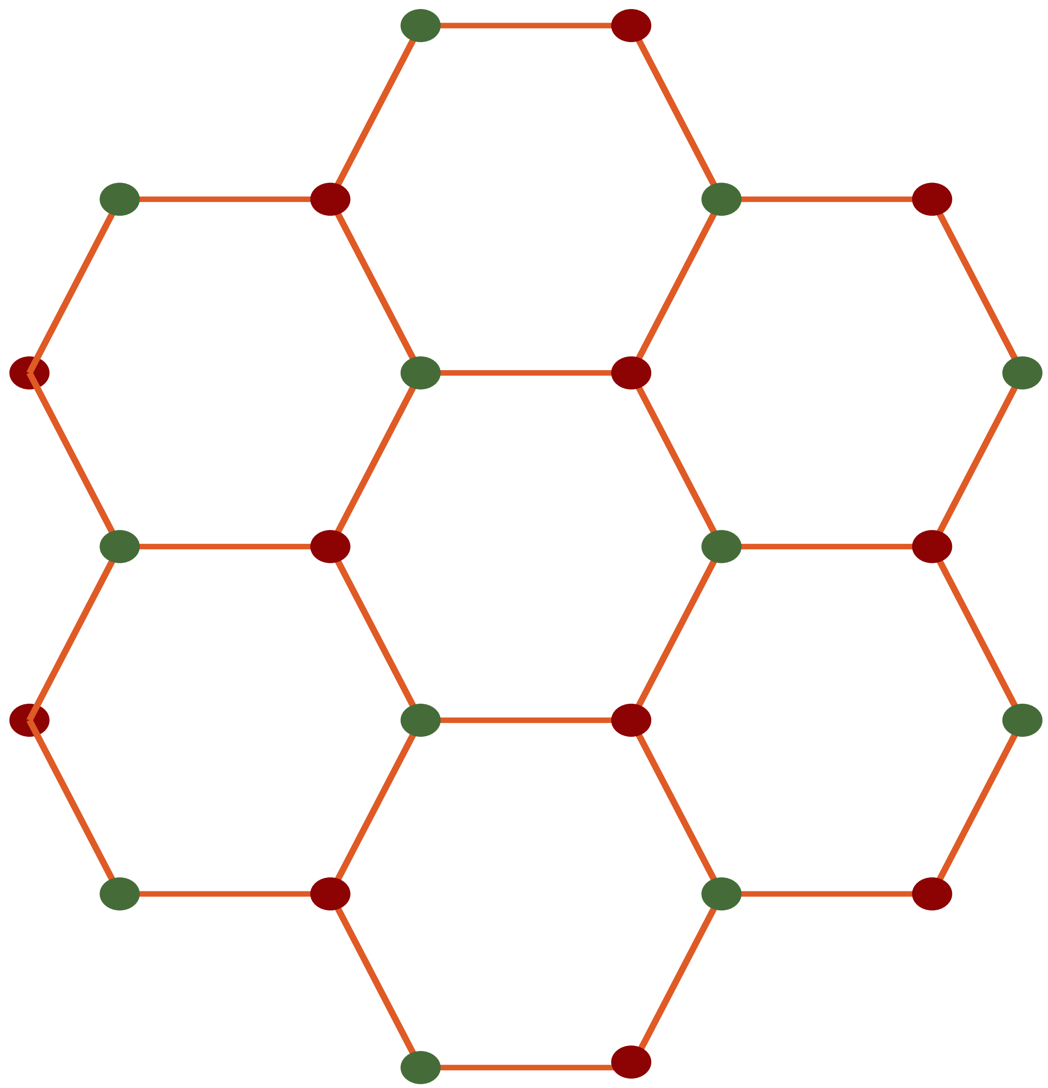
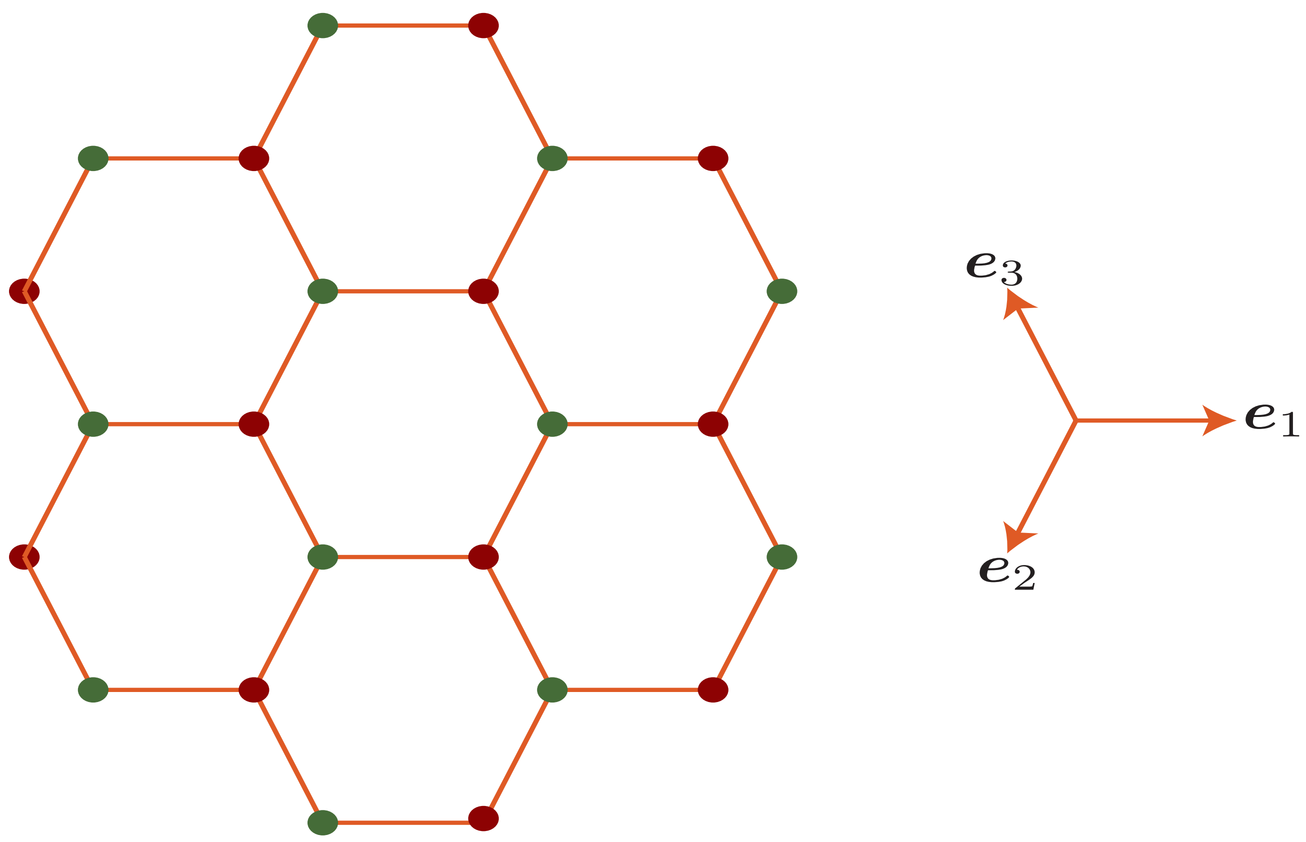
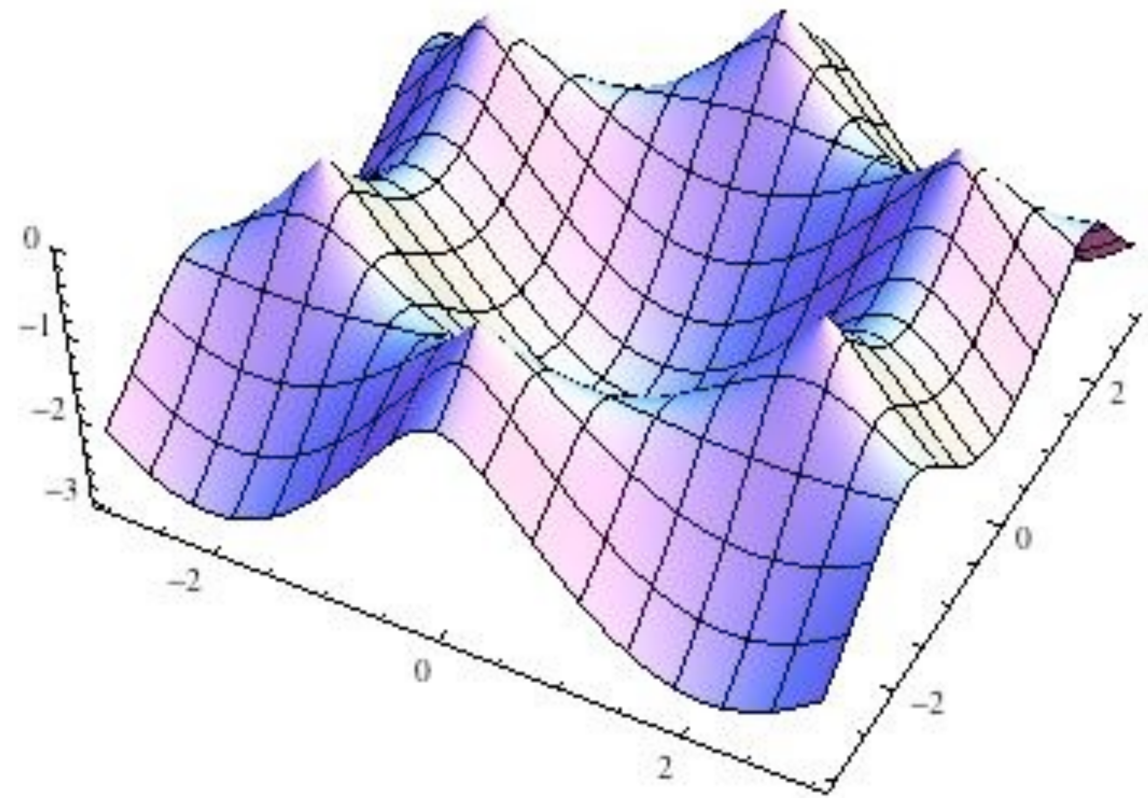
Honeycomb lattice

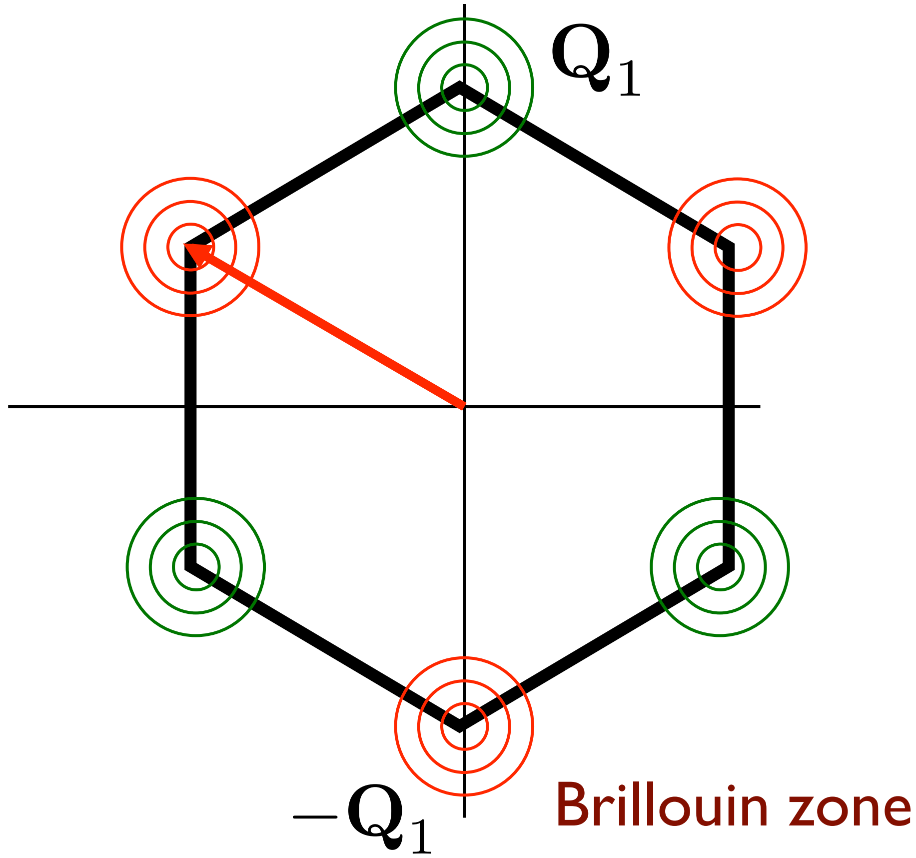
(describes graphene after adding long-range Coulomb interactions)



$$H = -t \sum_{\langle ij \rangle} c_{i\alpha}^{\dagger} c_{j\alpha} + U \sum_{i} \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right)$$







Semi-metal with massless Dirac fermions at small *U/t*

We define the Fourier transform of the fermions by

$$c_A(\mathbf{k}) = \sum_{\mathbf{r}} c_A(\mathbf{r}) e^{-i\mathbf{k}\cdot\mathbf{r}} \tag{4}$$

and similarly for c_B. **A and B are sublattice indices.**
The hopping Hamiltonian is

$$H_0 = -t \sum_{\langle ij \rangle} \left(c^\dagger_{Ai\alpha} c_{Bj\alpha} + c^\dagger_{Bj\alpha} c_{Ai\alpha} \right) \tag{5}$$

where α is a spin index. If we introduce Pauli matrices τ^a in sublattice space ($a = x, y, z$), this Hamiltonian can be written as

$$\begin{aligned} H_0 &= \int \frac{d^2k}{4\pi^2} c^\dagger(\mathbf{k}) \Big[-t \Big(\cos(\mathbf{k}\cdot\mathbf{e}_1) + \cos(\mathbf{k}\cdot\mathbf{e}_2) + \cos(\mathbf{k}\cdot\mathbf{e}_3) \Big) \tau^x \\ &\quad + t \Big(\sin(\mathbf{k}\cdot\mathbf{e}_1) + \sin(\mathbf{k}\cdot\mathbf{e}_2) + \sin(\mathbf{k}\cdot\mathbf{e}_3) \Big) \tau^y \Big] c(\mathbf{k}) \end{aligned} \tag{6}$$

The low energy excitations of this Hamiltonian are near $\mathbf{k} \approx \pm \mathbf{Q}_1$.

In terms of the fields near $\mathbf{Q}_1$ and $-\mathbf{Q}_1$, we define

$$\begin{aligned}
\Psi_{A1\alpha}(\mathbf{k}) &= c_{A\alpha}(\mathbf{Q}_1 + \mathbf{k}) \\
\Psi_{A2\alpha}(\mathbf{k}) &= c_{A\alpha}(-\mathbf{Q}_1 + \mathbf{k}) \\
\Psi_{B1\alpha}(\mathbf{k}) &= c_{B\alpha}(\mathbf{Q}_1 + \mathbf{k}) \\
\Psi_{B2\alpha}(\mathbf{k}) &= c_{B\alpha}(-\mathbf{Q}_1 + \mathbf{k}) \qquad (7)
\end{aligned}$$

We consider Ψ to be a 8 component vector, and introduce Pauli matrices ρ^a which act in the $1, 2$ valley space. Then the Hamiltonian is

$$H_0 = \int \frac{d^2k}{4\pi^2} \Psi^\dagger(\mathbf{k}) \Big(v\tau^y k_x + v\tau^x \rho^z k_y \Big) \Psi(\mathbf{k}), \qquad (8)$$

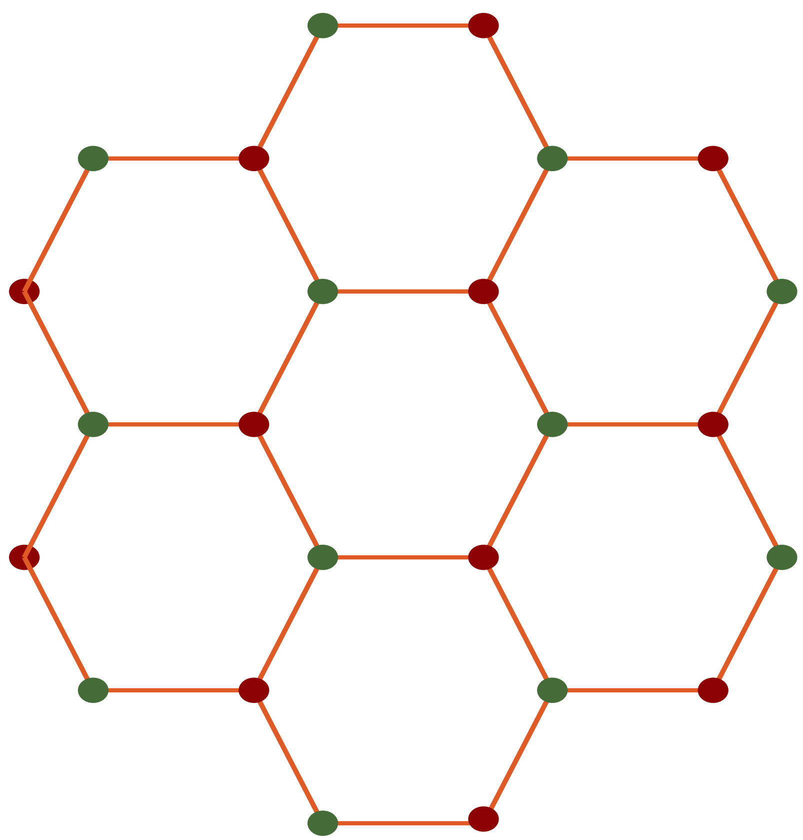
where $v = 3t/2$; below we set $v = 1$. Now define $\overline{\Psi} = \Psi^\dagger \rho^z \tau^z$. Then we can write the imaginary time Lagrangian as

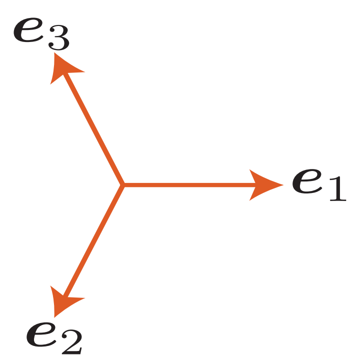
$$\mathcal{L}_0 = -i\overline{\Psi} \left(\omega\gamma_0 + k_x\gamma_1 + k_y\gamma_2 \right) \Psi \qquad (9)$$

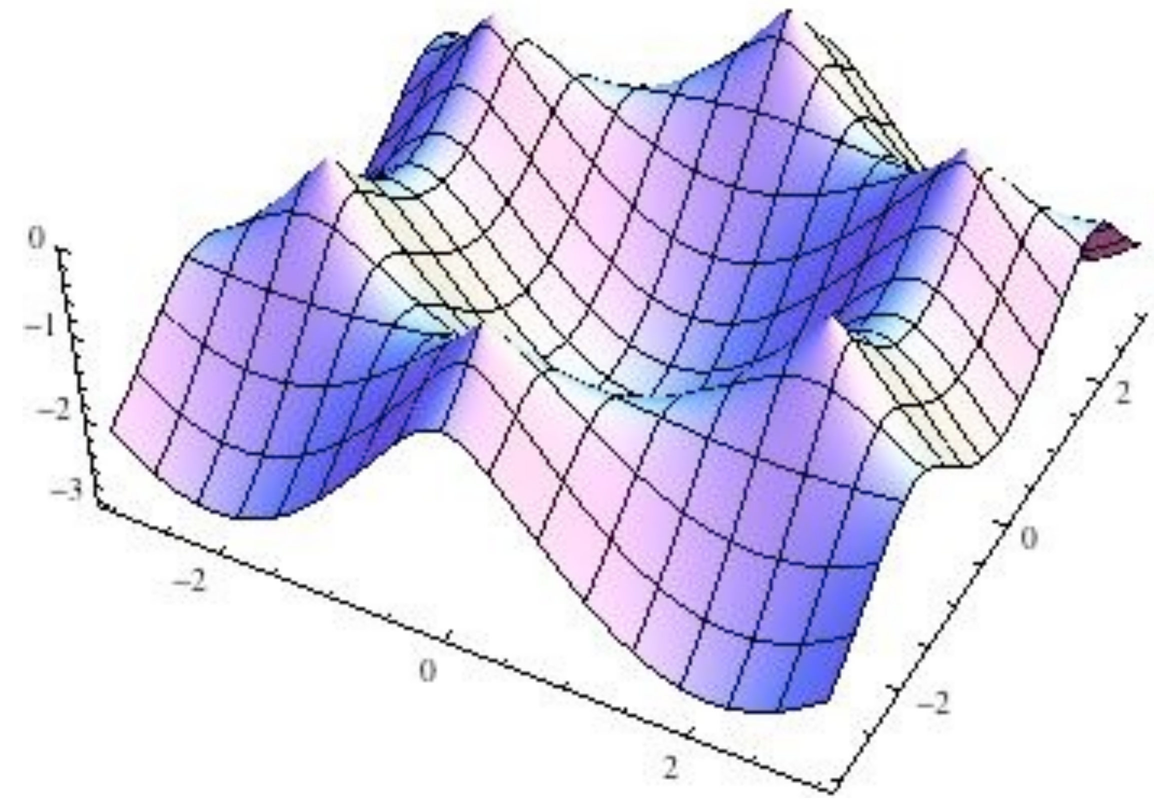
where

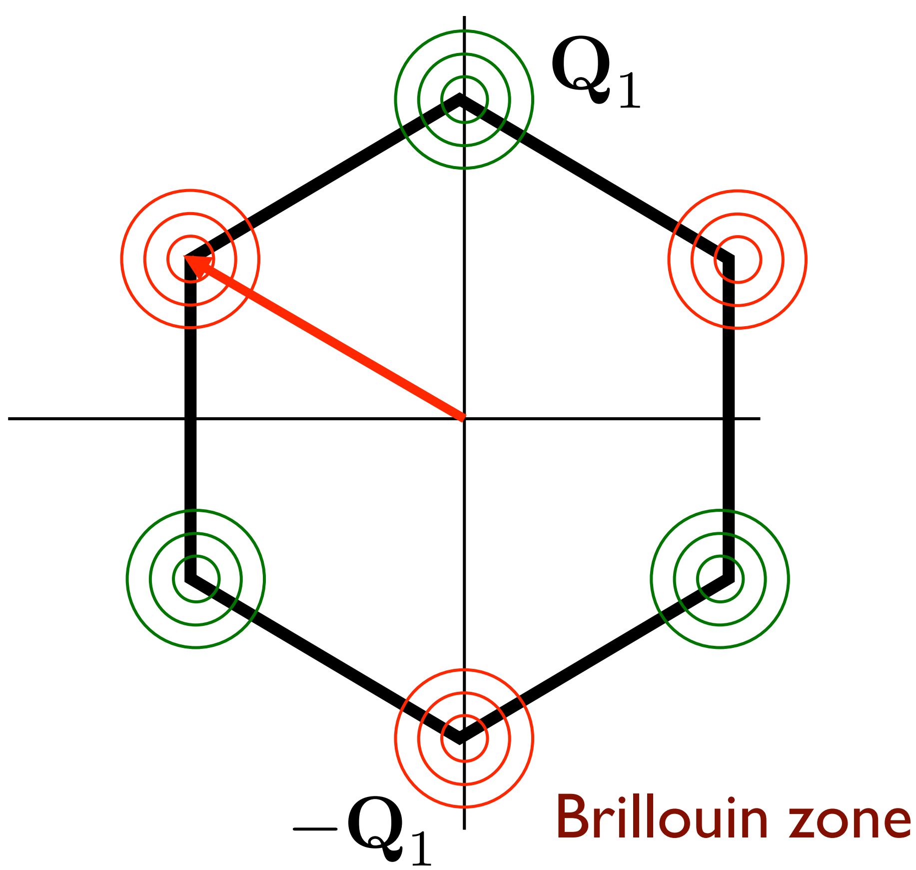
$$\gamma_0 = -\rho^z\tau^z \quad \gamma_1 = \rho^z\tau^x \quad \gamma_2 = -\tau^y \qquad (10)$$

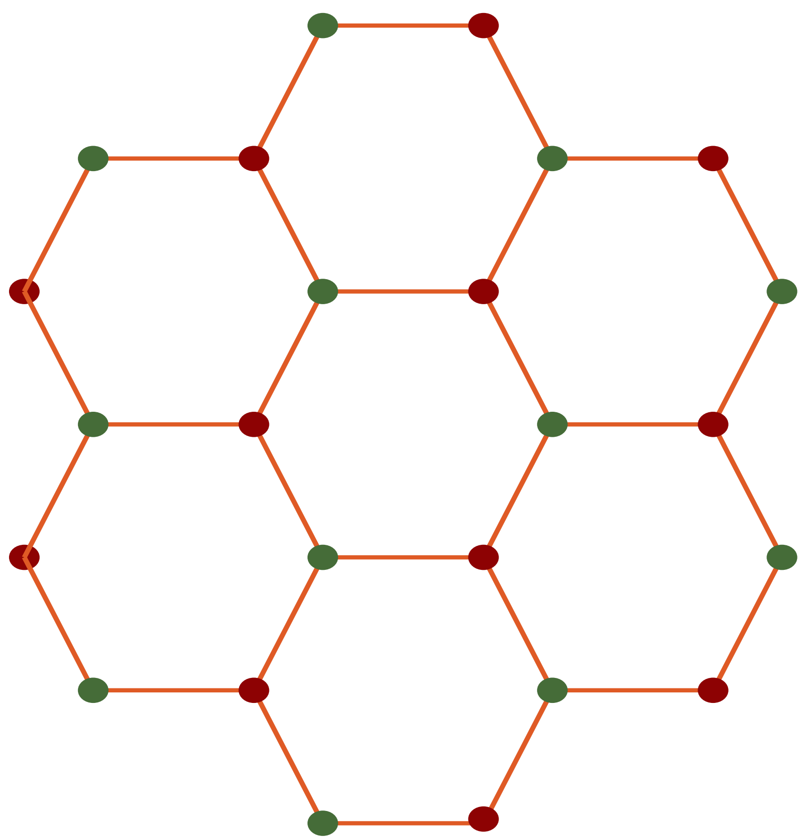
Exercise: Observe that $\mathcal{L}_0$ is invariant under the scaling transformation $x' = xe^{-\ell}$ and $\tau' = \tau e^{-\ell}$. Write the Hubbard interaction U in terms of the Dirac fermions, and show that it has the tree-level scaling transformation $U' = Ue^{-\ell}$. So argue that all short-range interactions are *irrelevant* in the Dirac semi-metal phase.

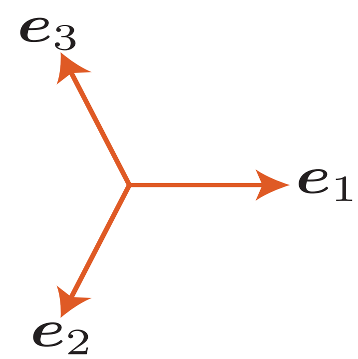


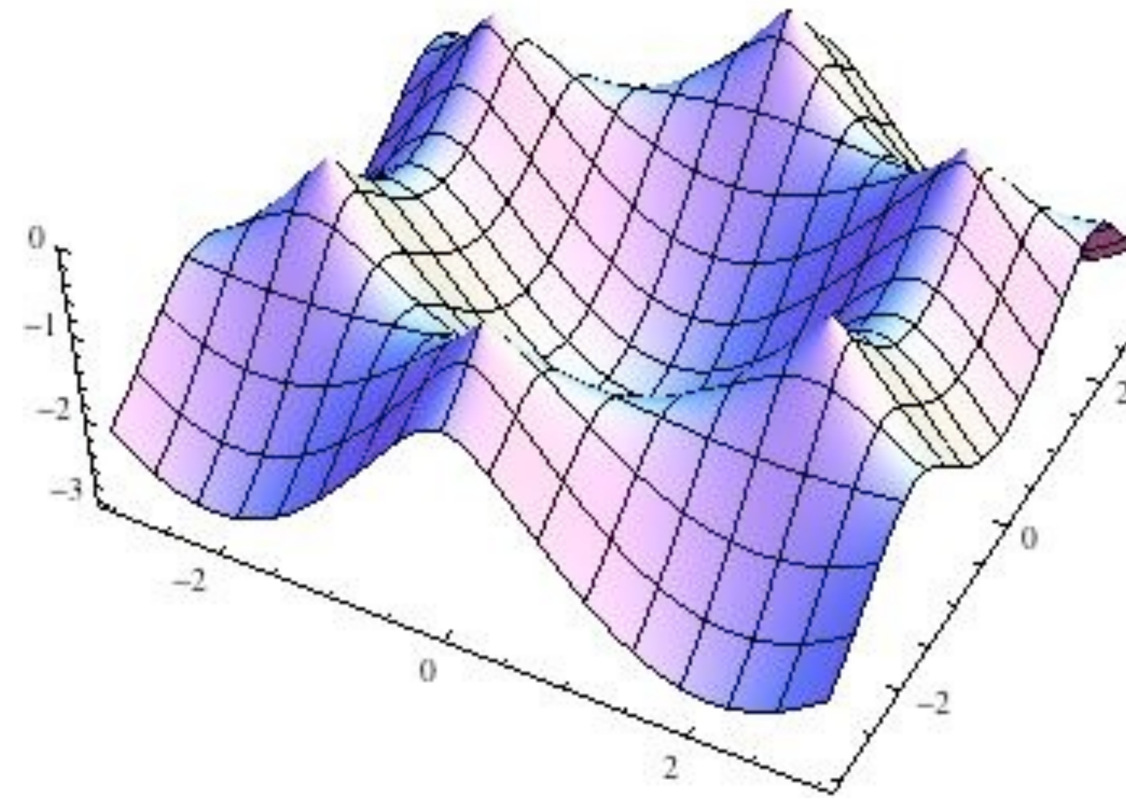

e_3
e_1
e_2

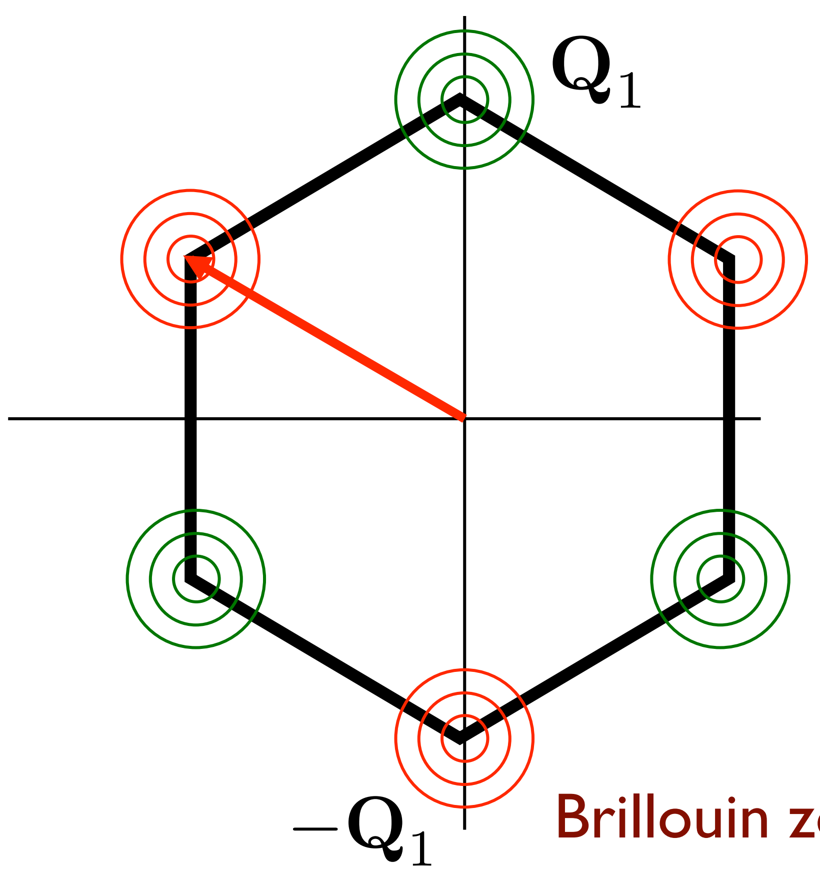

0
−1
−2
−3
−2
0
2
2
0
−2


$\mathbf{Q}_1$
$-\mathbf{Q}_1$
Brillouin zone









Brillouin zone

The theory of free Dirac fermions is invariant under conformal transformations of spacetime. This is a realization of a simple conformal field theory in 2+1 dimensions: a <u>CFT3</u>

The Hubbard Model at large U

$$H = -\sum_{i,j} t_{ij} c_{i\alpha}^\dagger c_{j\alpha} + U \sum_i \left(n_{i\uparrow} - \frac{1}{2}\right)\left(n_{i\downarrow} - \frac{1}{2}\right) - \mu \sum_i c_{i\alpha}^\dagger c_{i\alpha}$$

In the limit of large U, and at a density of one particle per site, this maps onto the Heisenberg antiferromagnet

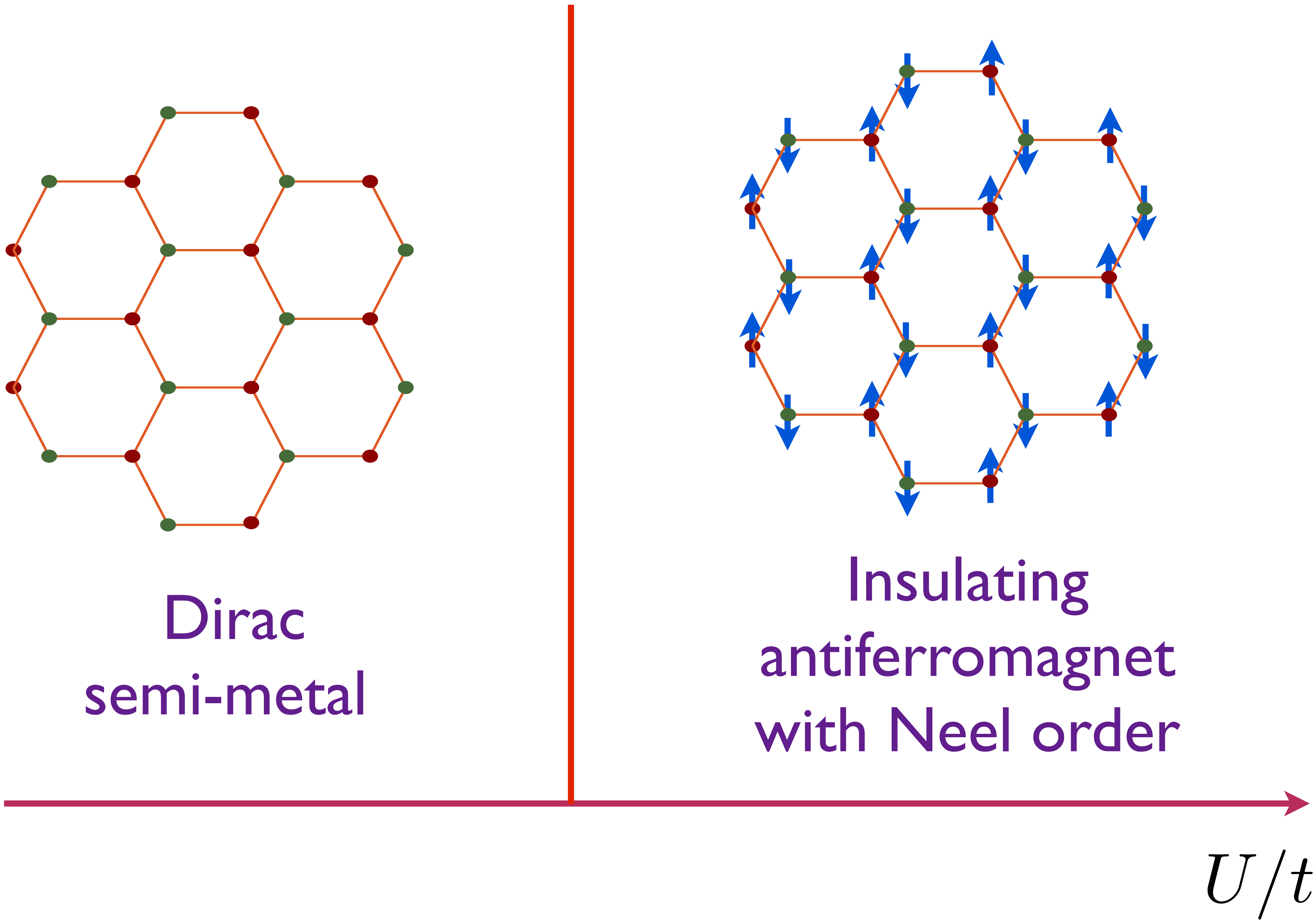
$$H_{AF} = \sum_{i<j} J_{ij} S_i^a S_j^a$$

where $a = x, y, z$,

$$S_i^a = \frac{1}{2} c_{i\alpha}^{a\dagger} \sigma_{\alpha\beta}^a c_{i\beta},$$

with σ^a the Pauli matrices and

$$J_{ij} = \frac{4t_{ij}^2}{U}$$


Dirac
semi-metal
Insulating
antiferromagnet
with Neel order
U/t

Antiferromagnetism

We use the operator equation (valid on each site i):

$$U\left(n_{\uparrow}-\frac{1}{2}\right)\left(n_{\downarrow}-\frac{1}{2}\right)=-\frac{2U}{3}S^{a2}+\frac{U}{4} \tag{11}$$

Then we decouple the interaction via

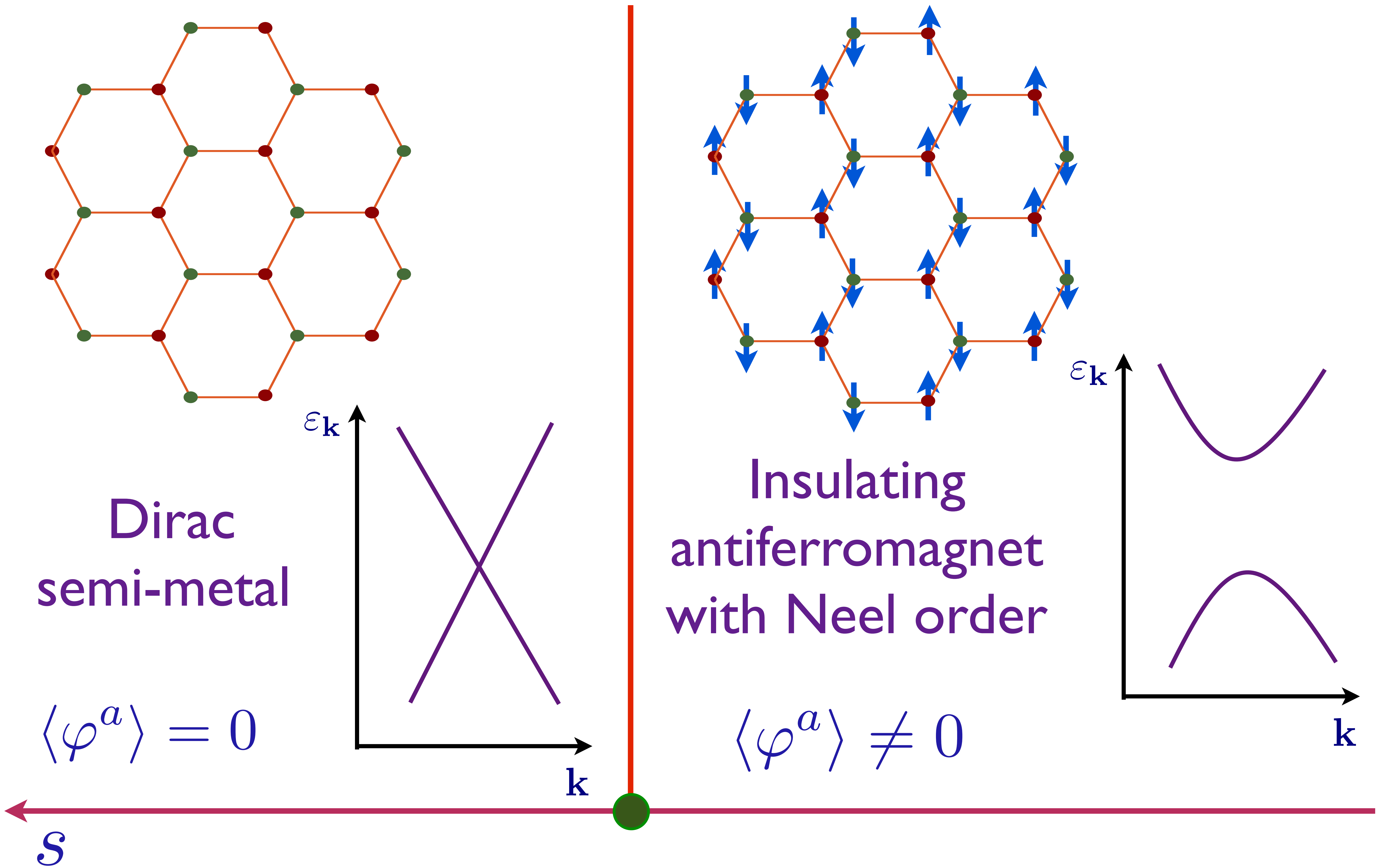
$$\exp\left(\frac{2U}{3}\sum_i\int d\tau S_i^{a2}\right)=\int\mathcal{D}J_i^a(\tau)\exp\left(-\sum_i\int d\tau\left[\frac{3}{8U}J_i^{a2}-J_i^aS_i^a\right]\right) \tag{12}$$

We now integrate out the fermions, and look for the saddle point of the resulting effective action for J_i^a.

Long wavelength fluctuations about this saddle point are described by a field theory of the Néel order parameter, φ^a, coupled to the Dirac fermions in the **Gross-Neveu** model.

$$\mathcal{L}=\overline{\Psi}\gamma_\mu\partial_\mu\Psi+\frac{1}{2}\left[(\partial_\mu\varphi^a)^2+s\varphi^{a2}\right]+\frac{u}{24}\left(\varphi^{a2}\right)^2-\lambda\varphi^a\overline{\Psi}\rho^z\sigma^a\Psi$$

I.F. Herbut, V. Juricic, and B. Roy, *Phys. Rev. B* **79**, 085116 (2009)



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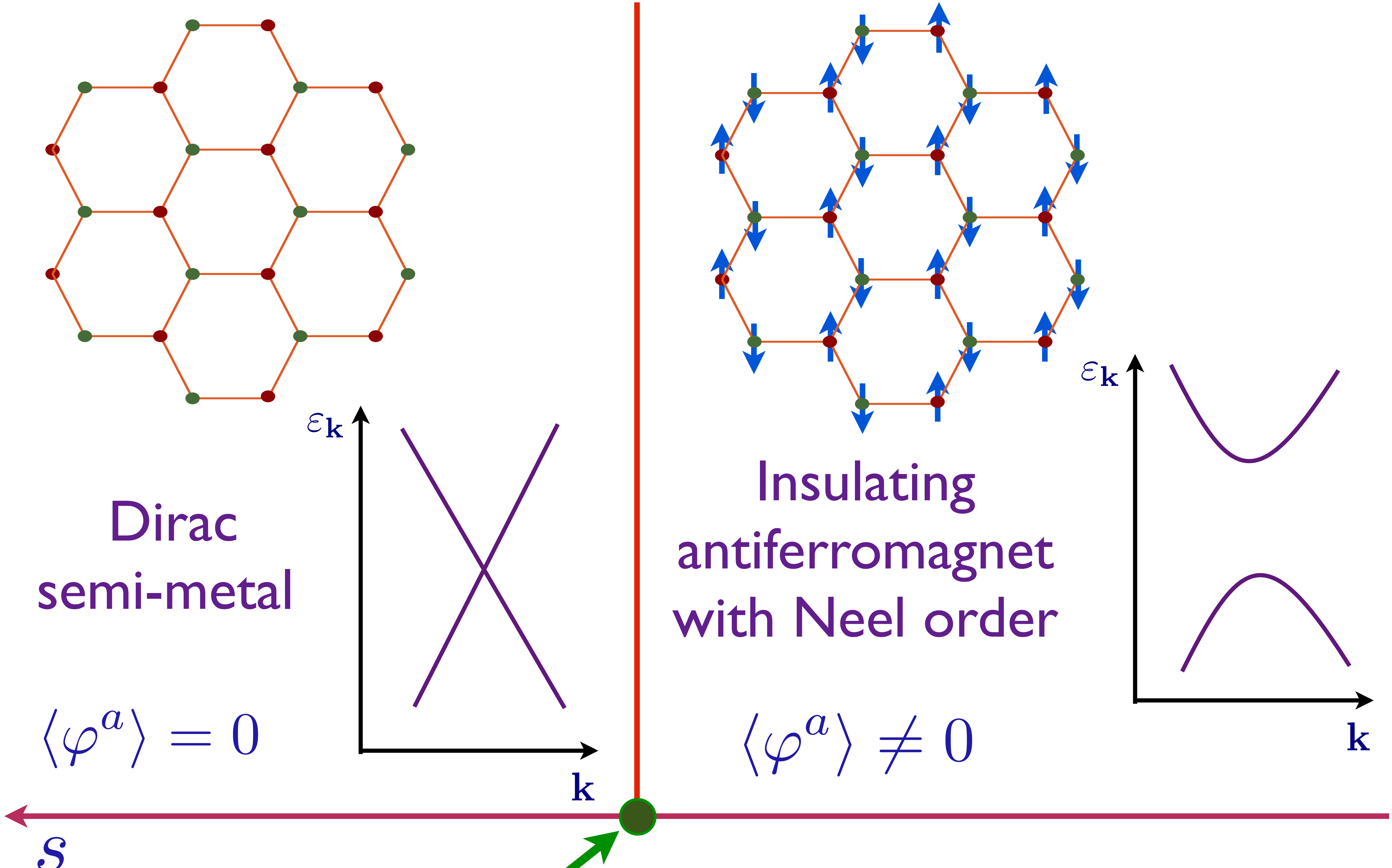
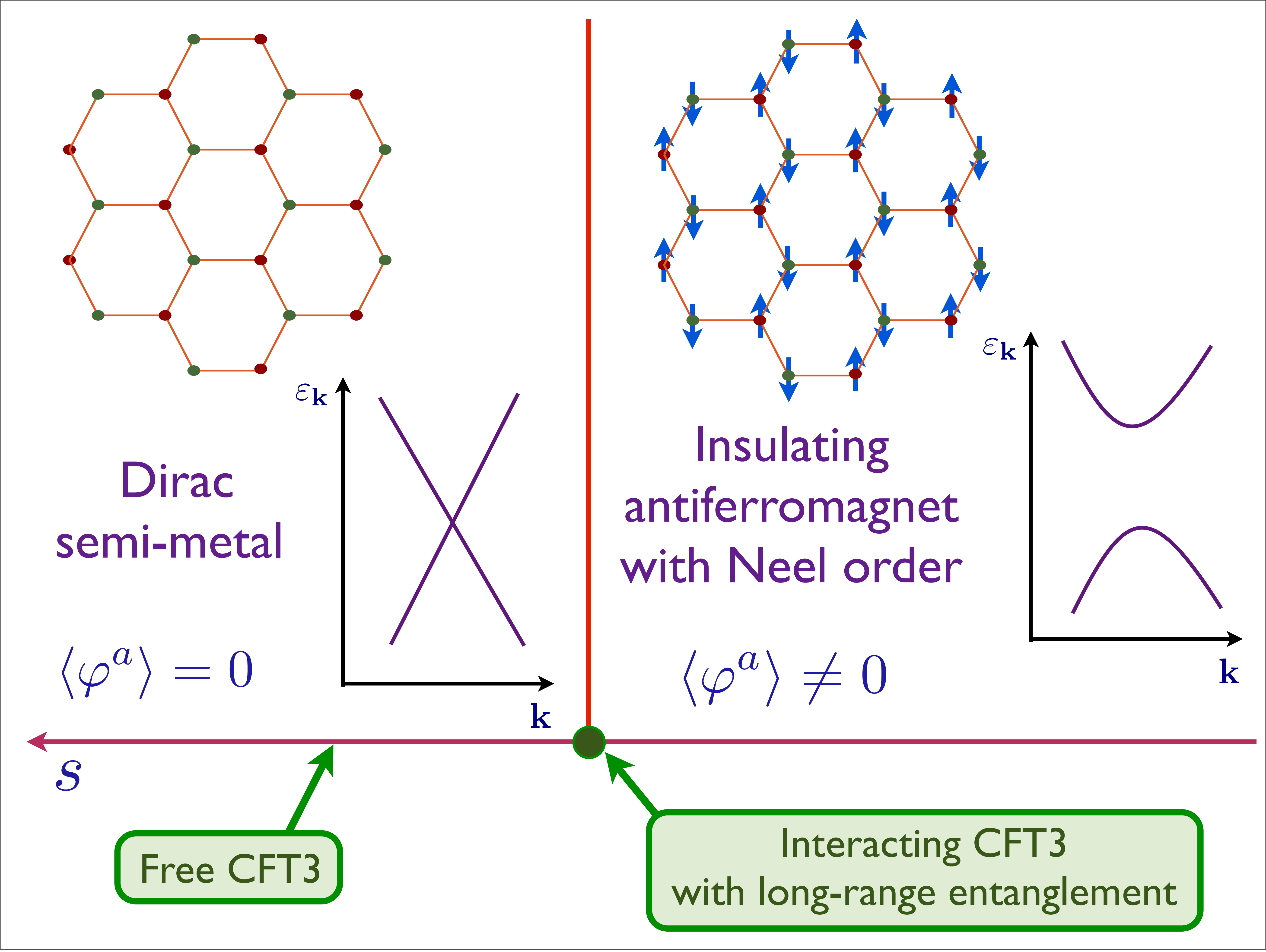


At the quantum critical point, the non-linear couplings λ and u in the Gross-Neveu model reach non-zero fixed-point values under the renormalization group flow. The critical theory is an *interacting* CFT3



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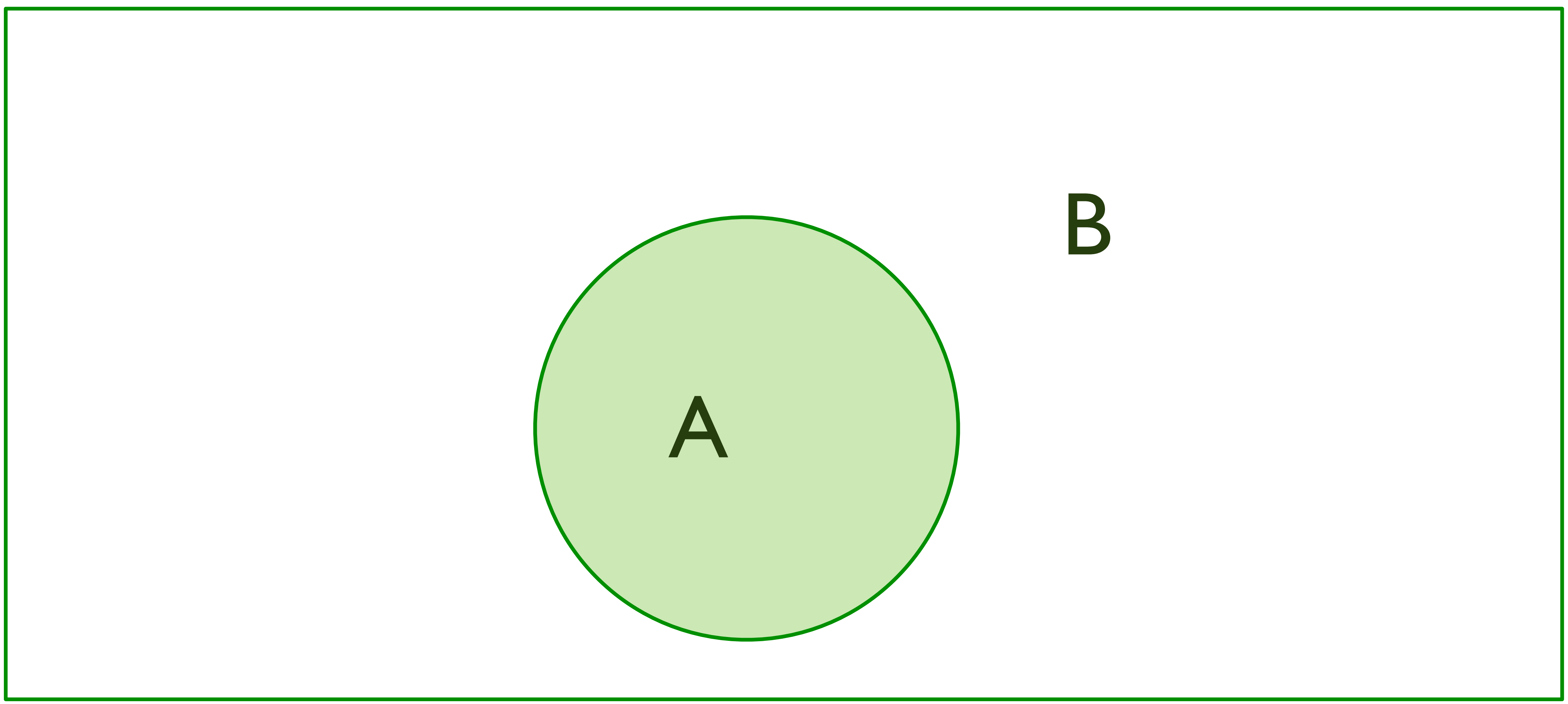
Long-range entanglement in a CFT3

- Long-range entanglement: entanglement entropy obeys $S_{EE} = aL - \gamma$, where γ is a universal number associated with the CFT3.

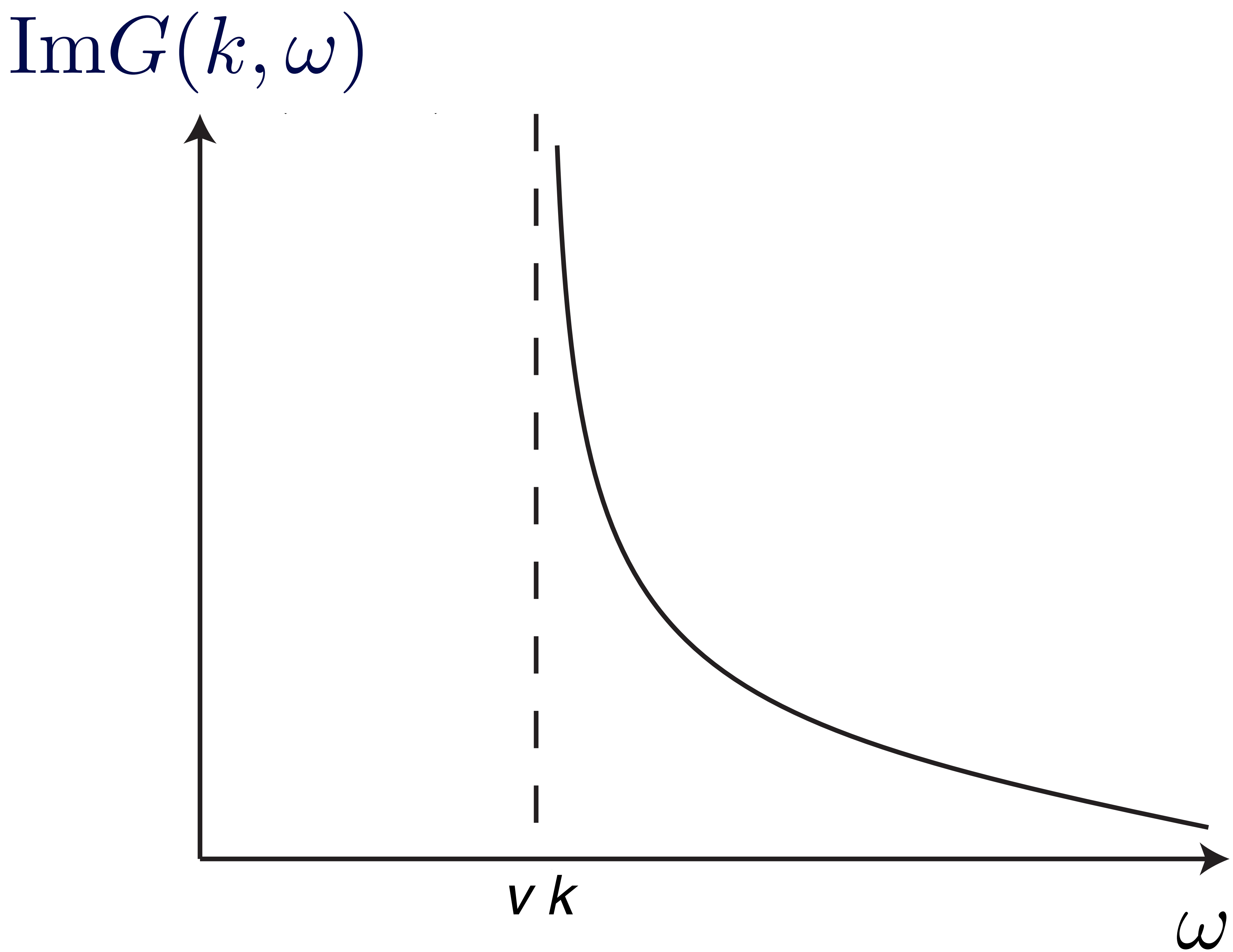


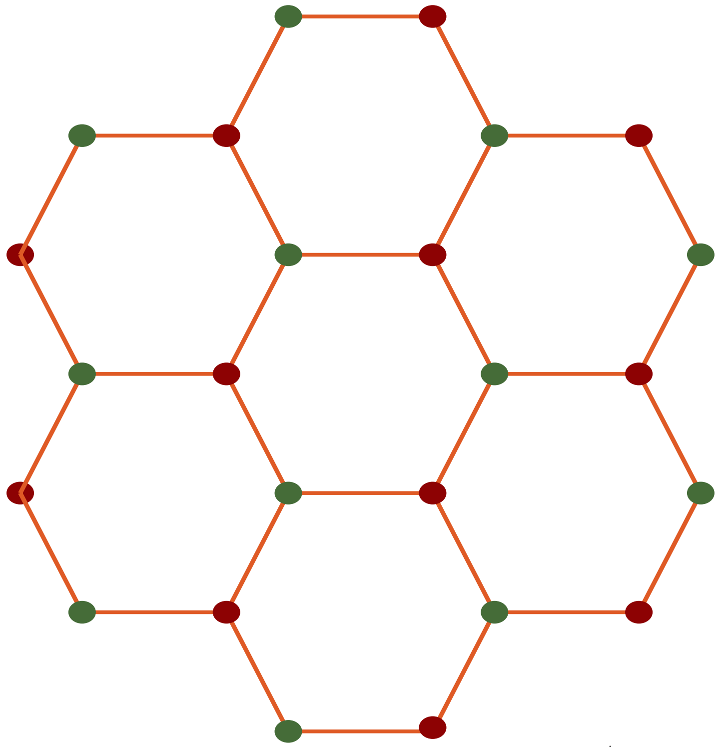
M.A. Metlitski, C.A. Fuertes, and S. Sachdev, Physical Review B 80, 115122 (2009).
H. Casini, M. Huerta, and R. Myers, JHEP 1105:036, (2011)
I. Klebanov, S. Pufu, and B. Safdi, arXiv:1105.4598

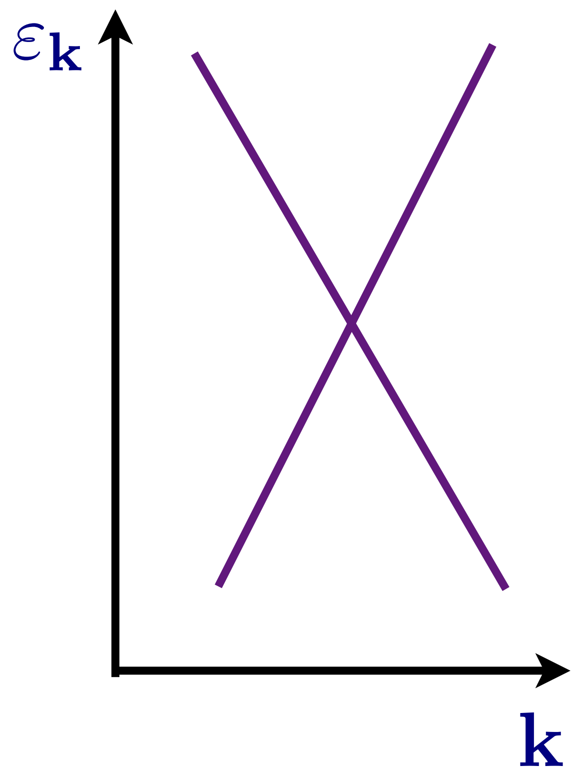
Electron Green's function for the interacting CFT3

$$G(k,\omega) = \left\langle \Psi(k,\omega); \Psi^\dagger(k,\omega) \right\rangle \sim \frac{i\omega + v k_x \tau^y + v k_y \tau^x \rho^z}{(\omega^2 + v^2 k_x^2 + v^2 k_y^2)^{1-\eta/2}}$$

where $\eta > 0$ is the *anomalous dimension* of the fermion. Note that this leads to a fermion spectral density which has no quasiparticle pole: thus the quantum critical point has no well-defined quasiparticle excitations.

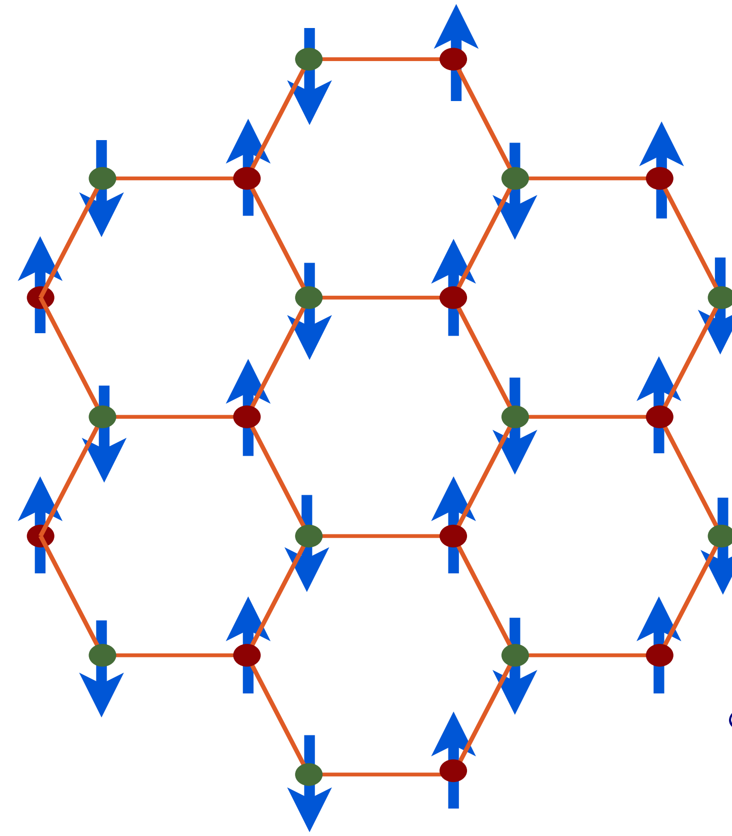

$\mathrm{Im}G(k,\omega)$
$v\,k$
ω



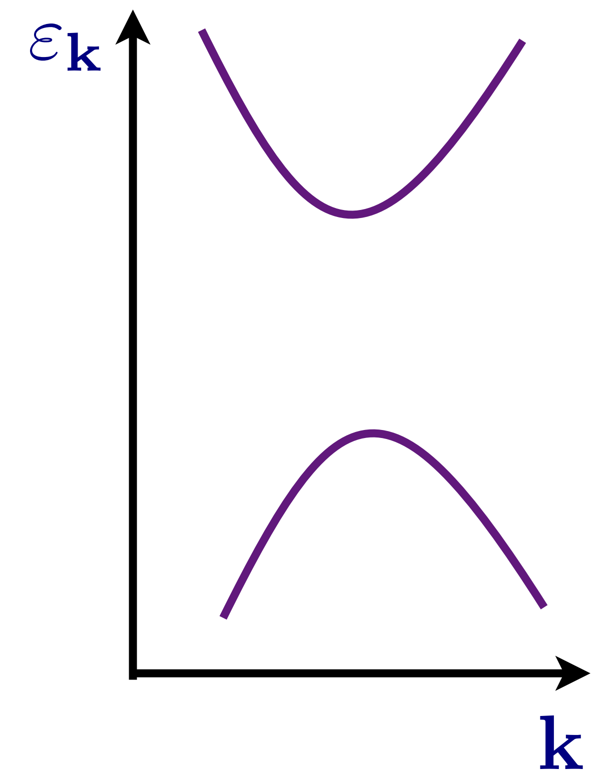


Dirac
semi-metal

$$\langle \varphi^a \rangle = 0$$



Insulating
antiferromagnet
with Neel order



$$\langle \varphi^a \rangle \neq 0$$

s

Quantum phase transition described by a strongly-coupled conformal field theory without well-defined quasiparticles

Electrical transport

The conserved electrical current is

$$J_\mu = -i\overline{\Psi}\gamma_\mu \Psi. \qquad (1)$$

Let us compute its two-point correlator, $K_{\mu\nu}(k)$ at a spacetime momentum k_μ at $T = 0$. At leading order, this is given by a one fermion loop diagram which evaluates to

$$\begin{aligned} K_{\mu\nu}(k) &= \int \frac{d^3p}{8\pi^3} \frac{\text{Tr}\left[\gamma_\mu(i\gamma_\lambda p_\lambda + m\rho^z\sigma^z)\gamma_\nu(i\gamma_\delta(k_\delta + p_\delta) + m\rho^z\sigma^z)\right]}{(p^2+m^2)((p+k)^2+m^2)} \\ &= -\frac{2}{\pi}\left(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\int_0^1 dx \frac{k^2 x(1-x)}{\sqrt{m^2 + k^2 x(1-x)}}, \end{aligned} \qquad (2)$$

where the mass $m = 0$ in the semi-metal and at the quantum critical point, while $m = |\lambda N_0|$ in the insulator. Note that the current correlation is purely transverse, and this follows from the requirement of current conservation

$$k_\mu K_{\mu\nu} = 0. \qquad (3)$$

Of particular interest to us is the K_{00} component, after analytic continuation to Minkowski space where the spacetime momentum k_μ is replaced by (ω, k). The conductivity is obtained from this correlator via the Kubo formula

$$\sigma(\omega) = \lim_{k \to 0} \frac{-i\omega}{k^2} K_{00}(\omega, k). \tag{4}$$

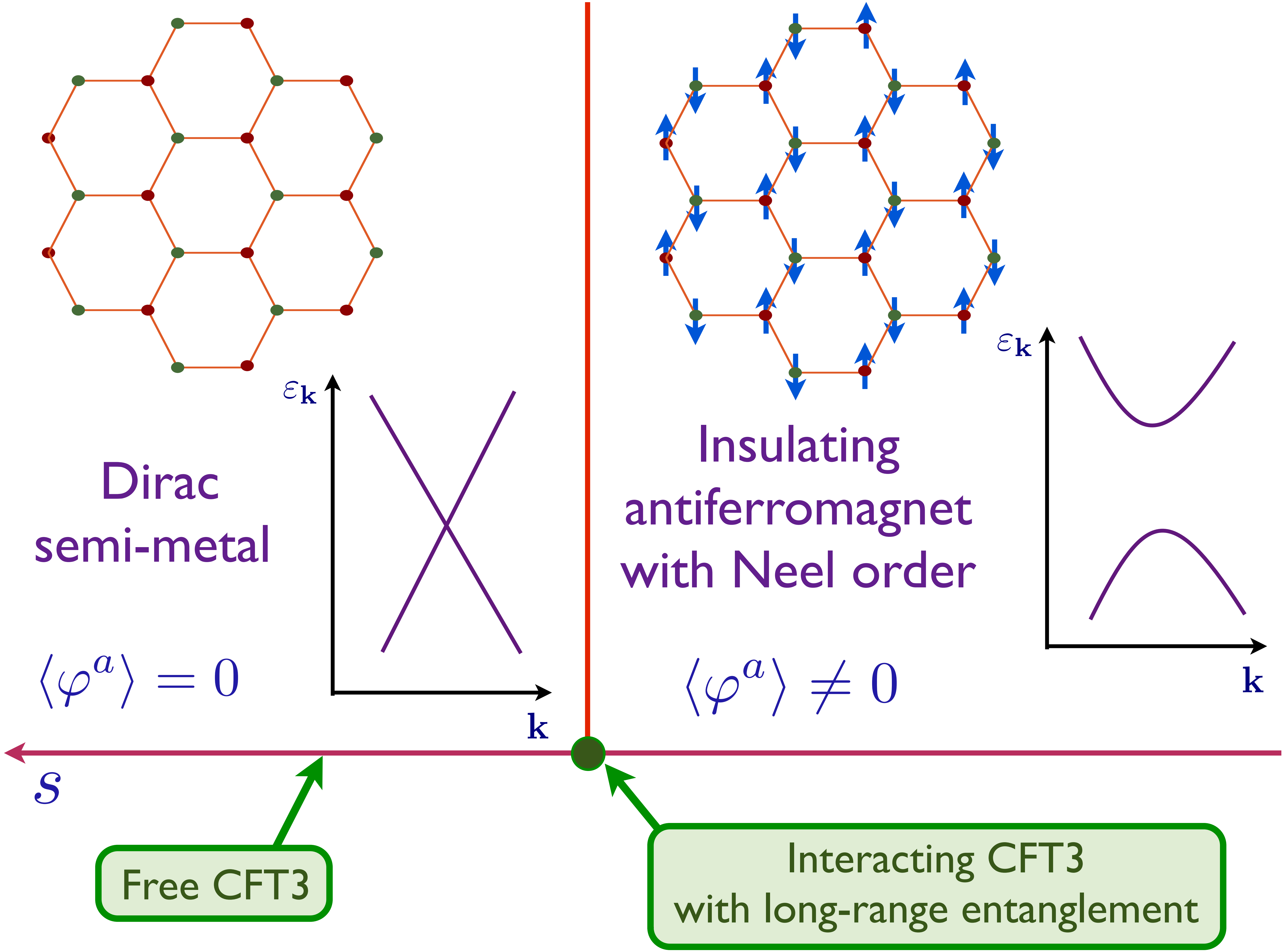
In the insulator, where $m > 0$, analysis of the integrand in Eq. (2) shows that that the spectral weight of the density correlator has a gap of $2m$ at $k = 0$, and the conductivity in Eq. (4) vanishes. These properties are as expected in any insulator.
In the metal, and at the critical point, where $m = 0$, the fermionic spectrum is gapless, and so is that of the charge correlator. The density correlator in Eq. (2) and the conductivity in Eq. (4) evaluate to the simple universal results

$$\begin{aligned} K_{00}(\omega, k) &= \frac{1}{4} \frac{k^2}{\sqrt{k^2 - \omega^2}} \\ \sigma(\omega) &= 1/4. \end{aligned} \tag{5}$$

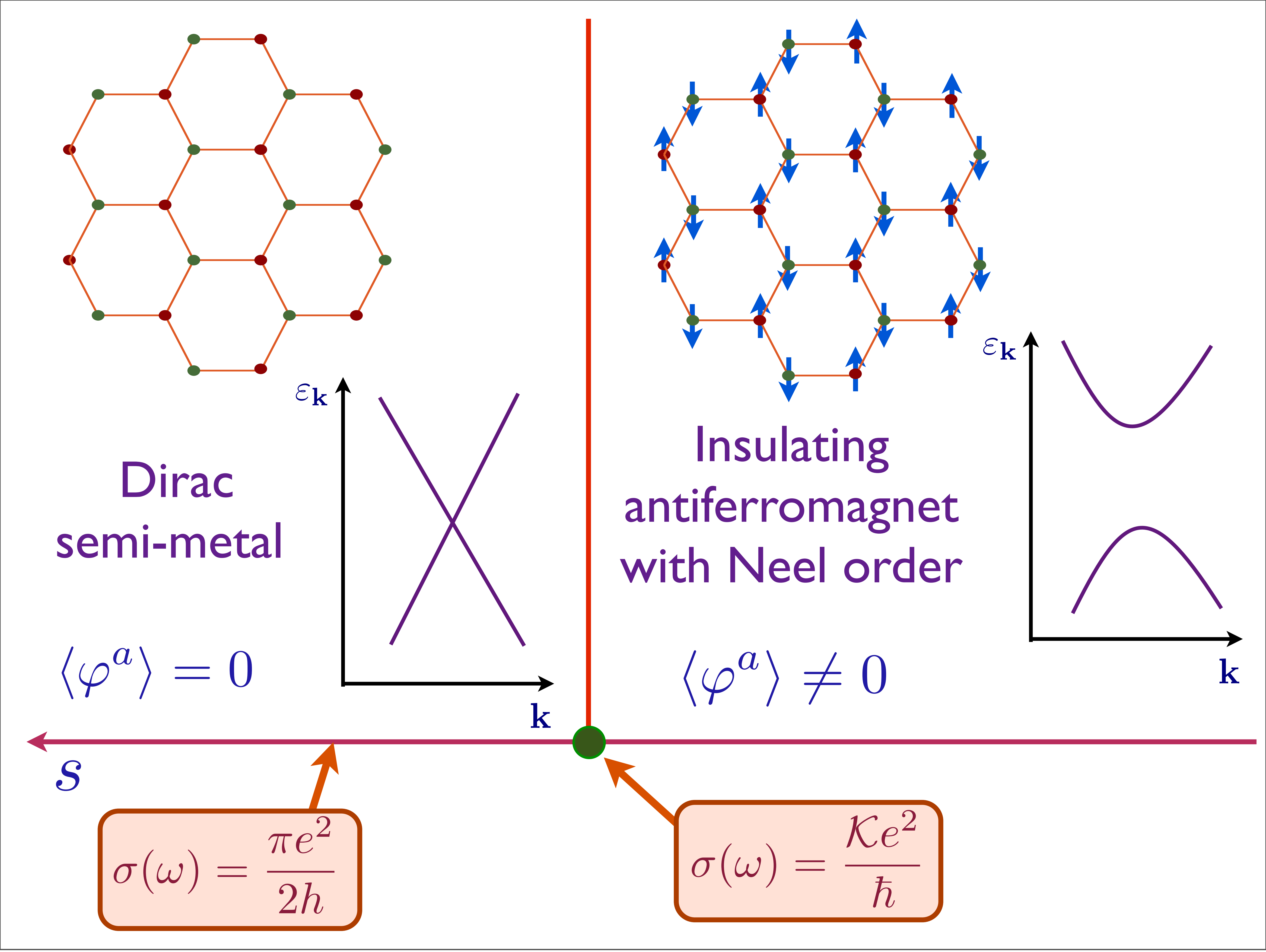
Going beyond one-loop, we find *no change* in these results in the semi-metal to all orders in perturbation theory. At the quantum critical point, there are no anomalous dimensions for the conserved current, but the amplitude does change yielding

$$
\begin{aligned}
K_{00}(\omega, k) &= \mathcal{K} \frac{k^2}{\sqrt{k^2 - \omega^2}} \\
\sigma(\omega) &= \mathcal{K},
\end{aligned}
\tag{6}
$$

where $\mathcal{K}$ is a universal number dependent only upon the universality class of the quantum critical point. The value of the $\mathcal{K}$ for the Gross-Neveu model is not known exactly, but can be estimated by computations in the $(3-d)$ or $1/N$ expansions.



$\varepsilon_{\mathbf{k}}$
Dirac
semi-metal
$\langle \varphi^a \rangle = 0$
$\mathbf{k}$
Insulating
antiferromagnet
with Neel order
$\varepsilon_{\mathbf{k}}$
$\langle \varphi^a \rangle \neq 0$
$\mathbf{k}$
s
Free CFT3
Interacting CFT3
with long-range entanglement



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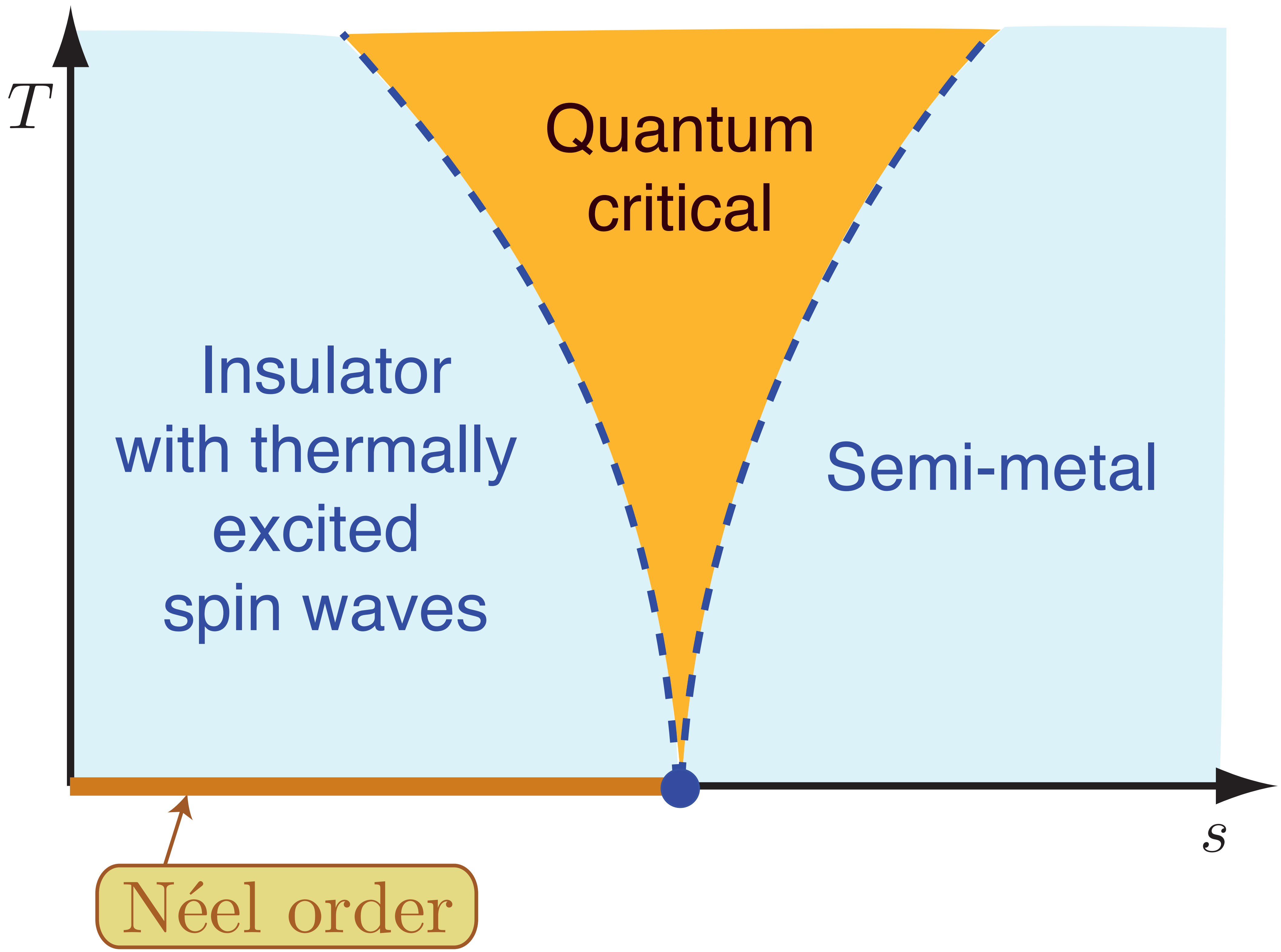
Phase diagram at non-zero temperatures



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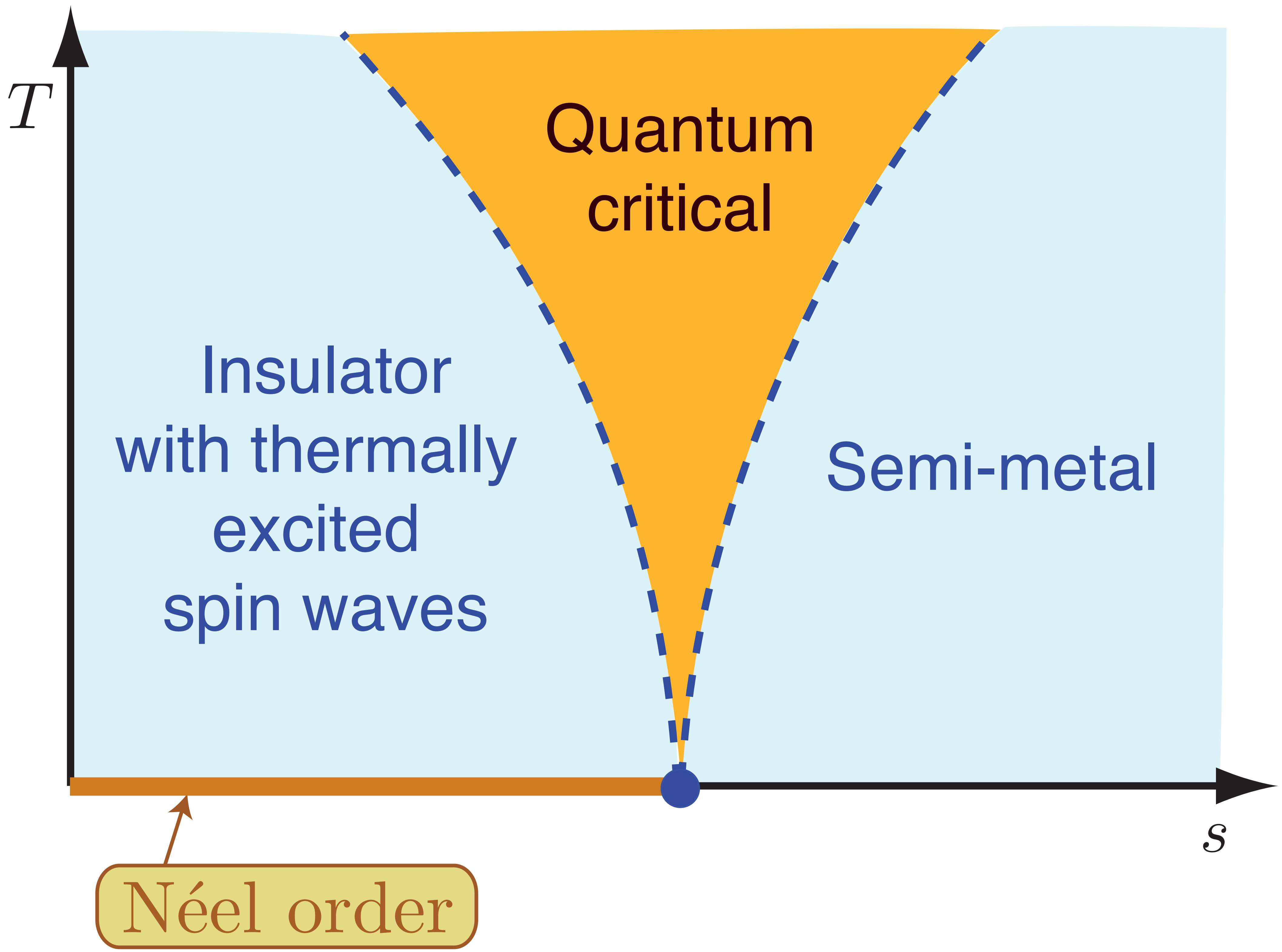

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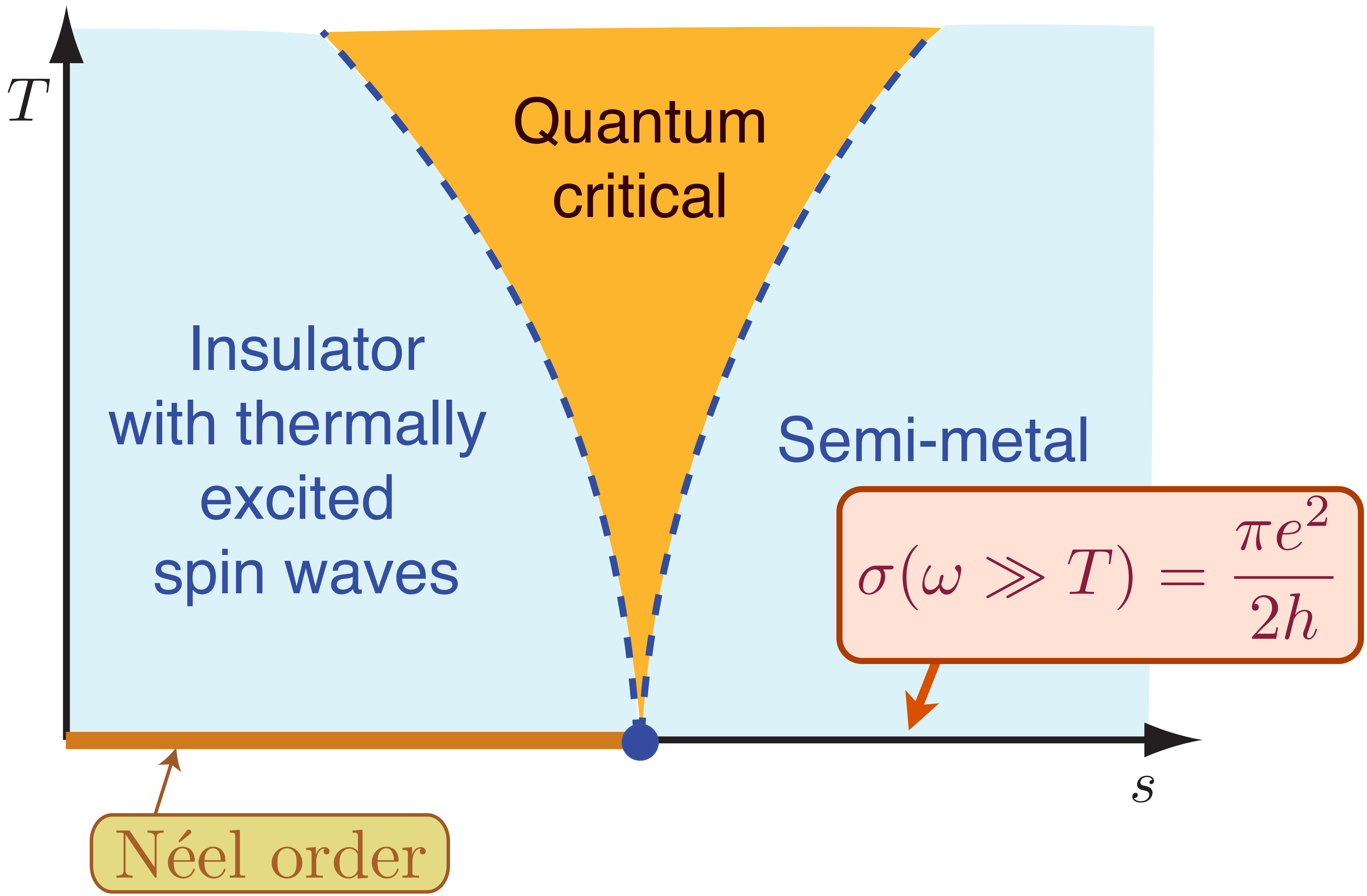
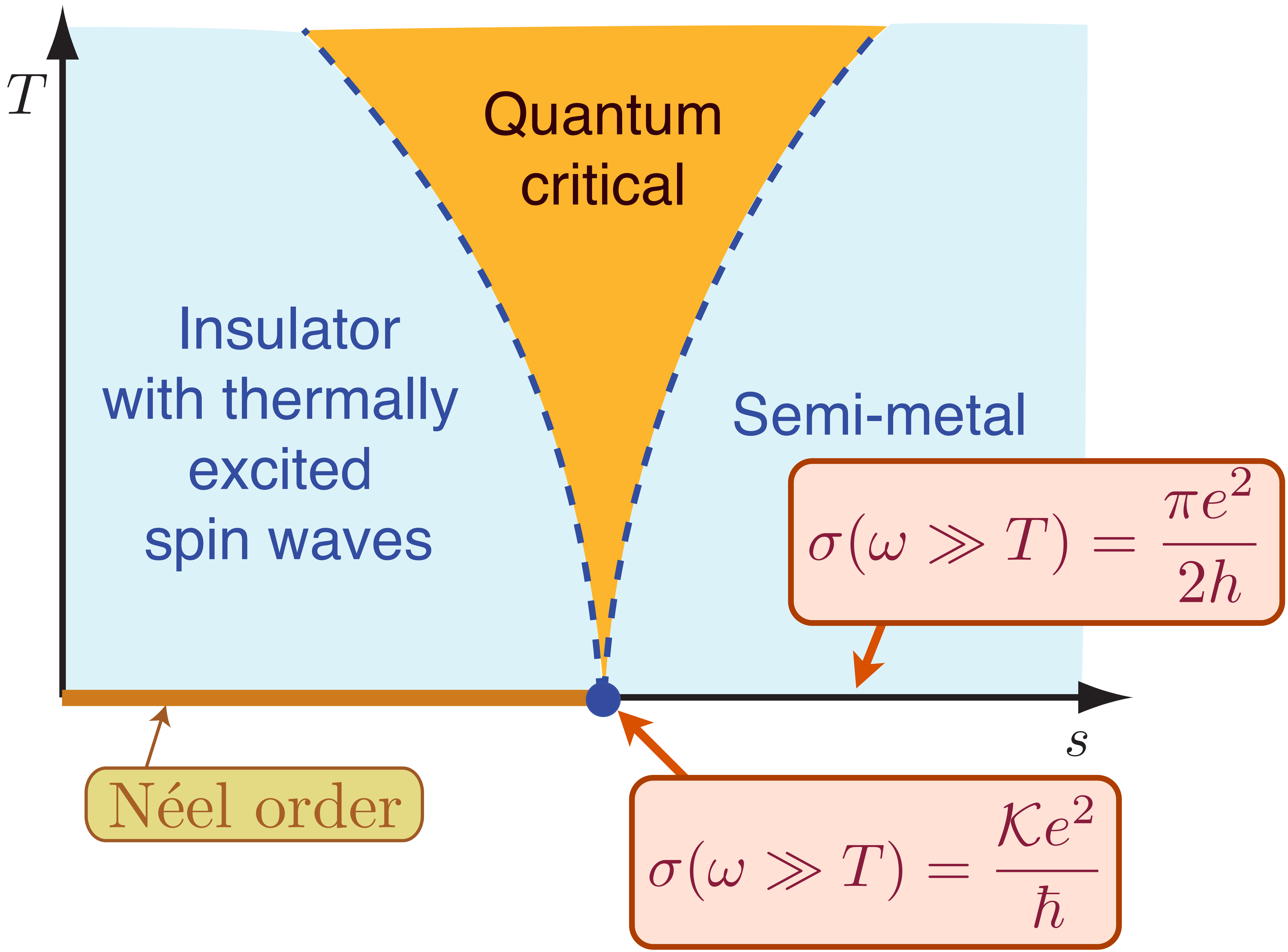
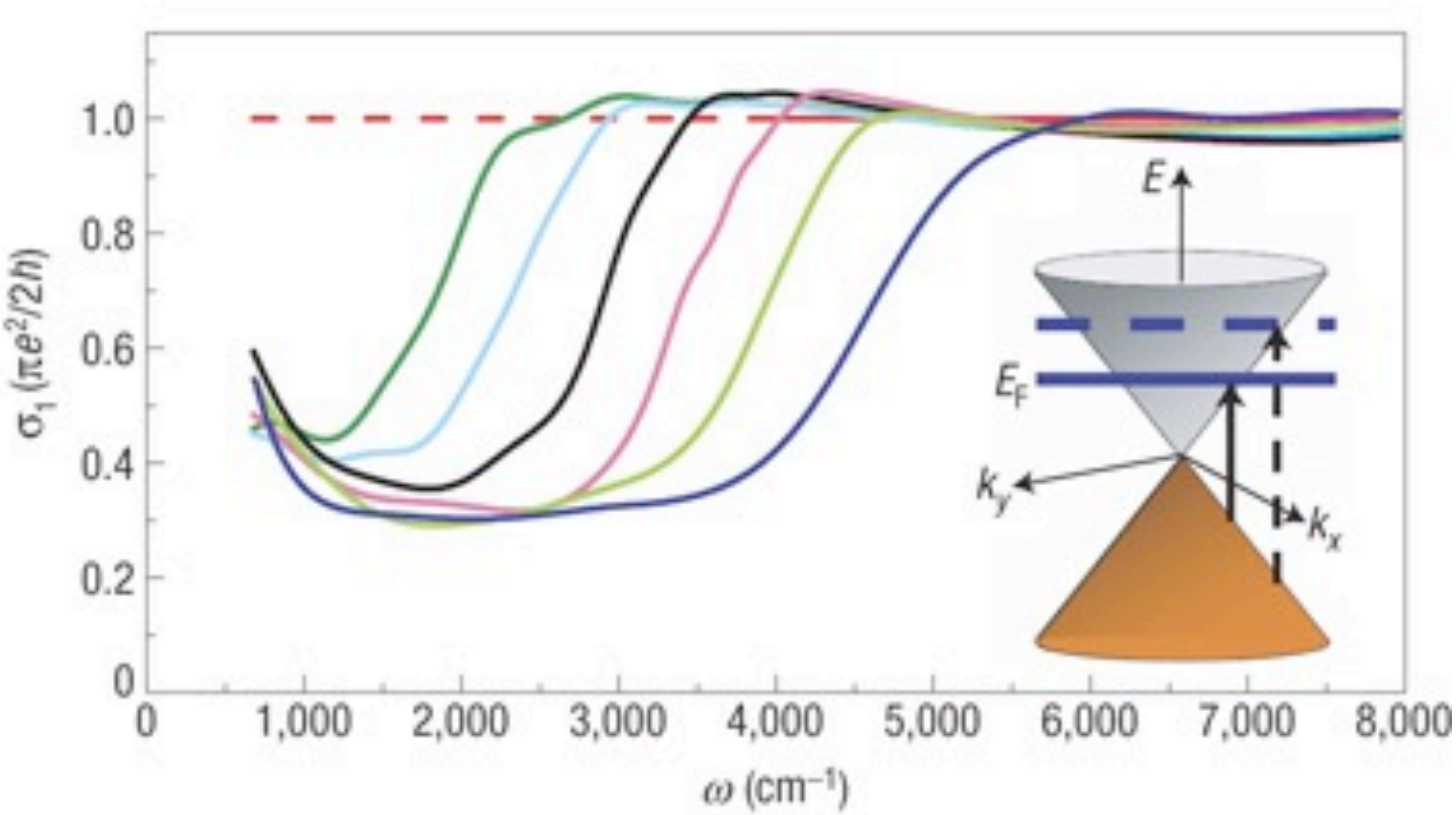


Phase diagram at non-zero temperatures



Optical conductivity of graphene



Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, Nature Physics 4, 532 (2008).

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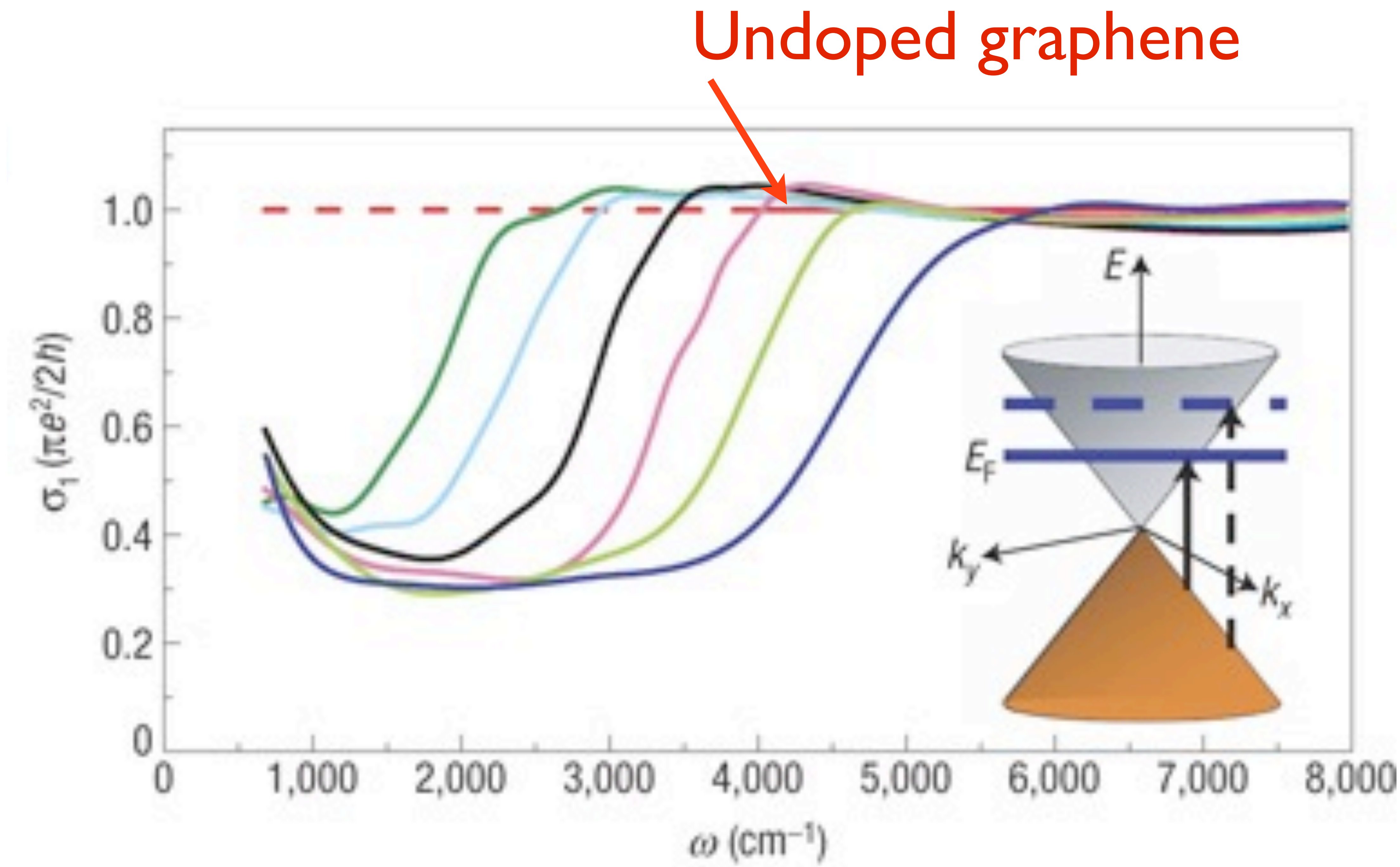


Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, *Nature Physics* **4**, 532 (2008).

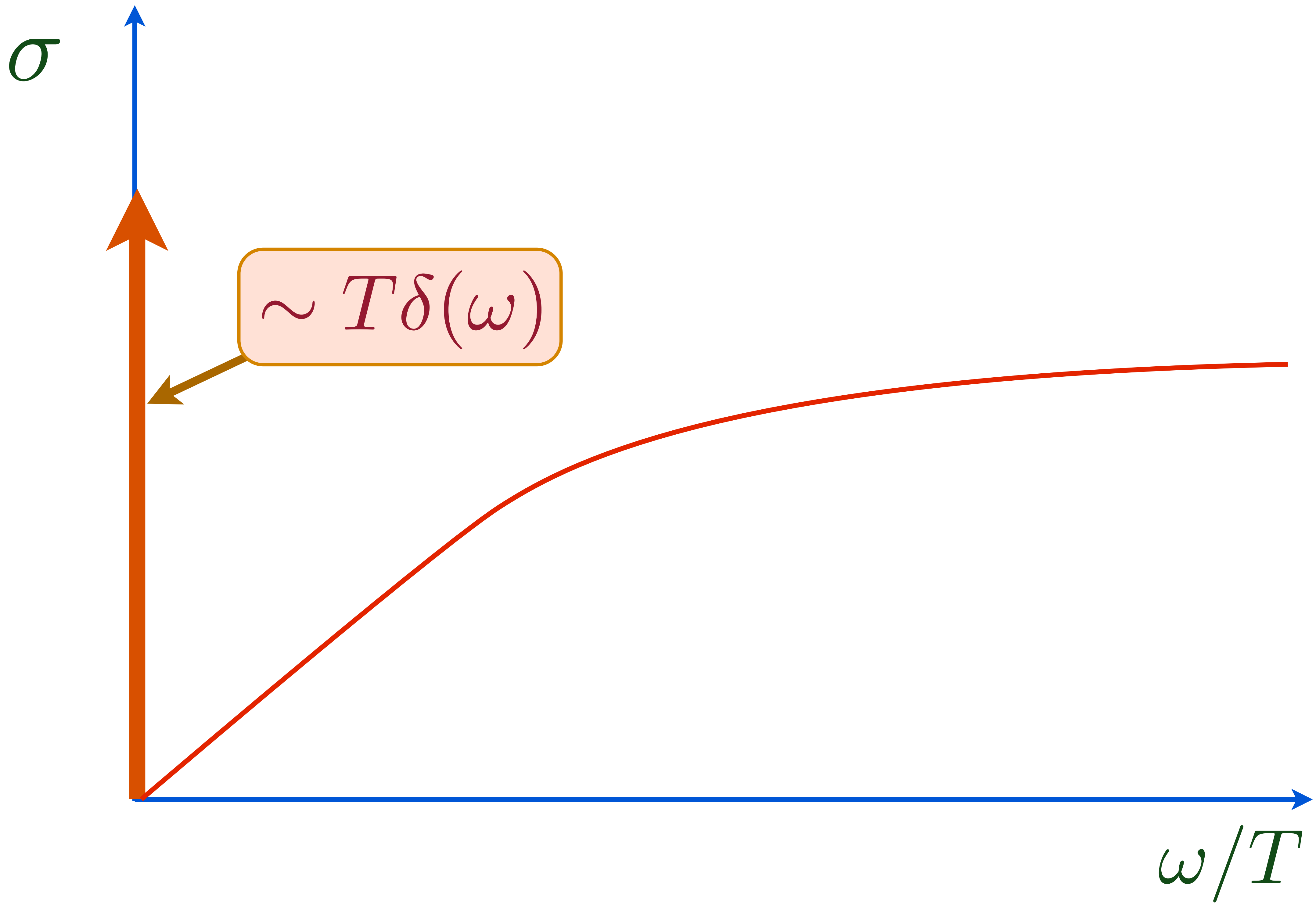
Non-zero temperatures

At the quantum-critical point at one-loop order, we can set $m = 0$, and then repeat the computation in Eq. (2) at $T > 0$. This only requires replacing the integral over the loop frequency by a summation over the Matsubara frequencies, which are quantized by odd multiples of πT. Such a computation, via Eq. (4) leads to the conductivity

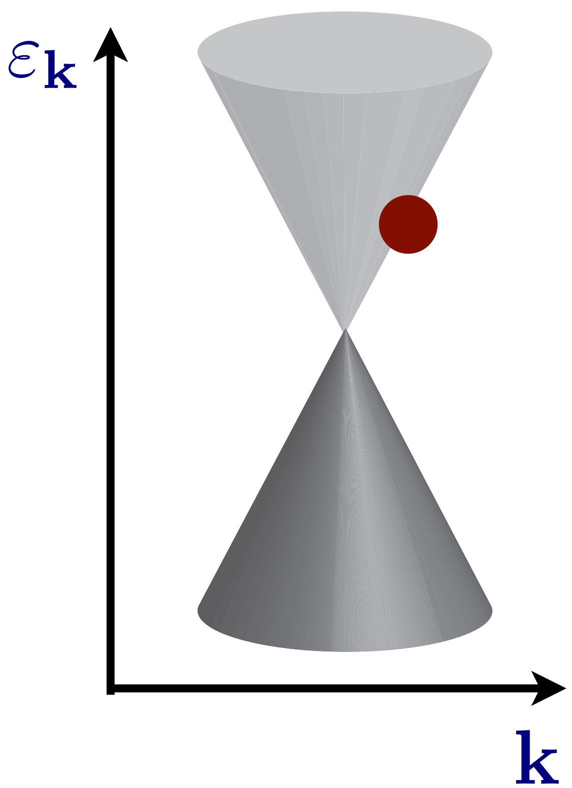
$$\text{Re}[\sigma(\omega)] = (2T \ln 2)\, \delta(\omega) + \frac{1}{4} \tanh\left(\frac{|\omega|}{4T}\right); \tag{7}$$

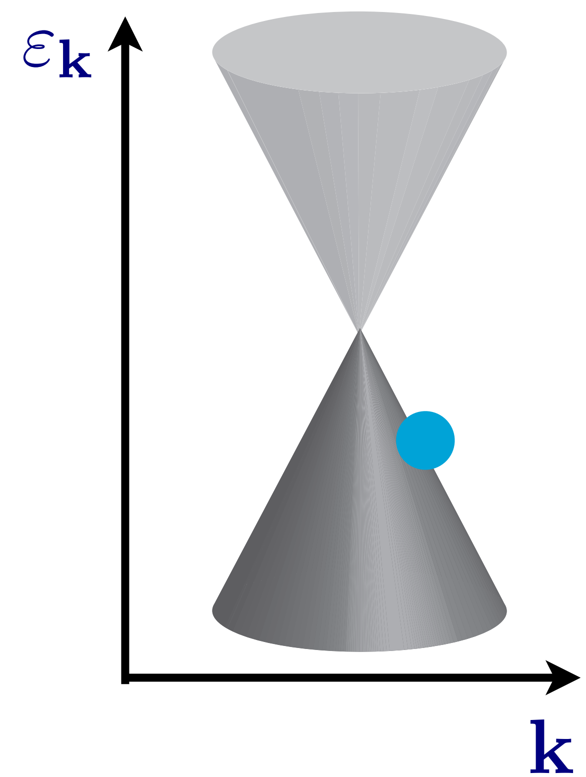
the imaginary part of $\sigma(\omega)$ is the Hilbert transform of $\text{Re}[\sigma(\omega)] - 1/4$. Note that this reduces to Eq. (5) in the limit $\omega \gg T$. However, the most important new feature of Eq. (7) arises for $\omega \ll T$, where we find a delta function at zero frequency in the real part. Thus the d.c. conductivity is infinite at this order, arising from the collisionless transport of thermally excited carriers.

Electrical transport in a free CFT3 for *T* > *0*



Particles	Holes
→ Momentum	← Momentum
→ Current	→ Current

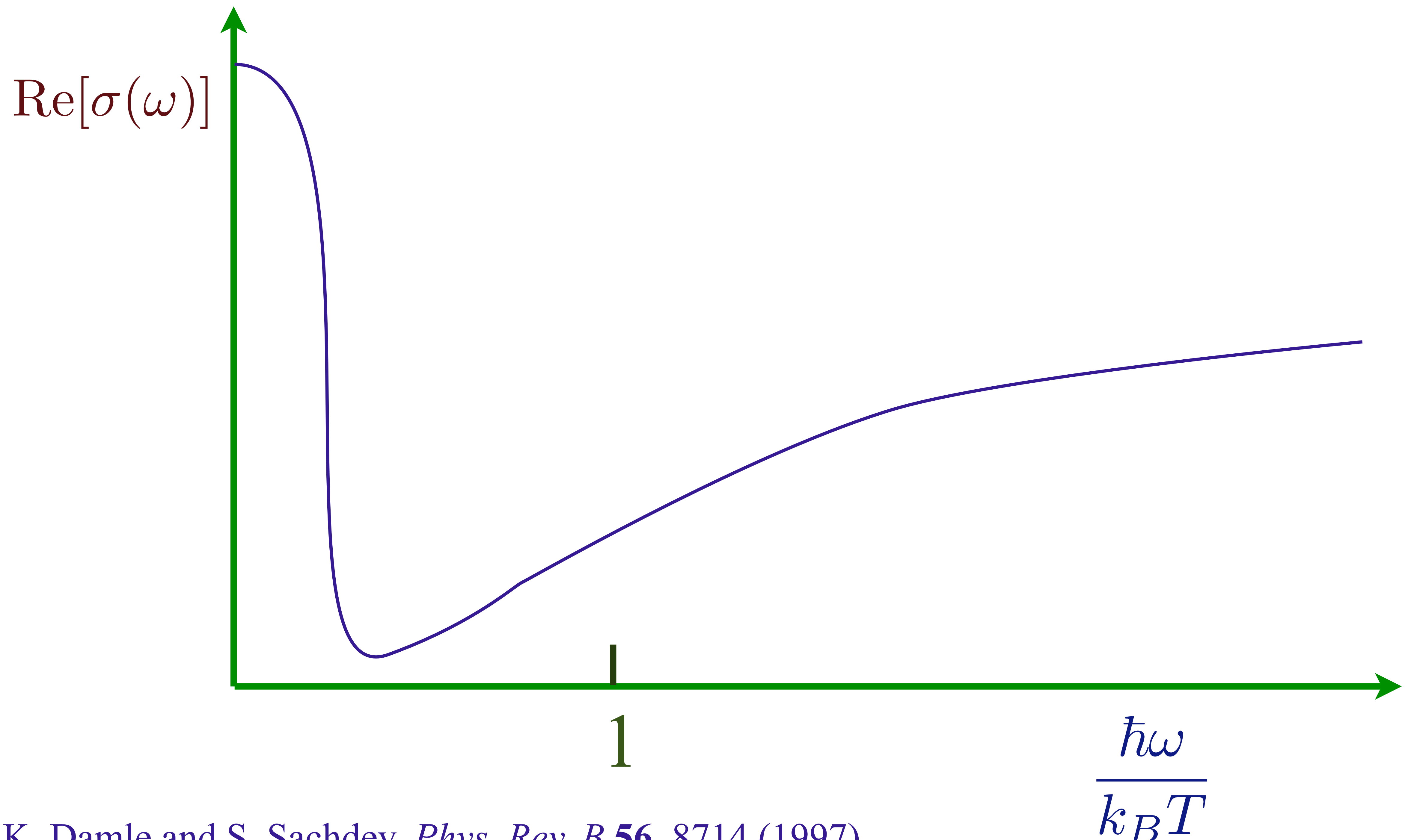




Particle hole symmetry: current carrying state has zero momentum, and collisions can relax current to zero

Electrical transport for a (weakly) interacting CFT3

$$\sigma(\omega, T) = \frac{e^2}{h}\Sigma\left(\frac{\hbar\omega}{k_B T}\right) \quad ; \quad \Sigma \to \text{a universal function}$$



K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

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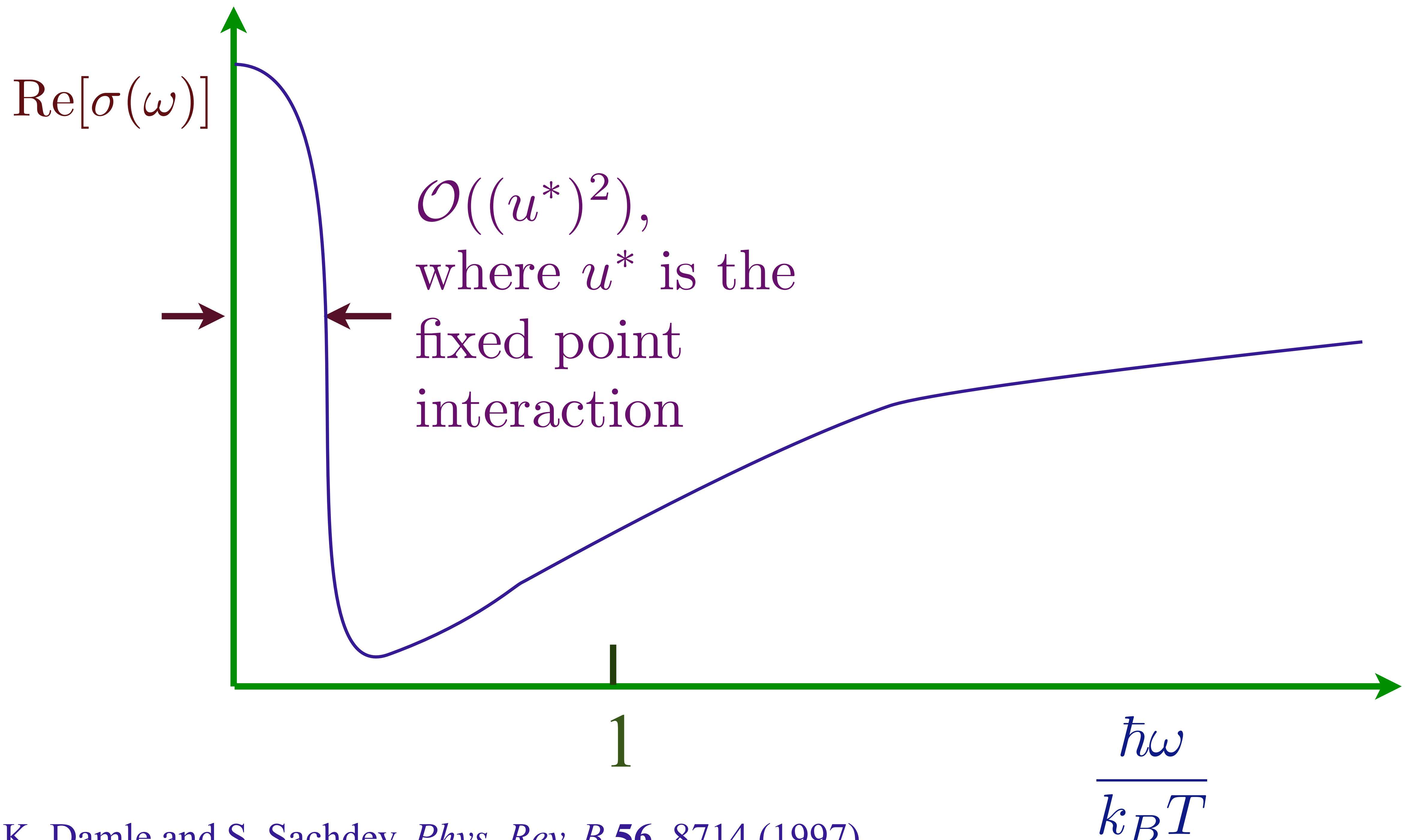

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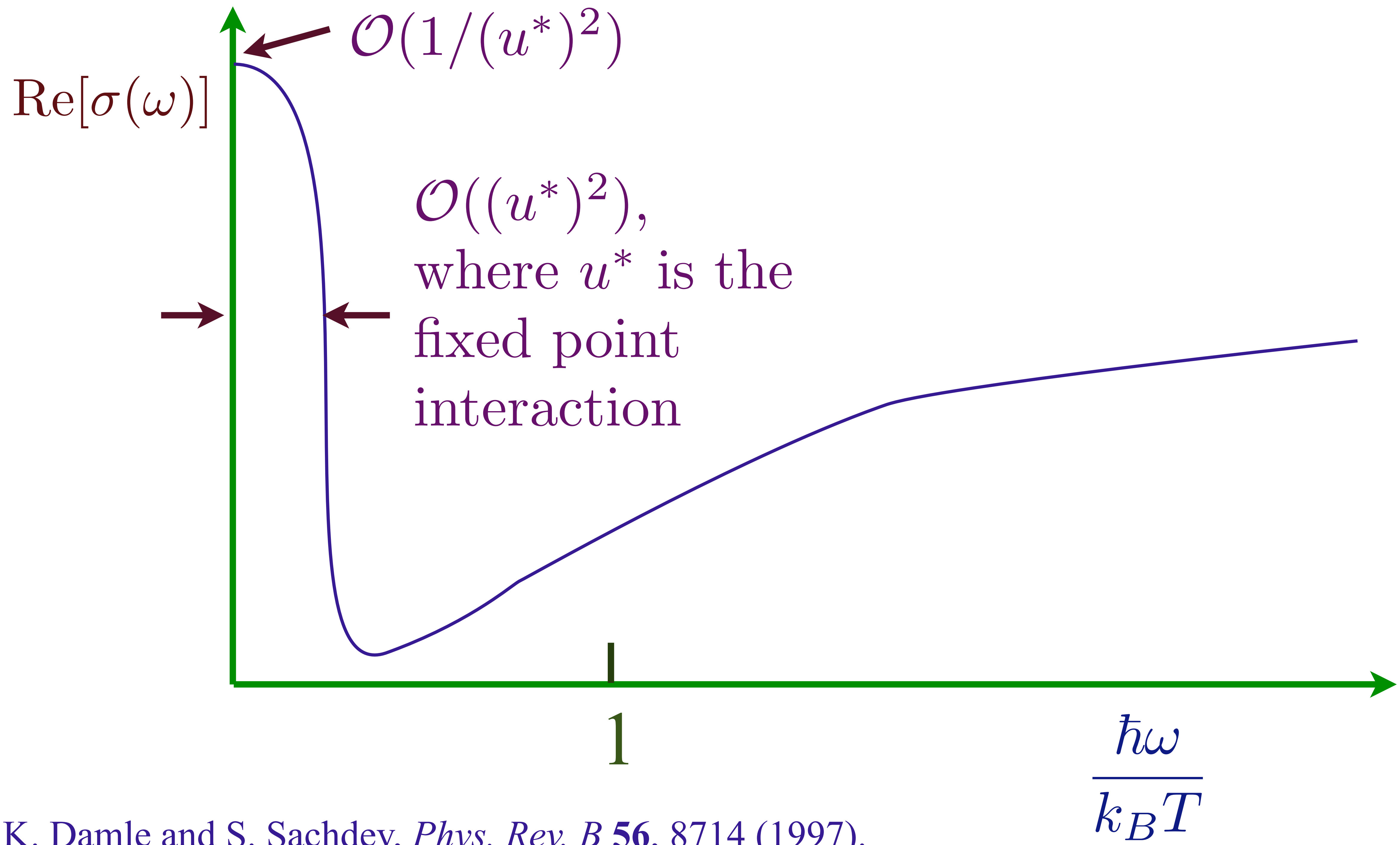

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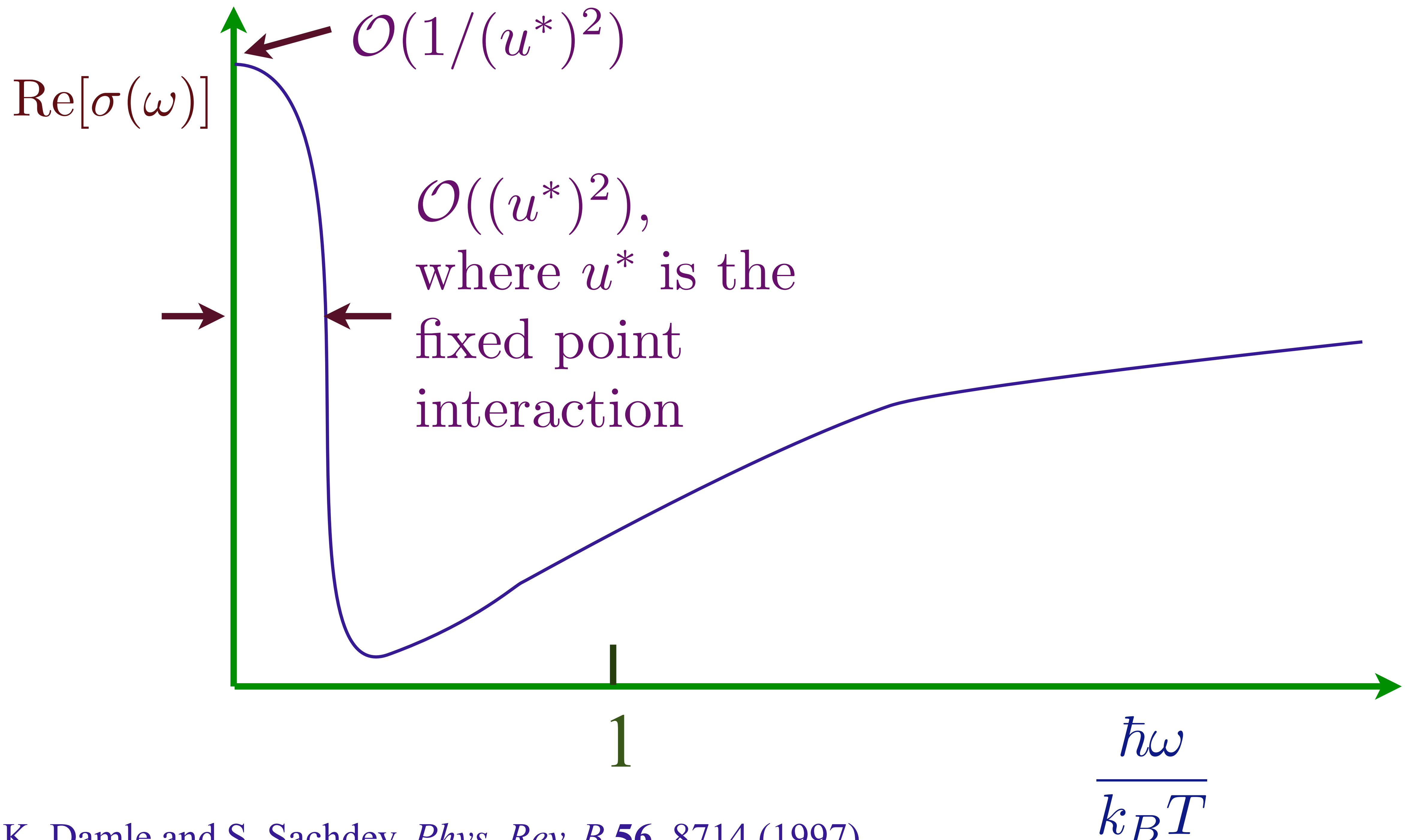

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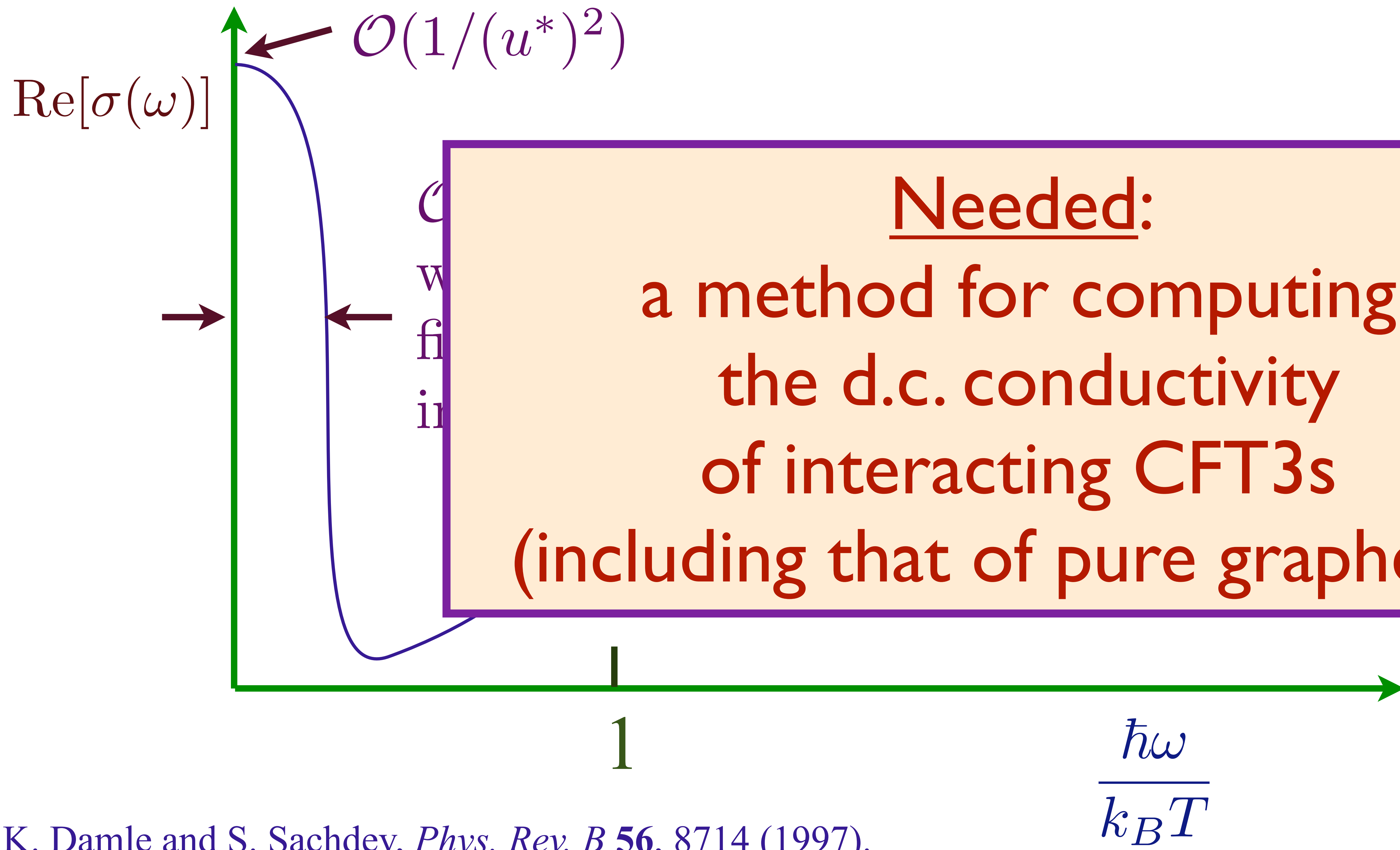


<u>Needed</u>:
a method for computing
the d.c. conductivity
of interacting CFT3s
(including that of pure graphene!)

K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Conformal quantum matter

A. Field theory:graphene

B. Field theory: superfluid-insulator transition

C. Field theory: antiferromagnets

D. Gauge-gravity duality

Conformal quantum matter

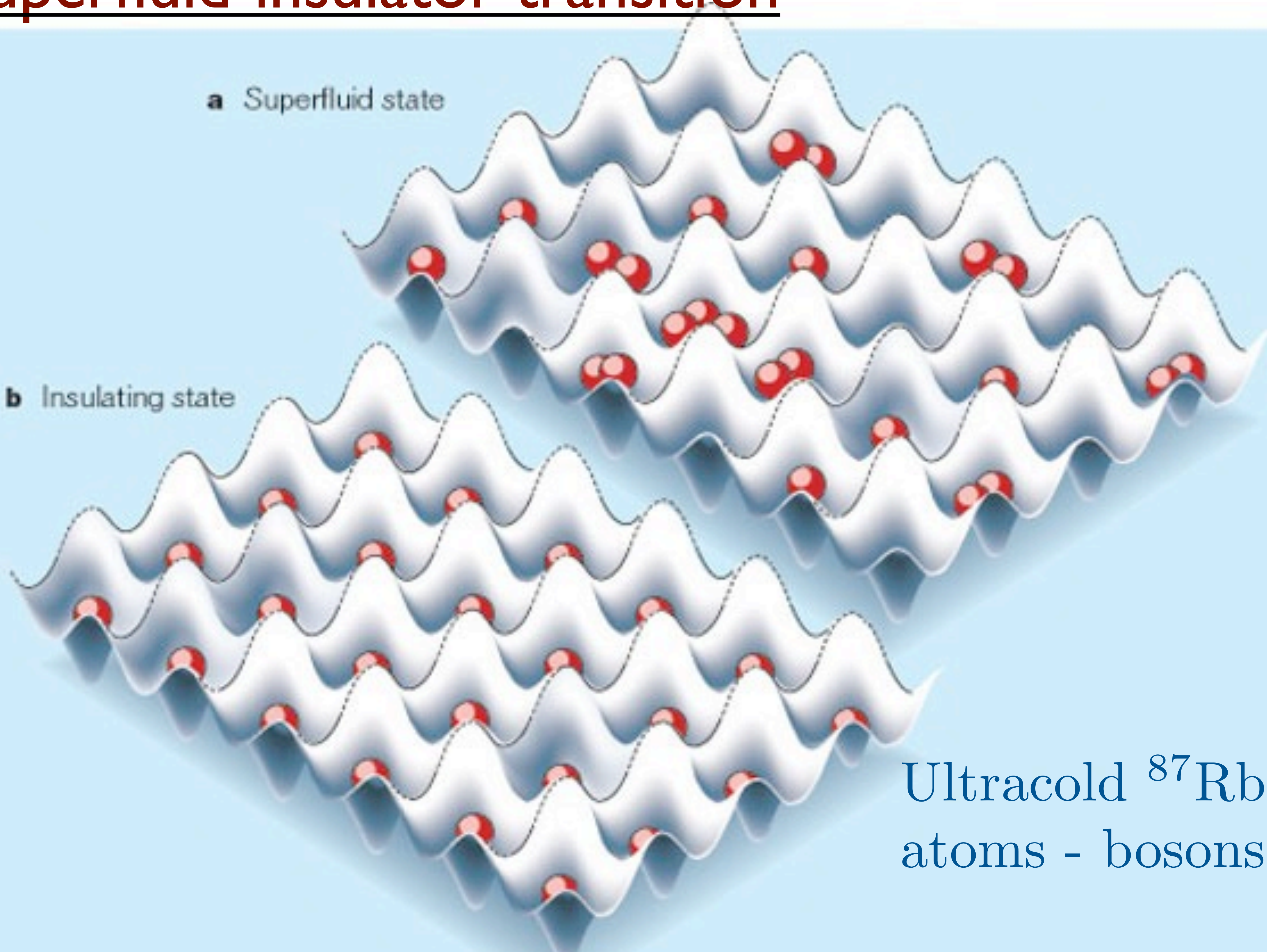
A. Field theory:graphene

B. Field theory: superfluid-insulator transition

C. Field theory: antiferromagnets

D. Gauge-gravity duality

Superfluid-insulator transition



Ultracold ^{87}Rb atoms - bosons

M. Greiner, O. Mandel, T. Esslinger, T. W. Hänsch, and I. Bloch, *Nature* **415**, 39 (2002).

The Superfluid-Insulator transition

Boson Hubbard model

Degrees of freedom: Bosons, $b_j^\dagger$, hopping between the sites, j, of a lattice, with short-range repulsive interactions.

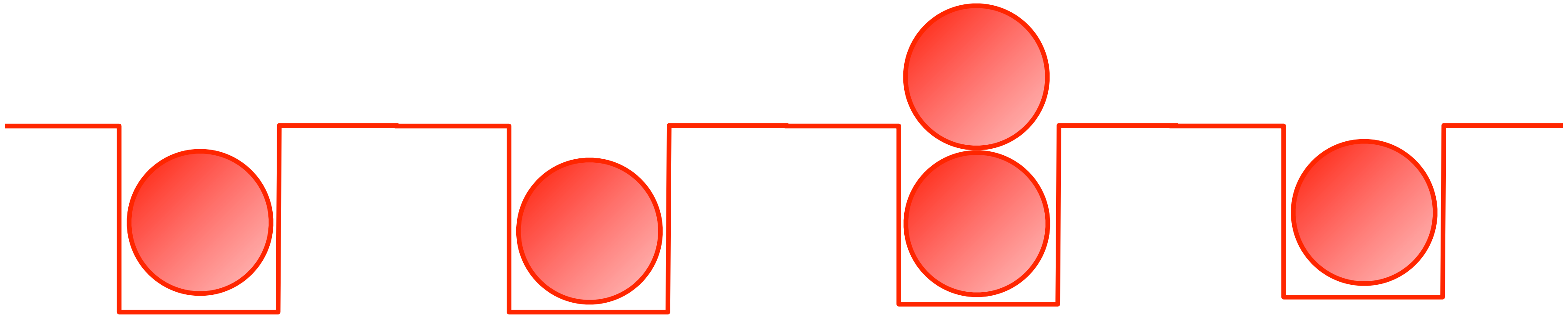
$$H = -t\sum_{\langle ij\rangle} b_i^\dagger b_j - \mu \sum_j n_j + \frac{U}{2}\sum_j n_j(n_j - 1) + \cdots$$

$$n_j \equiv b_j^\dagger b_j$$

$$[b_j, b_k^\dagger] = \delta_{jk}$$

M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).

Excitations of the insulator:



Particles ~ $\psi^\dagger$

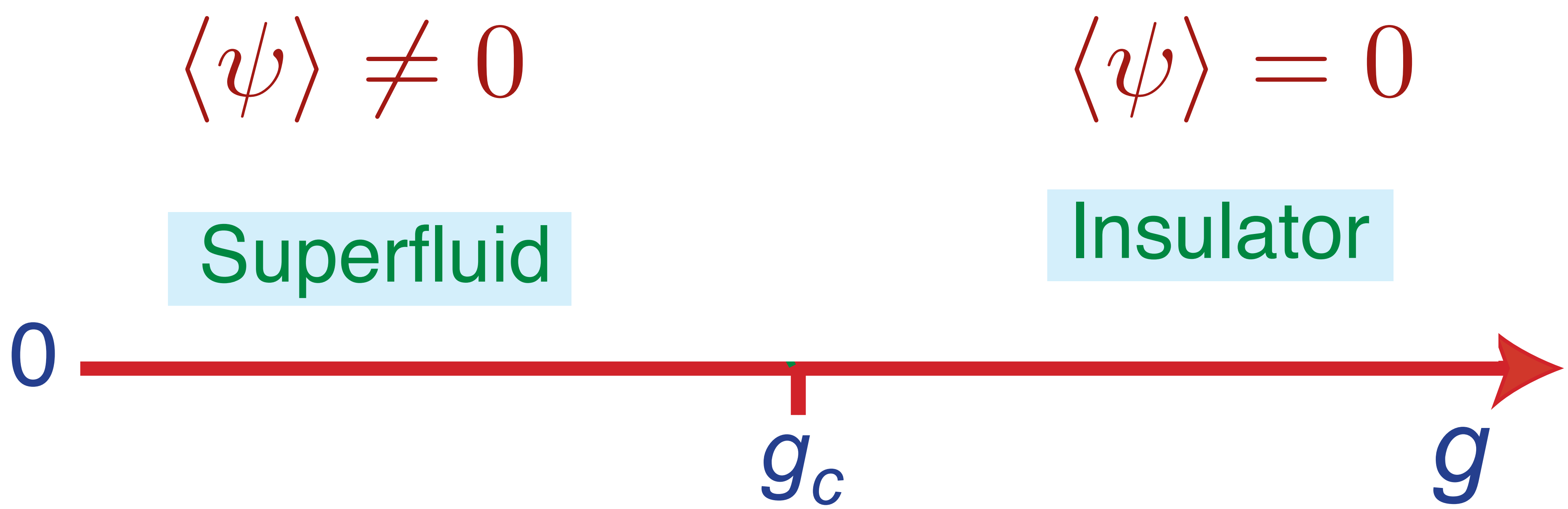
Holes ~ ψ

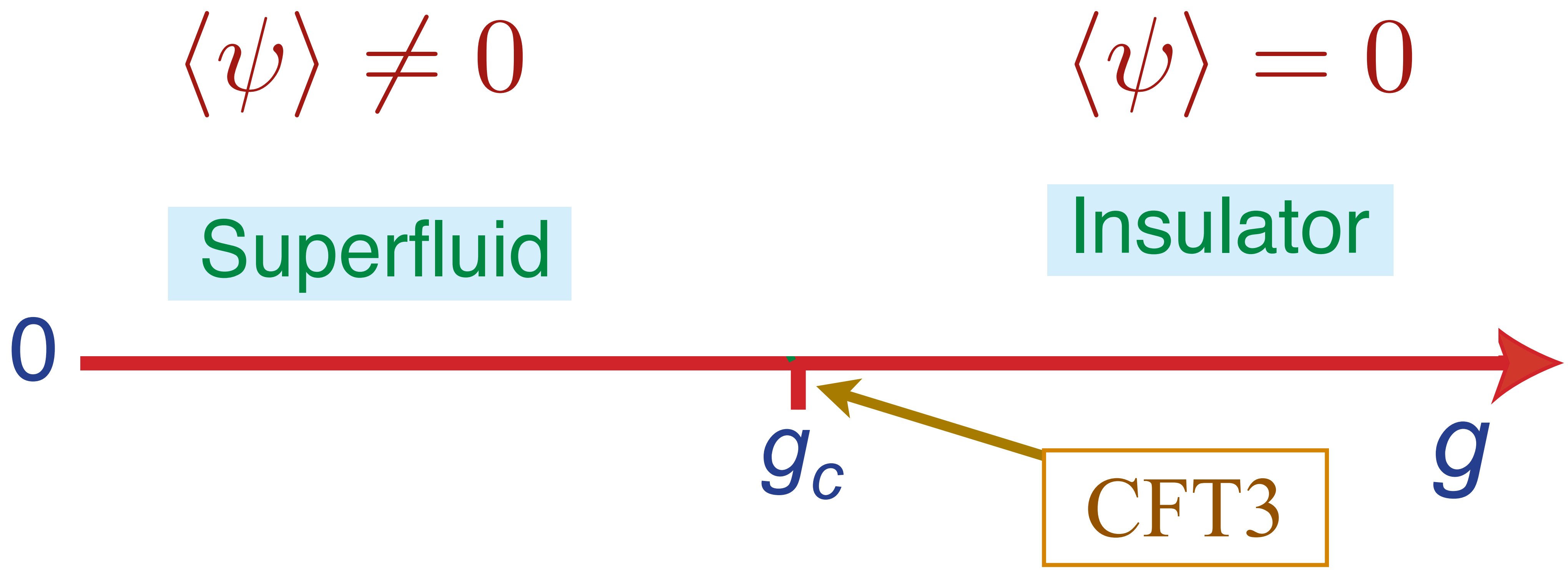
Density of particles = density of holes $\Rightarrow$
"relativistic" field theory for ψ:

.

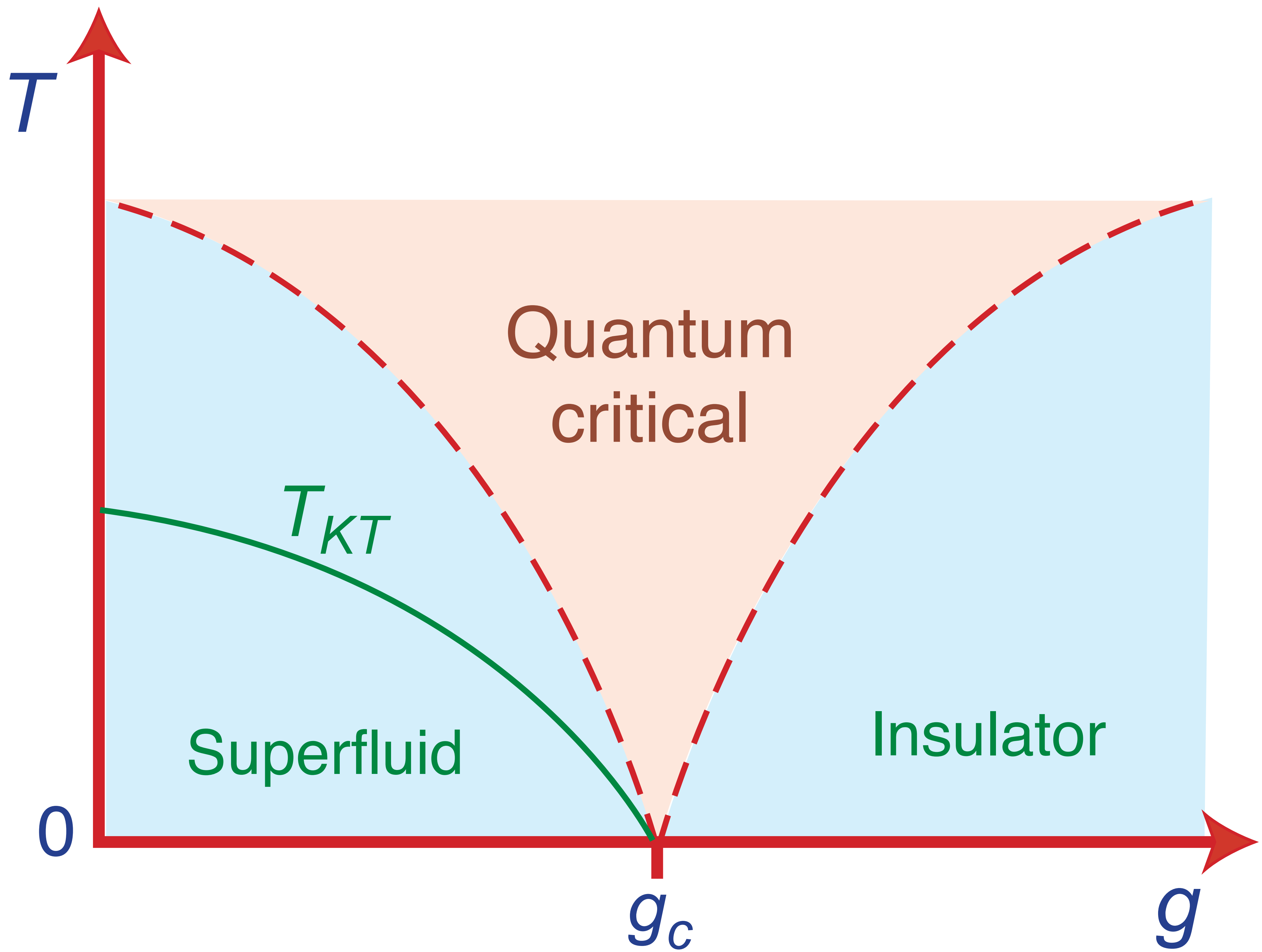
$$\mathcal{S} = \int d^2r d\tau \left[|\partial_\tau \psi|^2 + v^2 |\vec{\nabla}\psi|^2 + (g - g_c)|\psi|^2 + \frac{u}{2}|\psi|^4 \right]$$

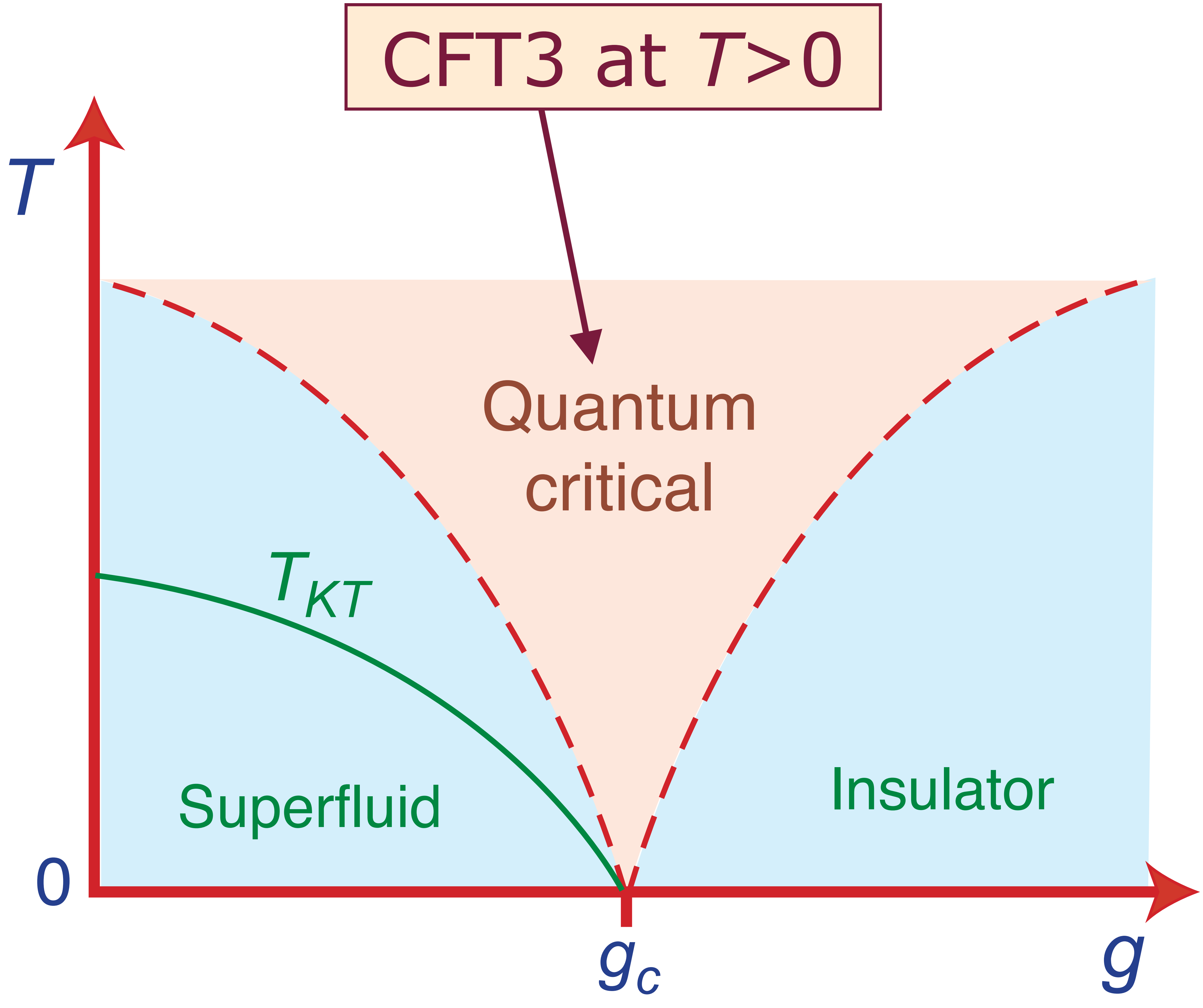
M.P. A. Fisher, P.B. Weichmann, G. Grinstein, and D.S. Fisher, *Phys. Rev. B* **40**, 546 (1989).


$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
g



$\langle \psi \rangle \neq 0$
$\langle \psi \rangle = 0$
Superfluid
Insulator
0
g_c
CFT3
g


T
Quantum
critical
T_KT
Superfluid
Insulator
0
g_c
g


CFT3 at $T>0$
T
Quantum
critical
T_{KT}
Superfluid
Insulator
0
g_c
g

Quantum critical transport

Quantum *"nearly perfect fluid"*
with shortest possible
equilibration time, $\tau_{\rm eq}$

$$\tau_{\rm eq} = \mathcal{C}\,\frac{\hbar}{k_B T}$$

where $\mathcal{C}$ is a *universal* constant

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

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where $\mathcal{C}$ is a *universal* constant

Zaanen: Planckian dissipation

S. Sachdev, *Quantum Phase Transitions,* Cambridge (1999).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Conductivity

$$\sigma = \frac{Q^2}{h} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

(Q is the "charge" of one boson)

M.P.A. Fisher, G. Grinstein, and S.M. Girvin, *Phys. Rev. Lett*. **64**, 587 (1990)
K. Damle and S. Sachdev, *Phys. Rev. B* **56**, 8714 (1997).

Quantum critical transport

Transport co-oefficients not determined by collision rate, but by universal constants of nature

Momentum transport

$$\frac{\eta}{s} \equiv \frac{\text{viscosity}}{\text{entropy density}}$$
$$= \frac{\hbar}{k_B} \times [\text{Universal constant } \mathcal{O}(1)\ ]$$

P. Kovtun, D. T. Son, and A. Starinets, *Phys. Rev. Lett.* **94,** 11601 (2005)

Quantum critical transport

Describe charge transport using Boltzmann theory of interacting bosons:

$$\frac{dv}{dt} + \frac{v}{\tau_c} = F.$$

This gives a frequency (ω) dependent conductivity

$$\sigma(\omega) = \frac{\sigma_0}{1 - i\,\omega\,\tau_c}$$

where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

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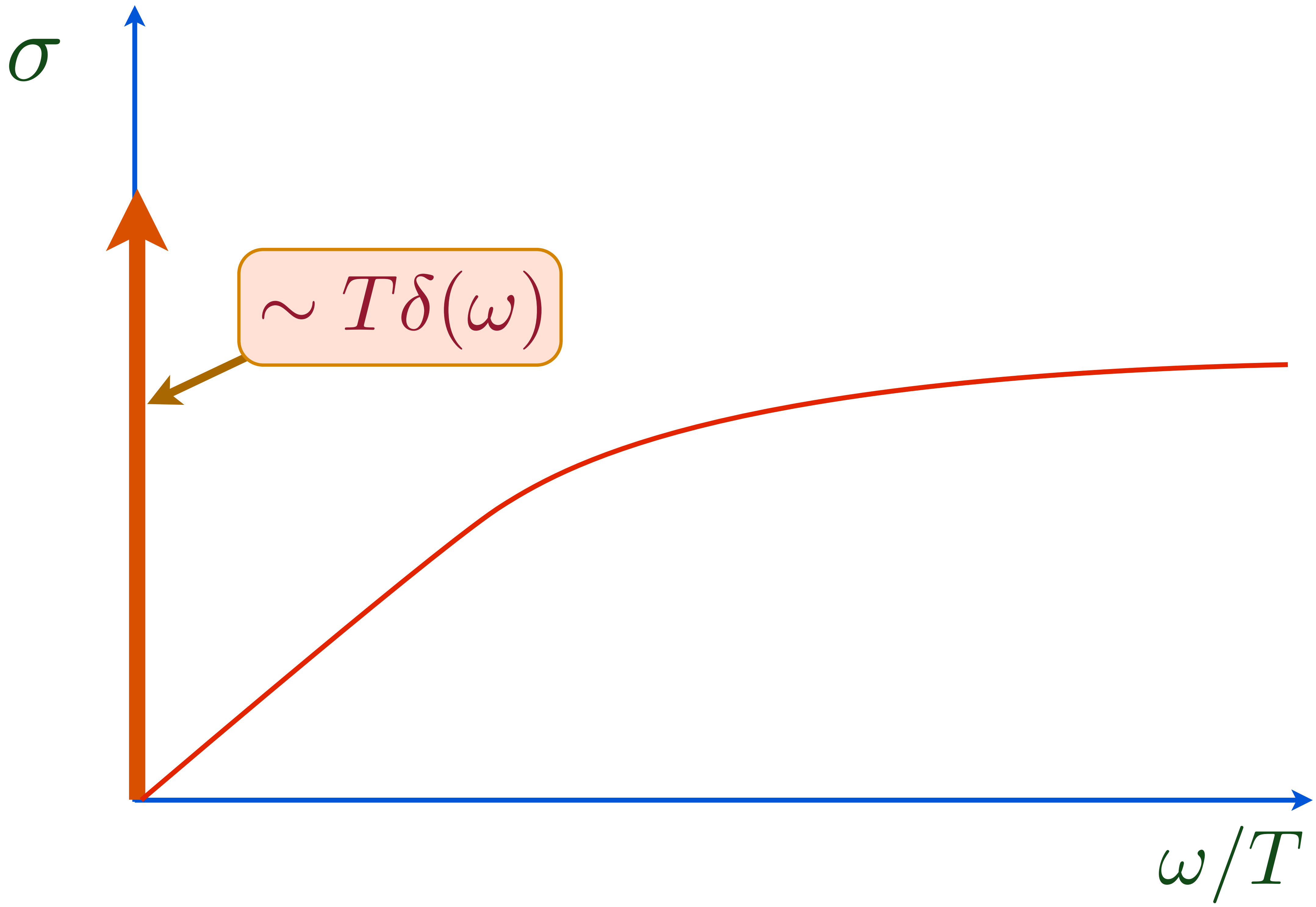
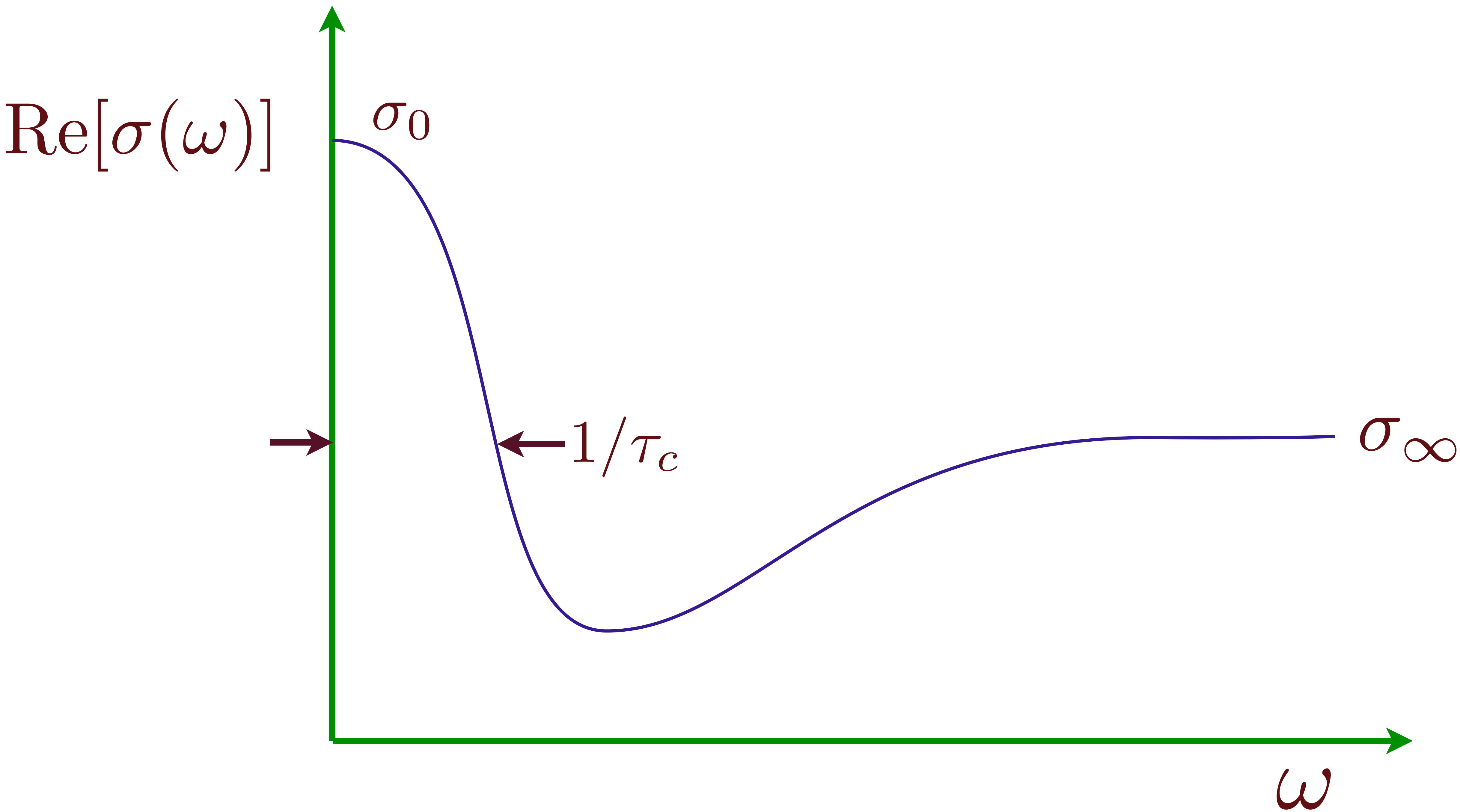
where $\tau_c \sim \hbar/(k_B T)$ is the time between boson collisions.

Also, we have $\sigma(\omega \to \infty) = \sigma_\infty$, associated with the density of states for particle-hole creation (the "optical conductivity") in the CFT3.

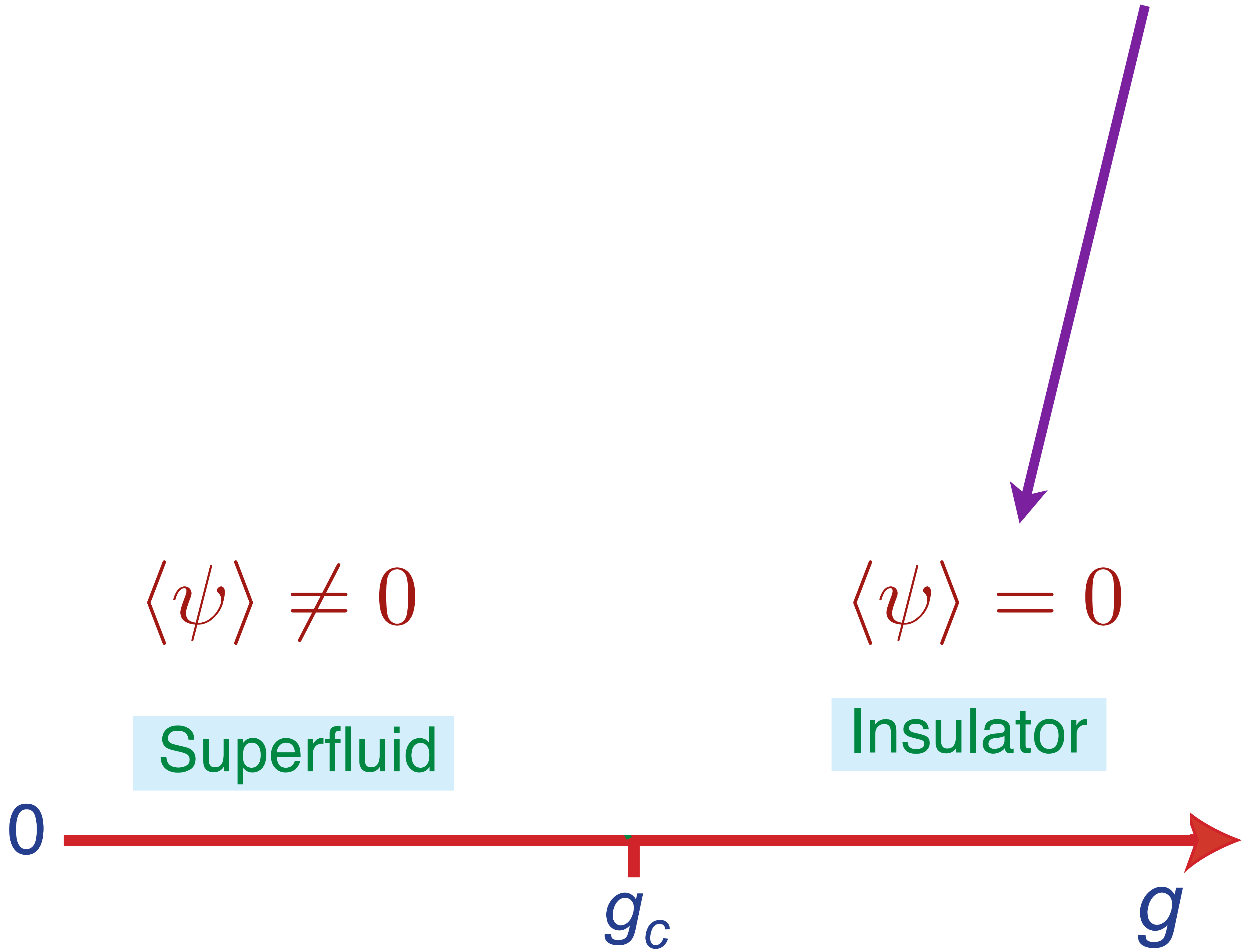
Electrical transport in a free-field theory for *T* > *0*



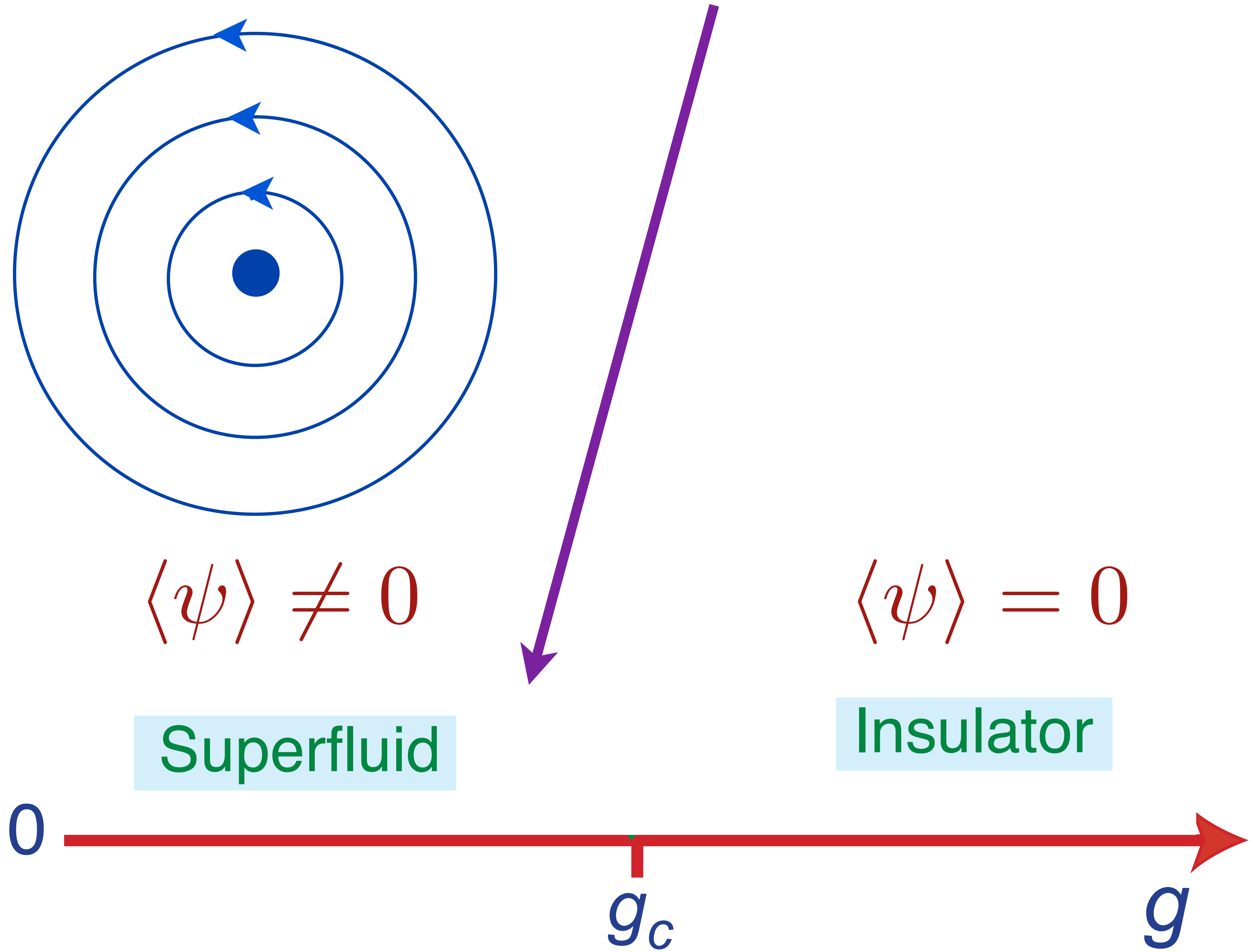
Boltzmann theory of bosons



So far, we have described the quantum critical point using the boson particle and hole excitations of the insulator.



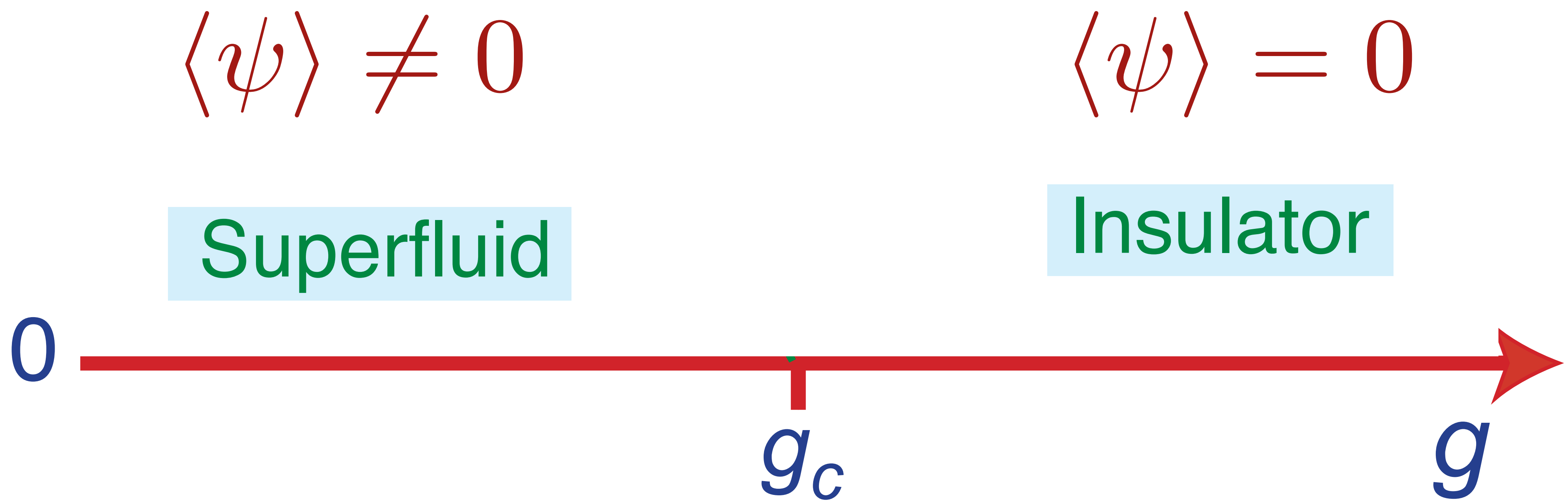
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.



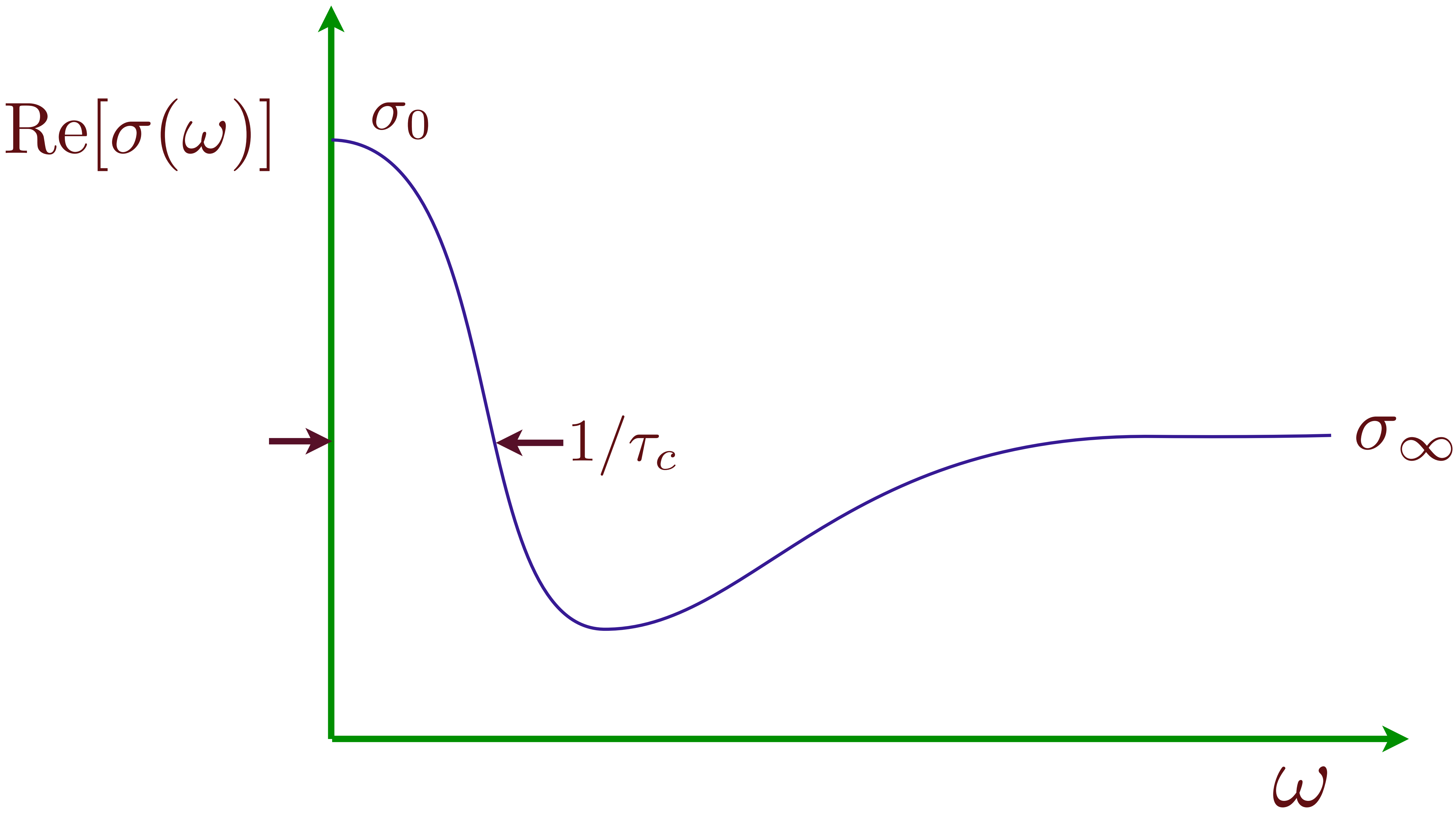
However, we could equally well describe the conductivity using the excitations of the superfluid, which are *vortices*.

These are quantum particles (in 2+1 dimensions) which described by a (mirror/e.m.) “dual” CFT3 with an emergent U(1) gauge field. Their $T > 0$ dynamics can also be described by a Boltzmann equation:

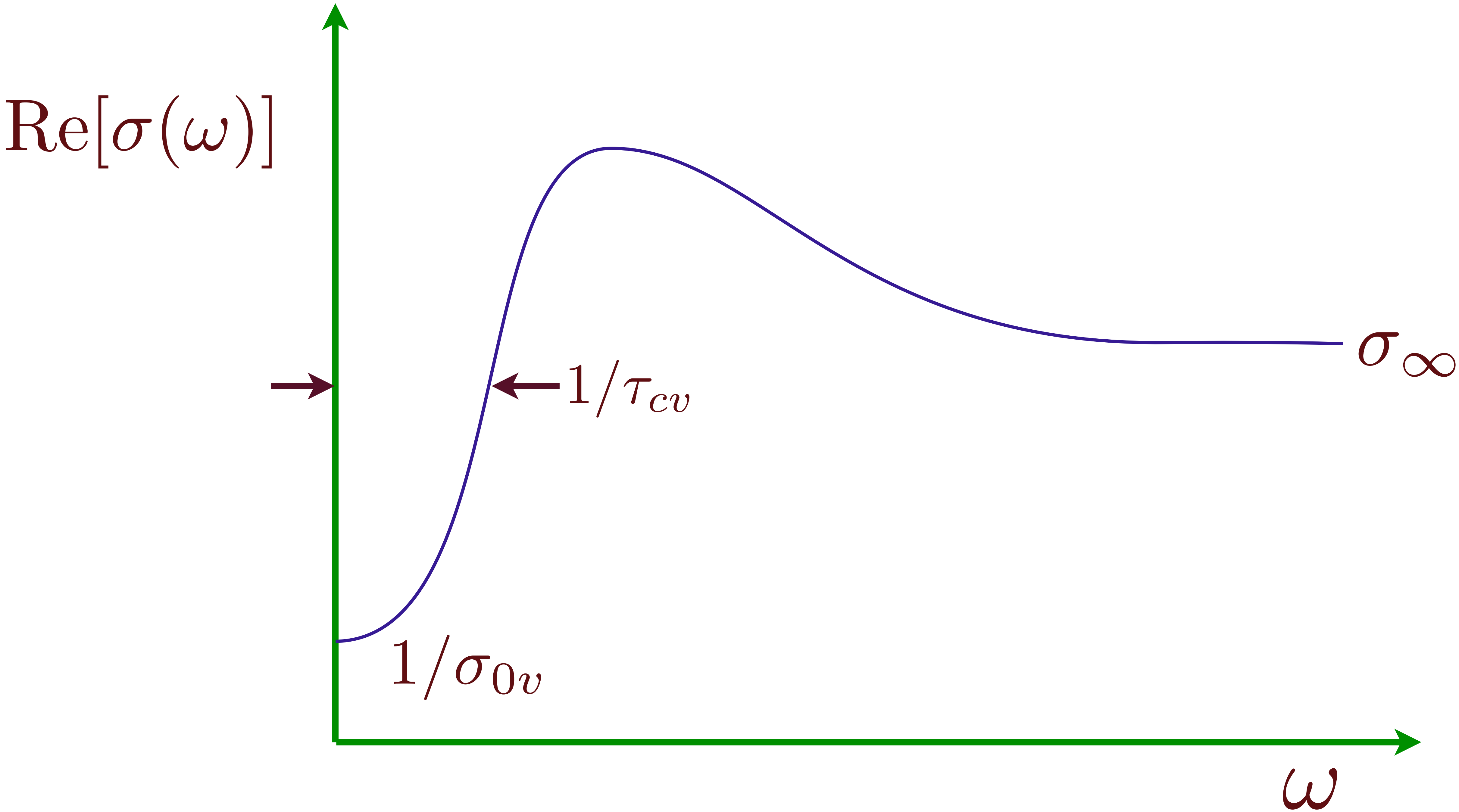
$$\text{Conductivity} = \text{Resistivity of vortices}$$



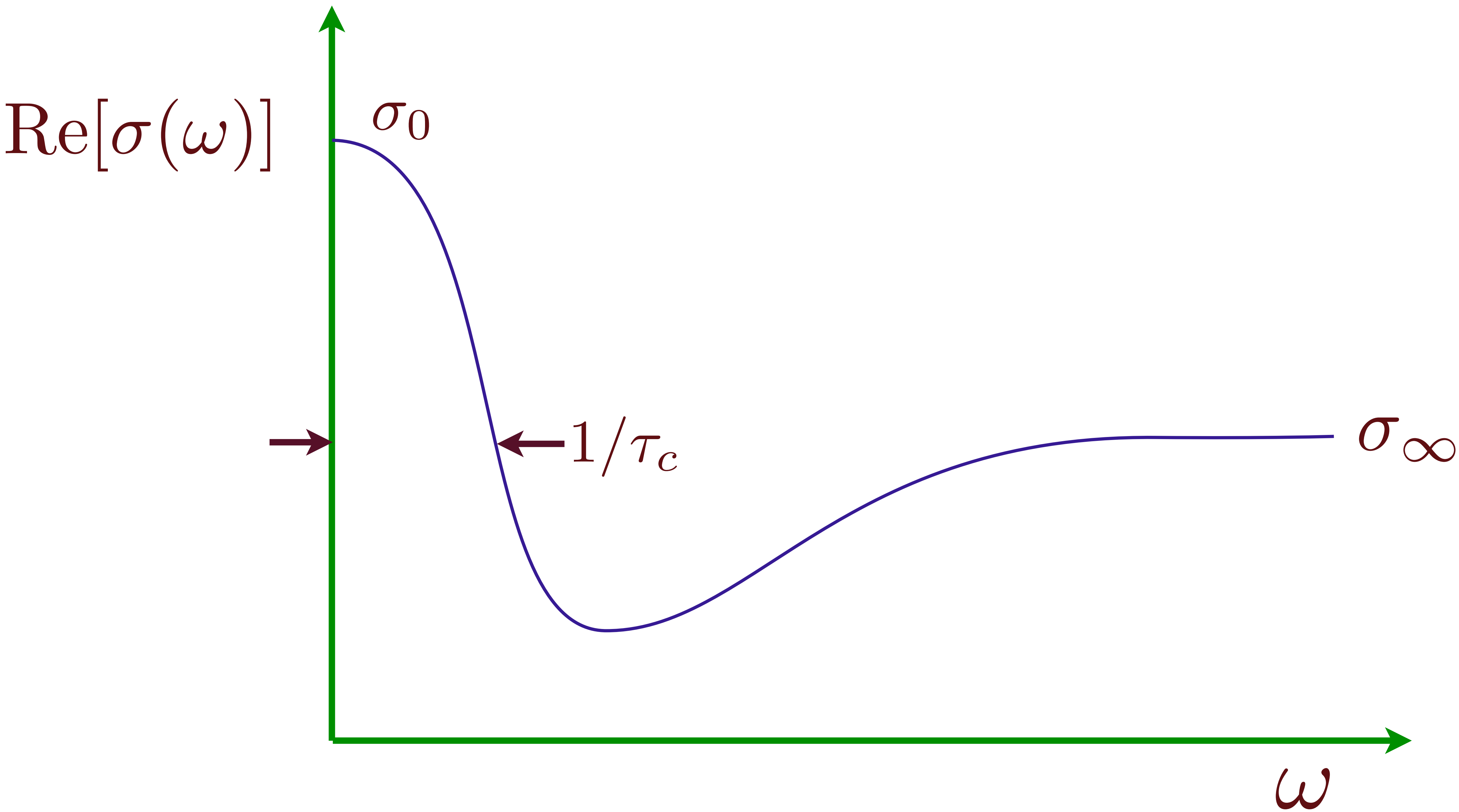
Boltzmann theory of bosons



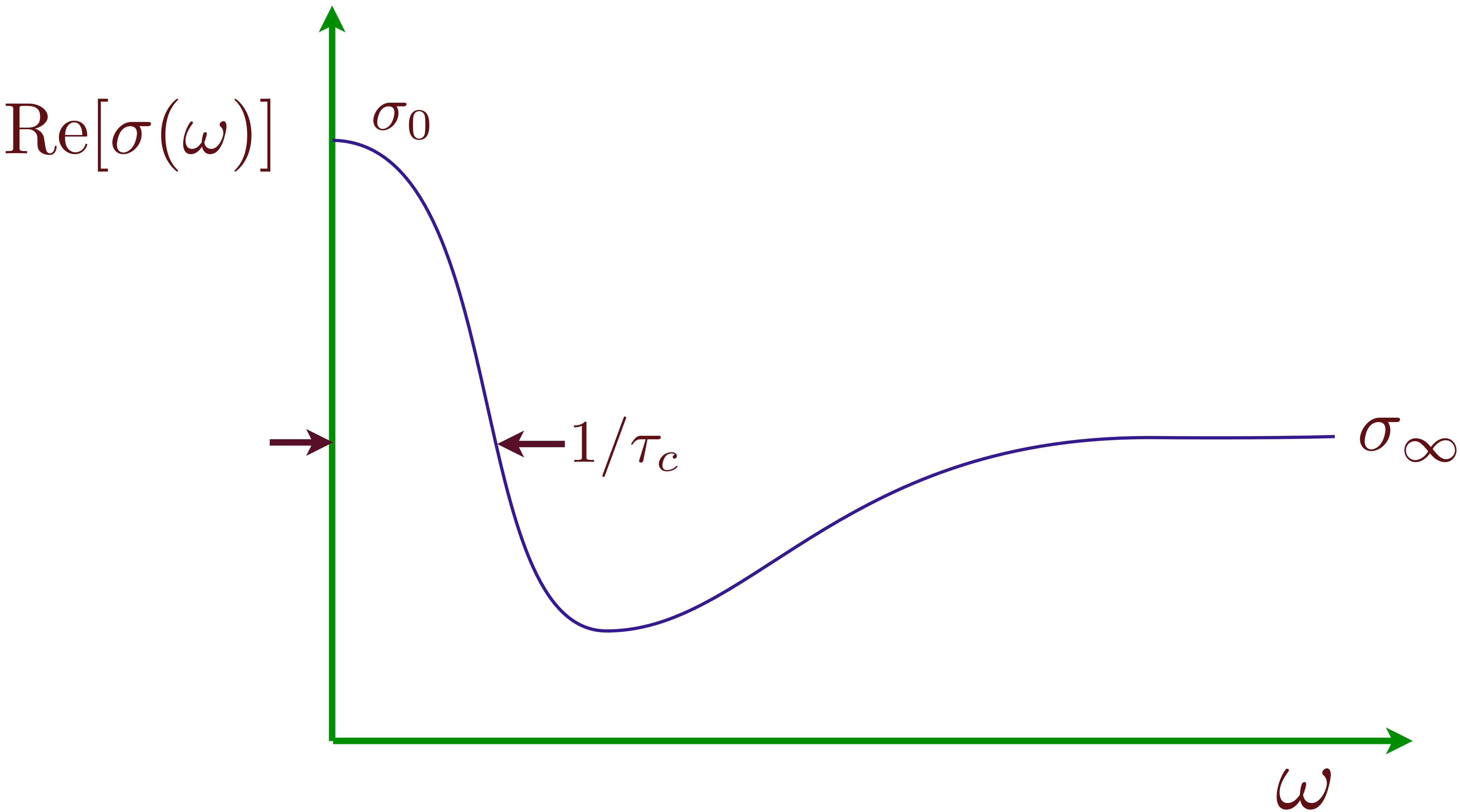
Boltzmann theory of vortices



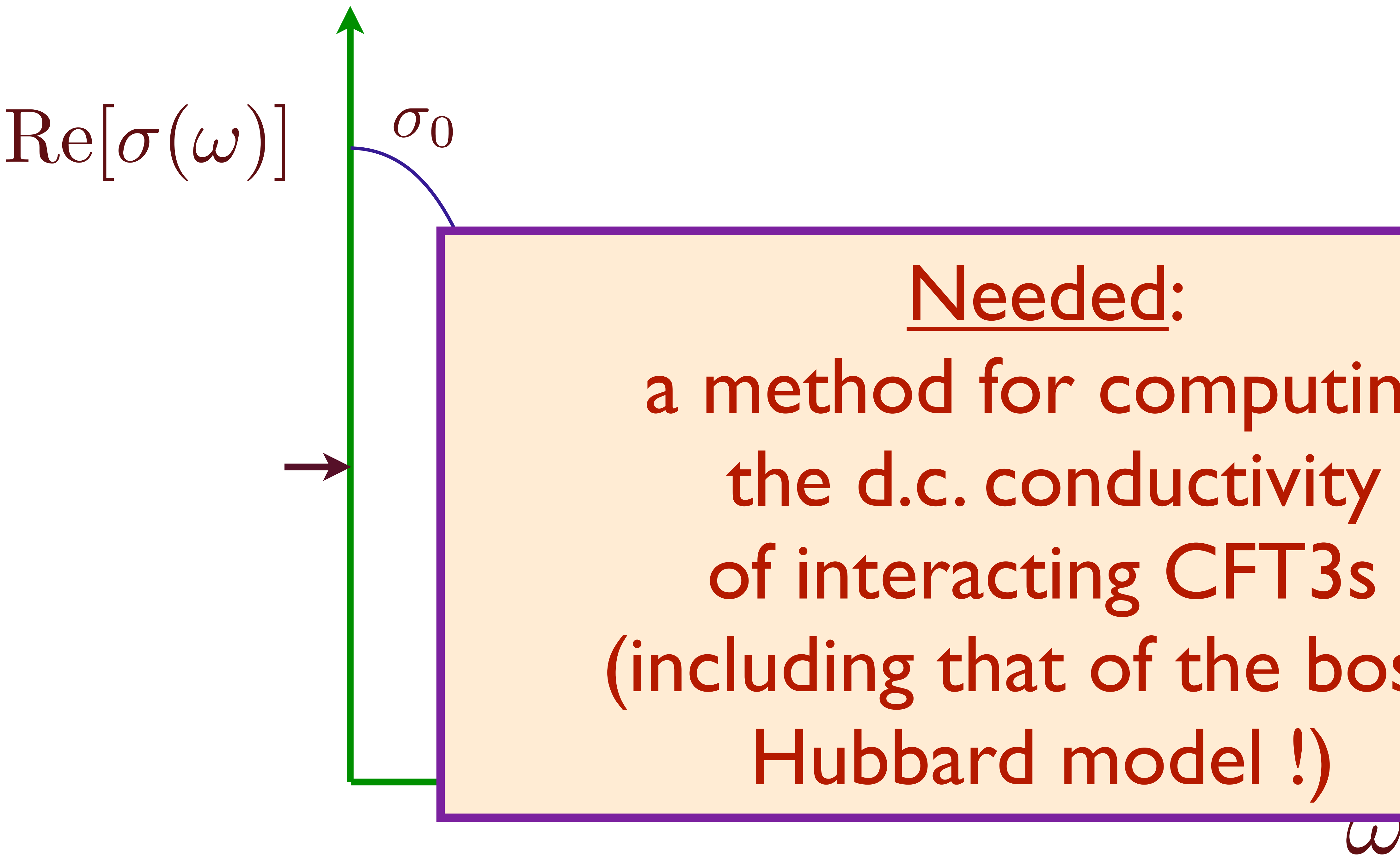
Boltzmann theory of bosons



Boltzmann theory of bosons



Boltzmann theory of bosons



Needed:

a method for computing the d.c. conductivity of interacting CFT3s (including that of the boson Hubbard model !)

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A. Field theory:graphene

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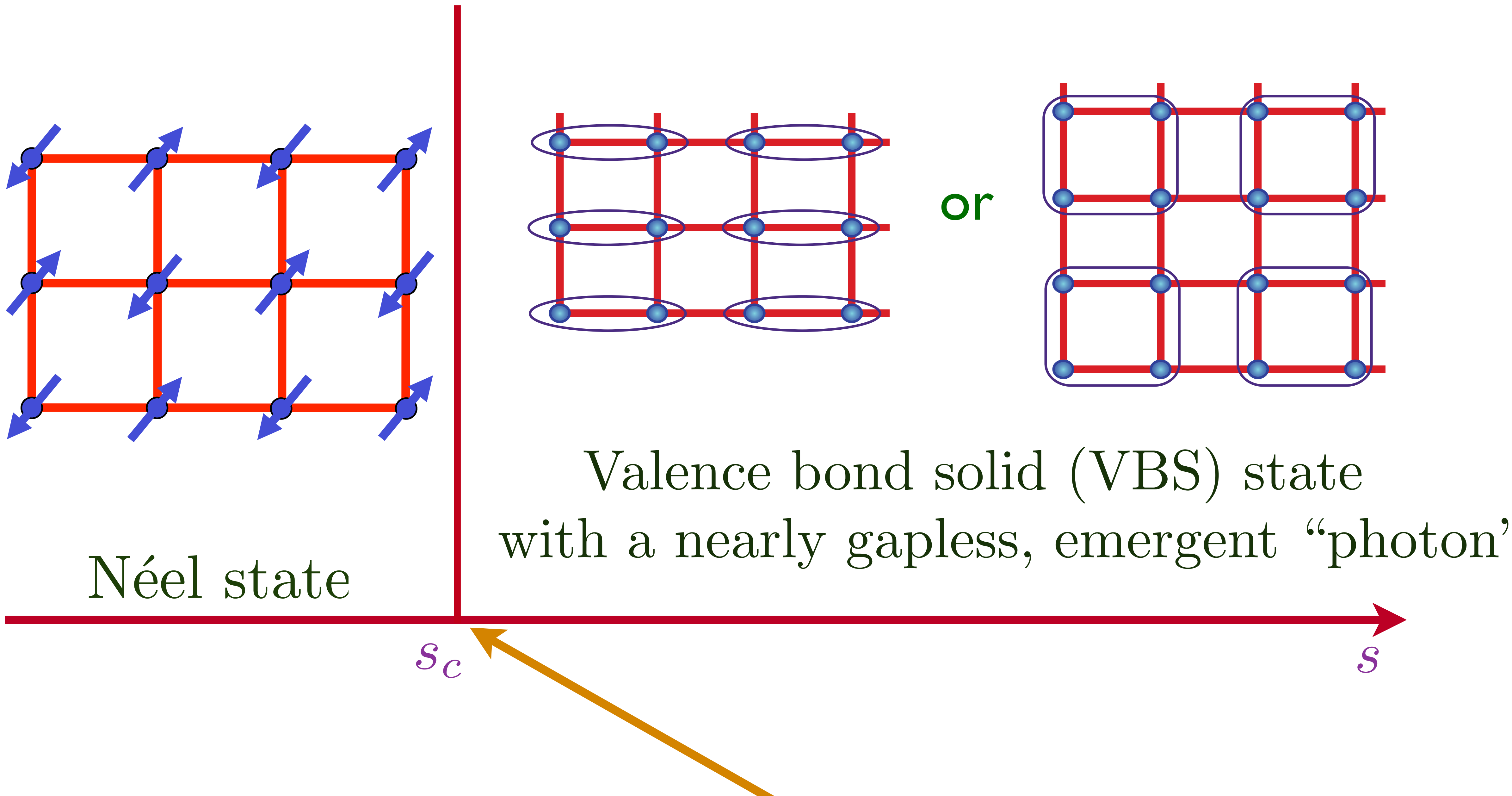
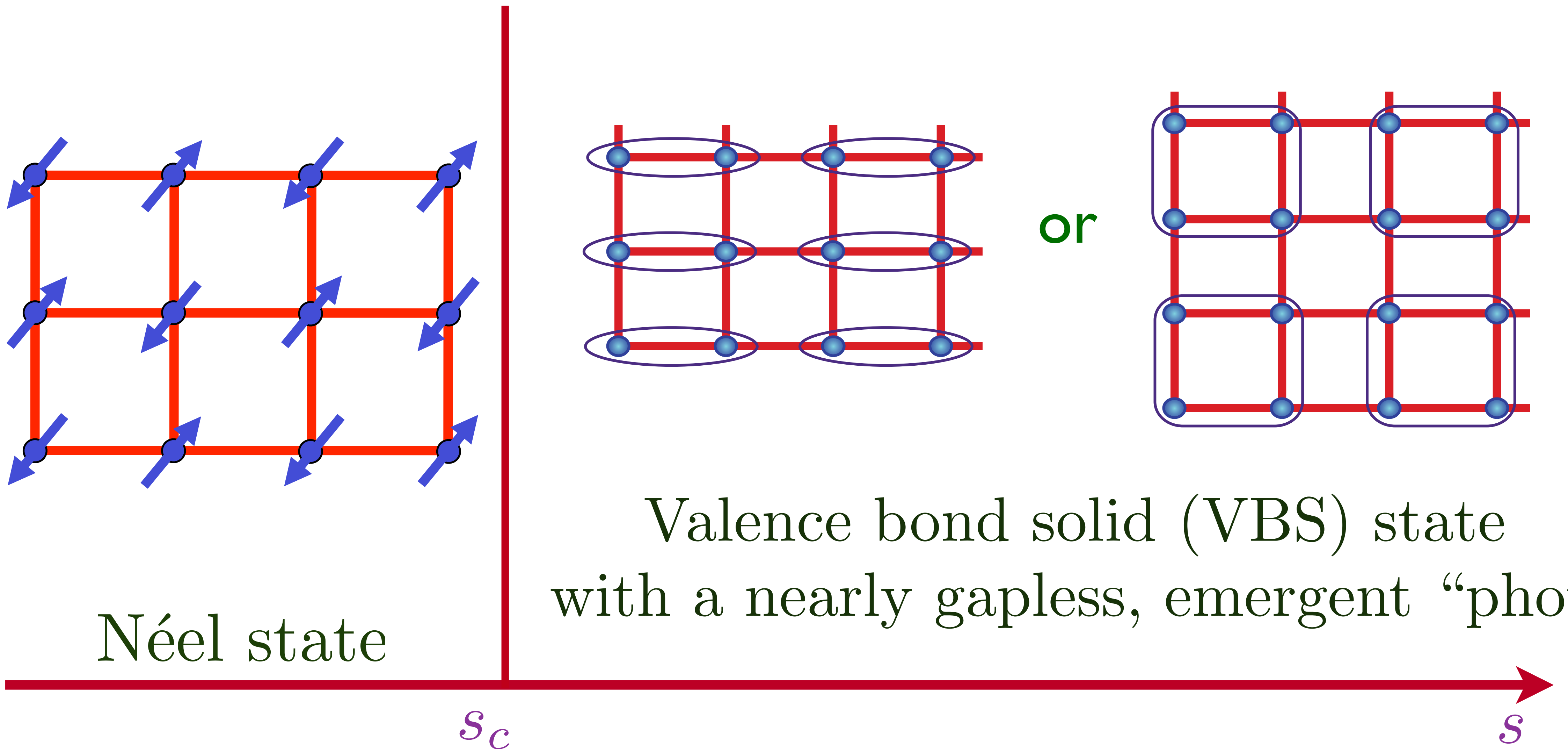
Quantum critical point in a frustrated square lattice antiferromagnet



Long-range entanglement described by a CFT3
with an emergent U(1) "photon"

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

Quantum critical point in a frustrated square lattice antiferromagnet

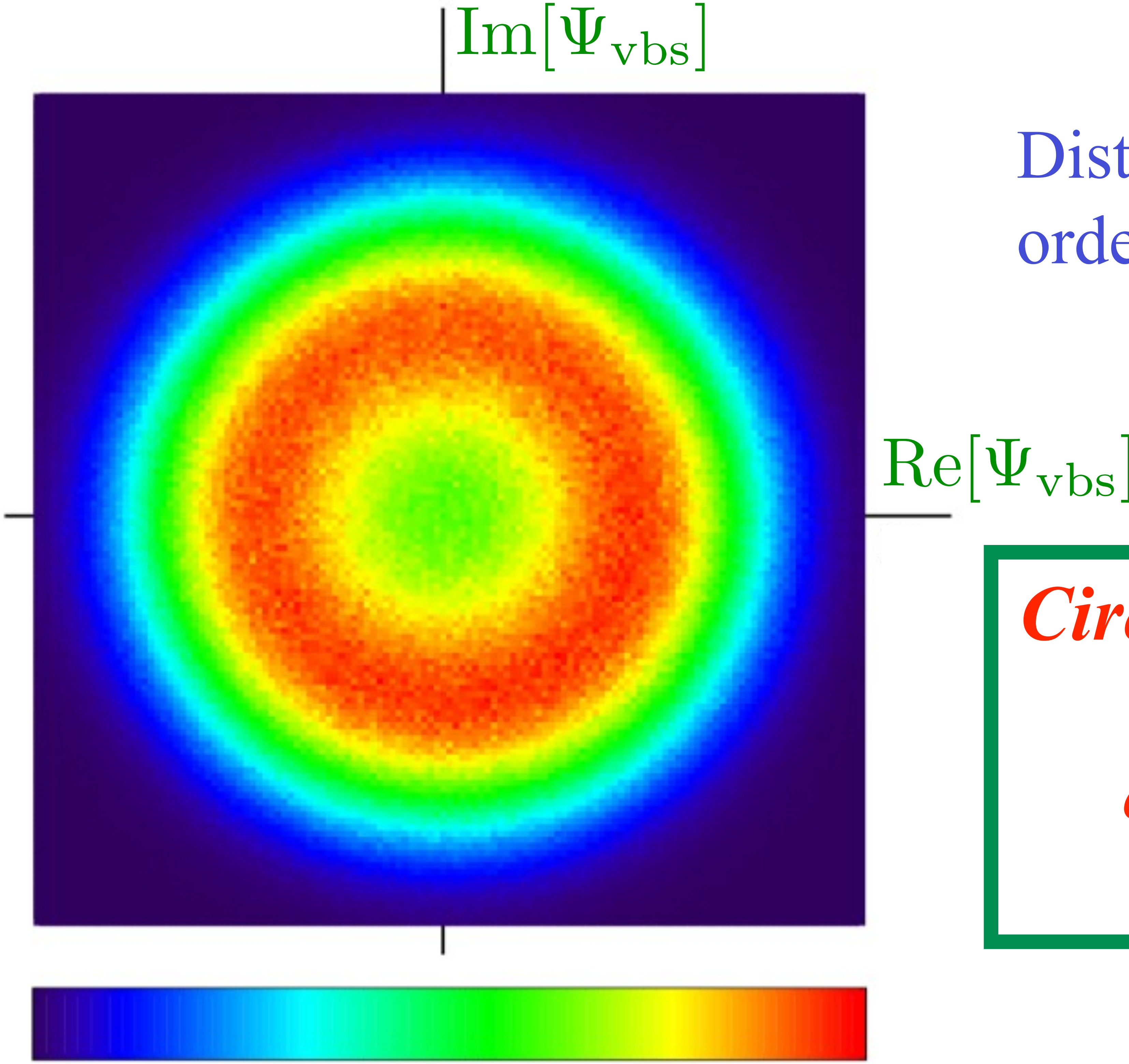


Critical theory for photons and deconfined spinons:

$$\mathcal{S}_z = \int d^2 r d\tau \left[|(\partial_\mu - i A_\mu) z_\alpha|^2 + s|z_\alpha|^2 + u(|z_\alpha|^2)^2 + \frac{1}{2e_0^2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 \right]$$

O.I. Motrunich and A. Vishwanath, *Phys. Rev. B* **70**, 075104 (2004).
T. Senthil, A. Vishwanath, L. Balents, S. Sachdev and M.P.A. Fisher, *Science* **303**, 1490 (2004).

$$\mathcal{H}_{\mathrm{SU}(2)} = J \sum_{\langle ij \rangle} \mathbf{S}_i \cdot \mathbf{S}_j - Q \sum_{\langle ijkl \rangle} \left(\mathbf{S}_i \cdot \mathbf{S}_j - \tfrac{1}{4}\right) \left(\mathbf{S}_k \cdot \mathbf{S}_l - \tfrac{1}{4}\right)$$



Distribution of VBS order Ψ_{vbs} at large Q

Circular symmetry is evidence for emergent U(1) photon

A.W. Sandvik, *Phys. Rev. Lett.* **98**, 2272020 (2007).

Conformal quantum matter

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Conformal quantum matter

A. Field theory:graphene

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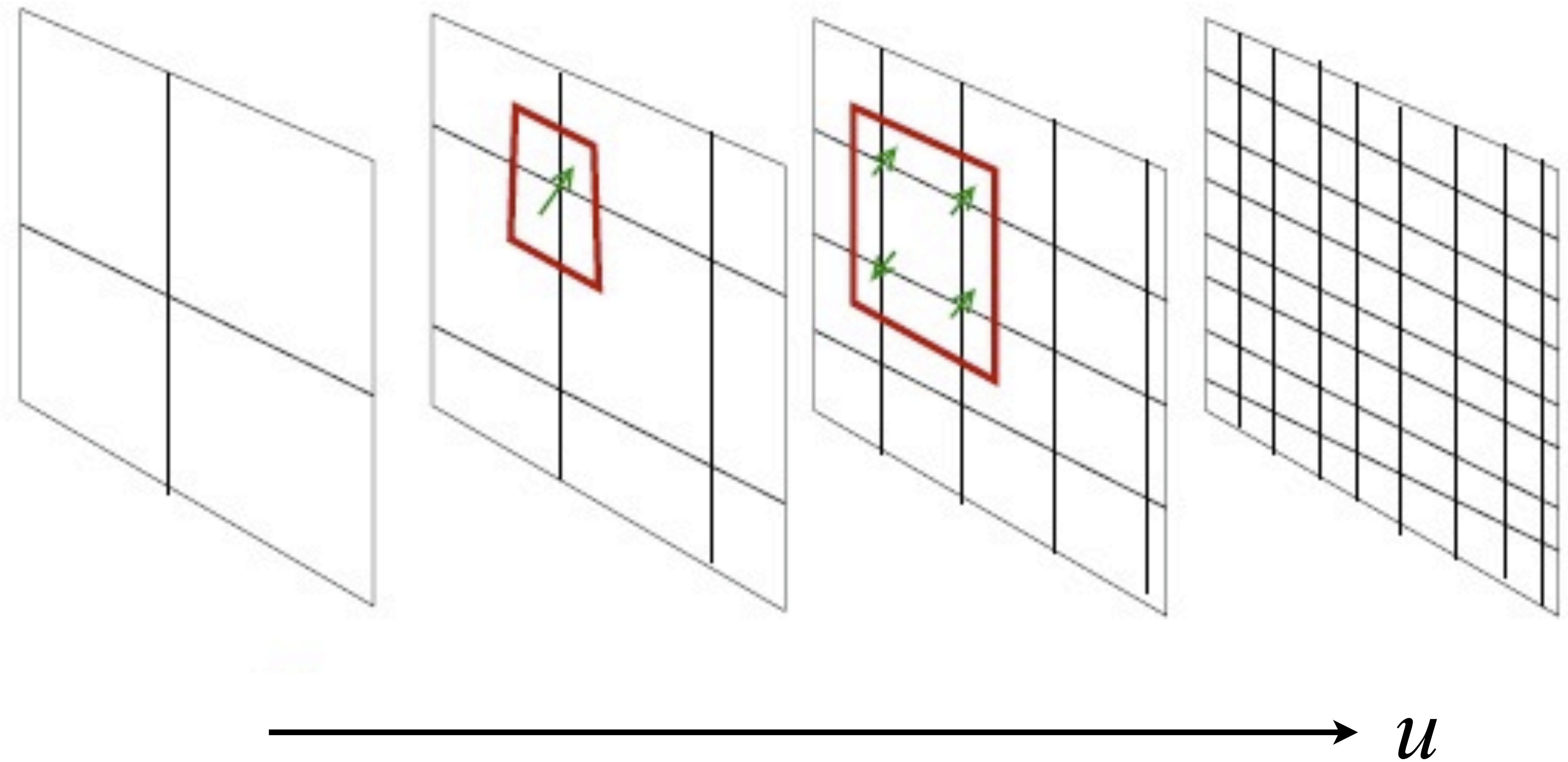
D. Gauge-gravity duality

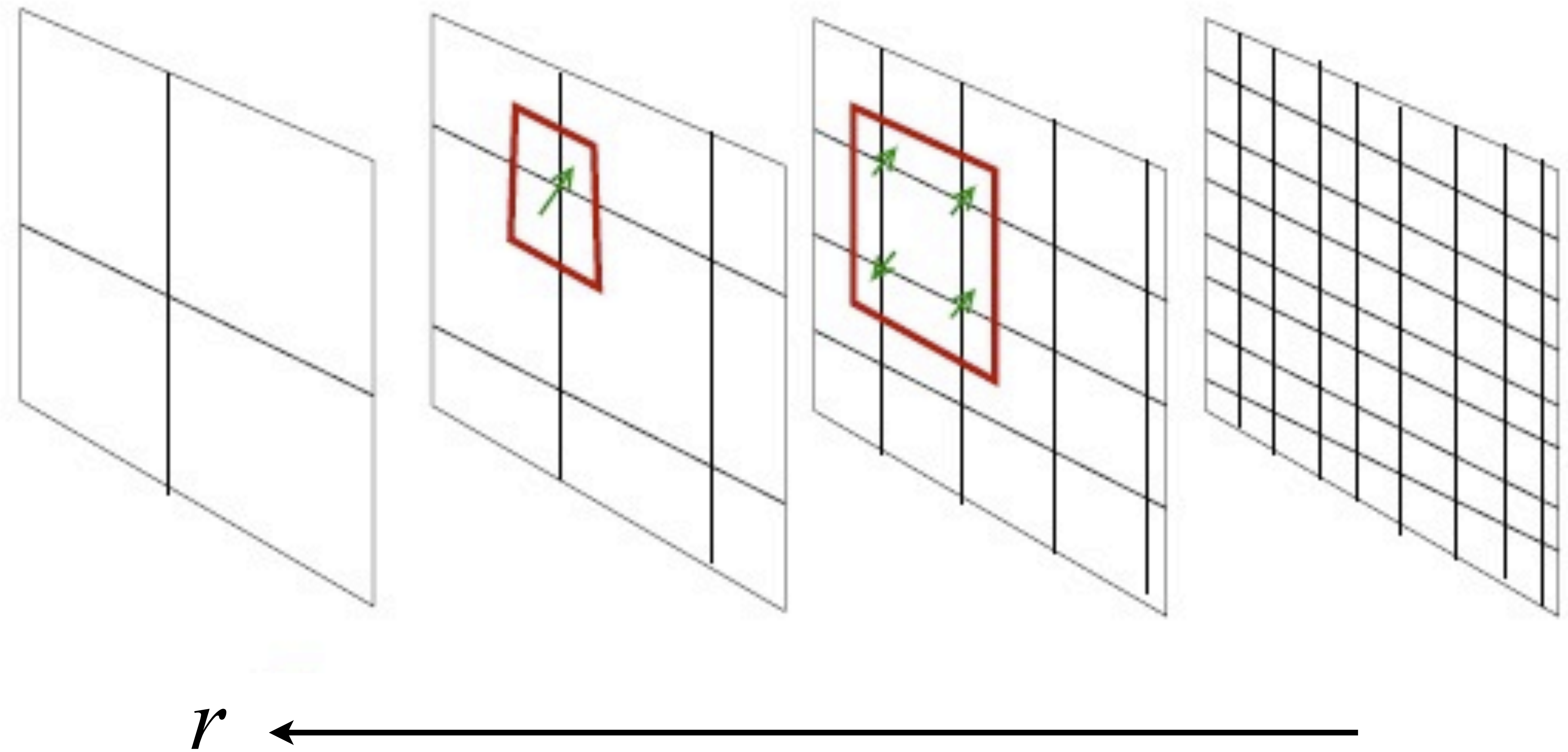
Field theories in $d+1$ spacetime dimensions are characterized by couplings g which obey the renormalization group equation

$$u \frac{dg}{du} = \beta(g)$$

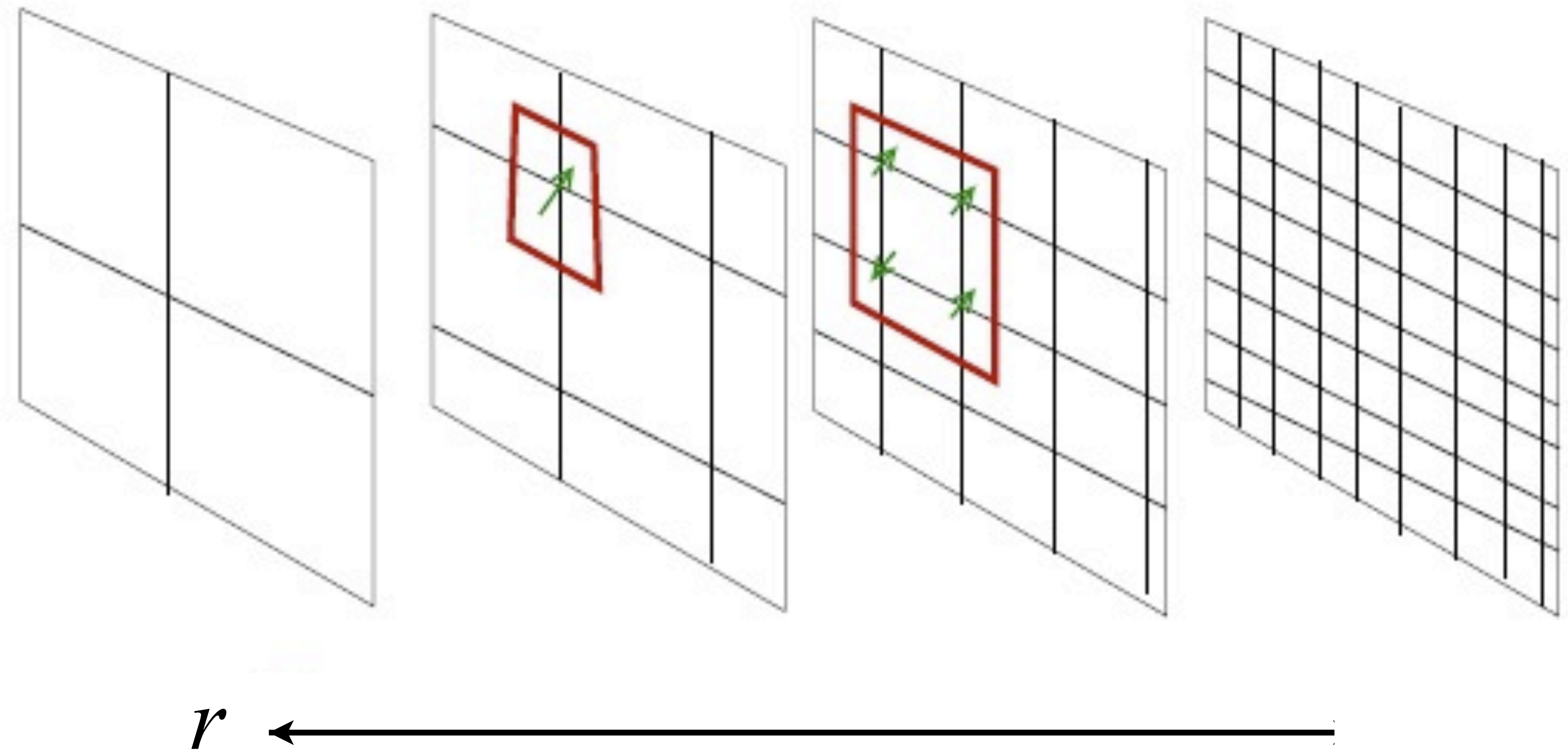
where u is the energy scale. The RG equation is *local* in energy scale, *i.e.* the RHS does not depend upon u.

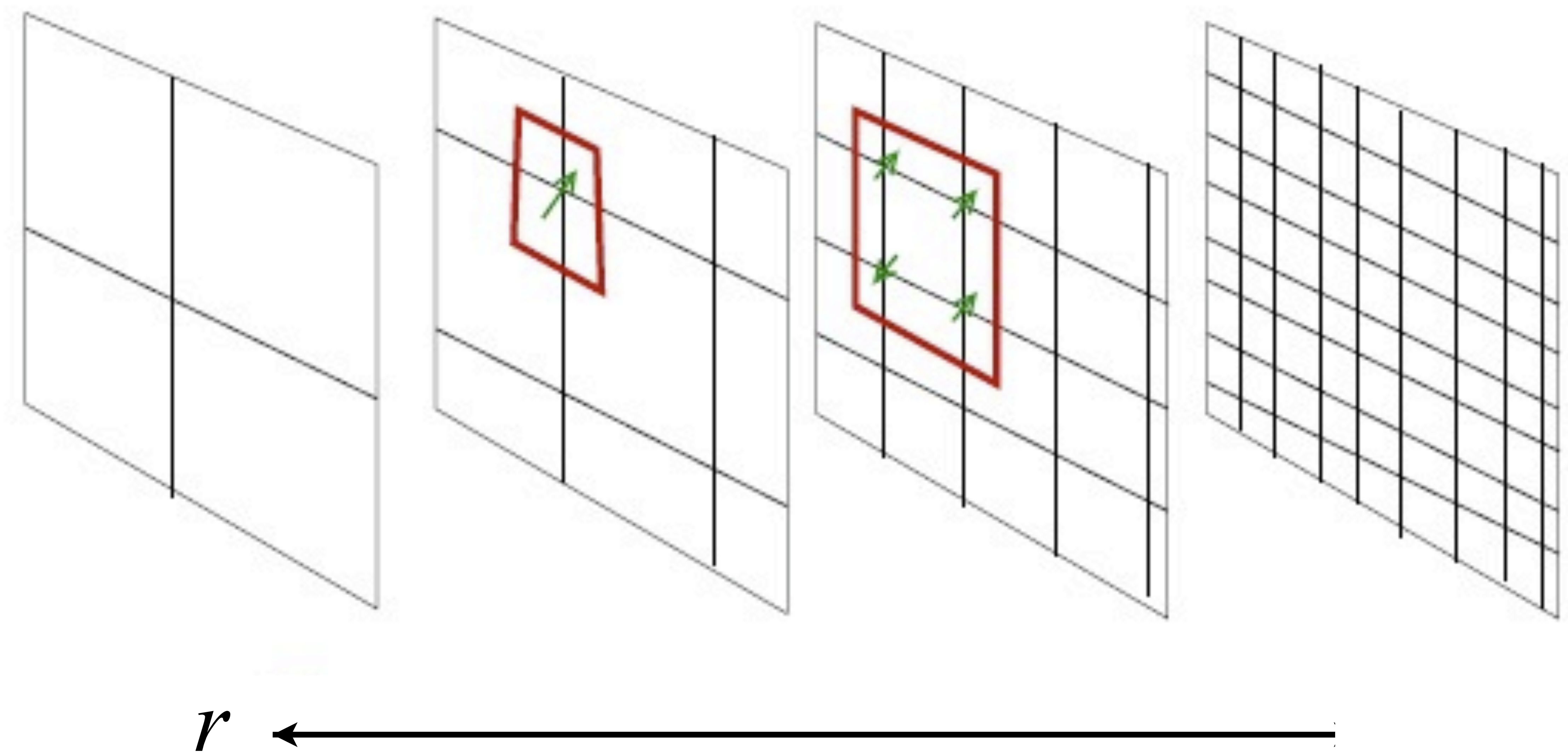


J. McGreevy, arXiv0909.0518



J. McGreevy, arXiv0909.0518


r



Key idea: $\Rightarrow$ Implement r as an extra dimension, and map to a local theory in $d+2$ spacetime dimensions.

For a relativistic CFT in d spatial dimensions, the metric in the holographic space is uniquely fixed by demanding the following scale transformaion $(i = 1 \dots d)$

$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

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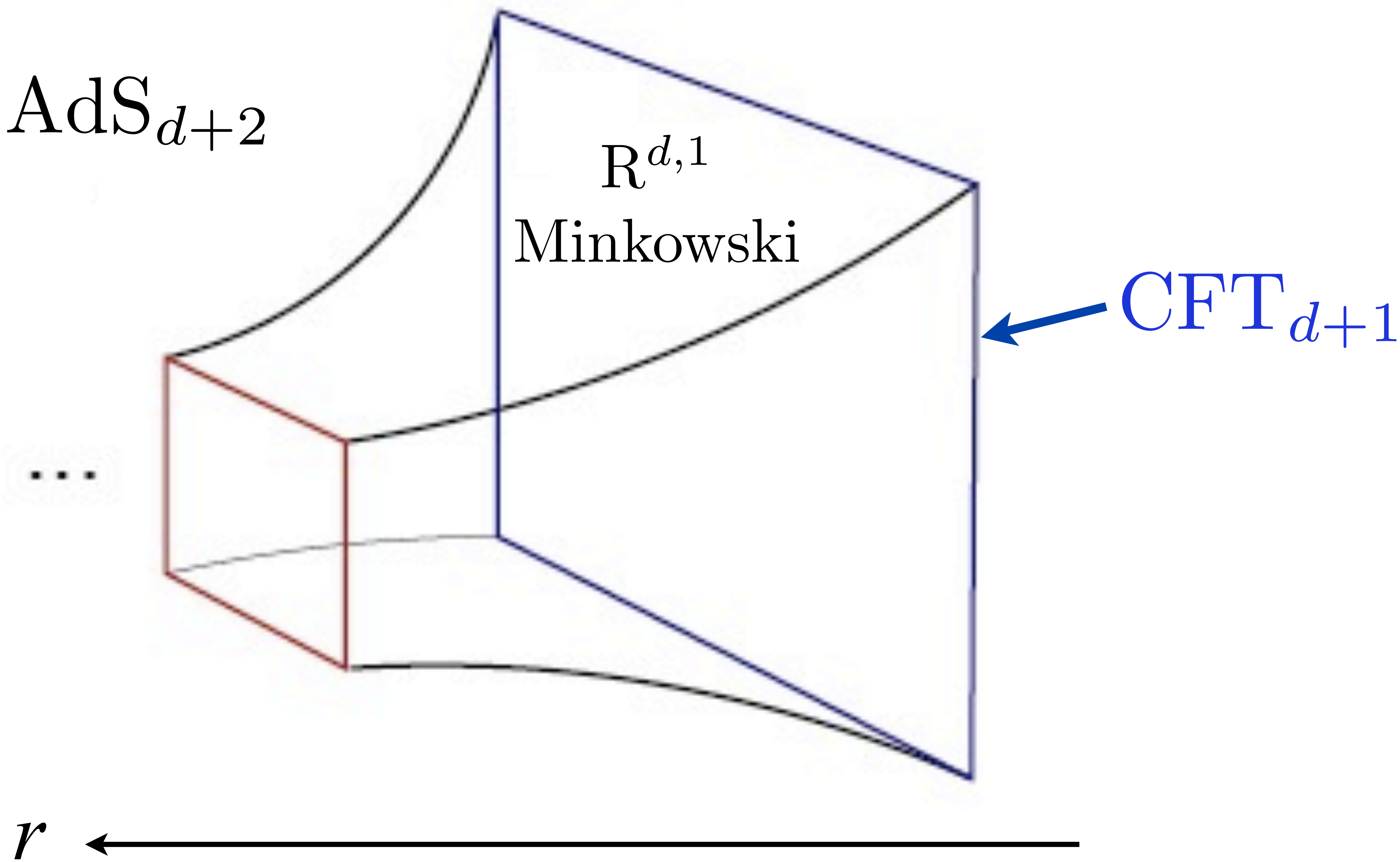
$$x_i \to \zeta x_i \quad , \quad t \to \zeta t \quad , \quad ds \to ds$$

This gives the unique metric

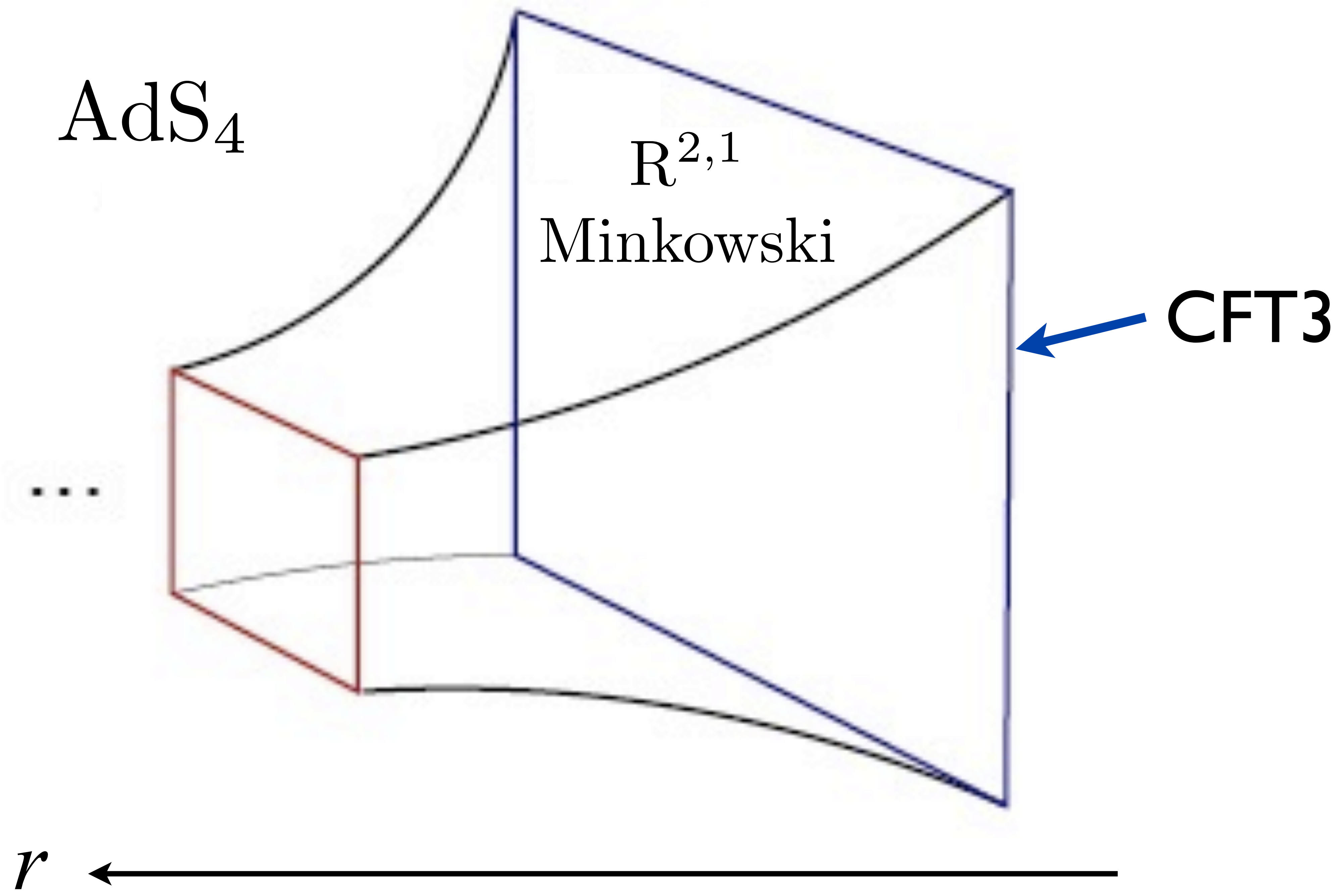
$$ds^2 = \frac{1}{r^2}\left(-dt^2 + dr^2 + dx_i^2\right)$$

Reparametrization invariance in r has been used to the prefactor of dx_i^2 equal to $1/r^2$. This fixes $r \to \zeta r$ under the scale transformation. This is the metric of the space AdS_{d+2}.

AdS/CFT correspondence



AdS/CFT correspondence



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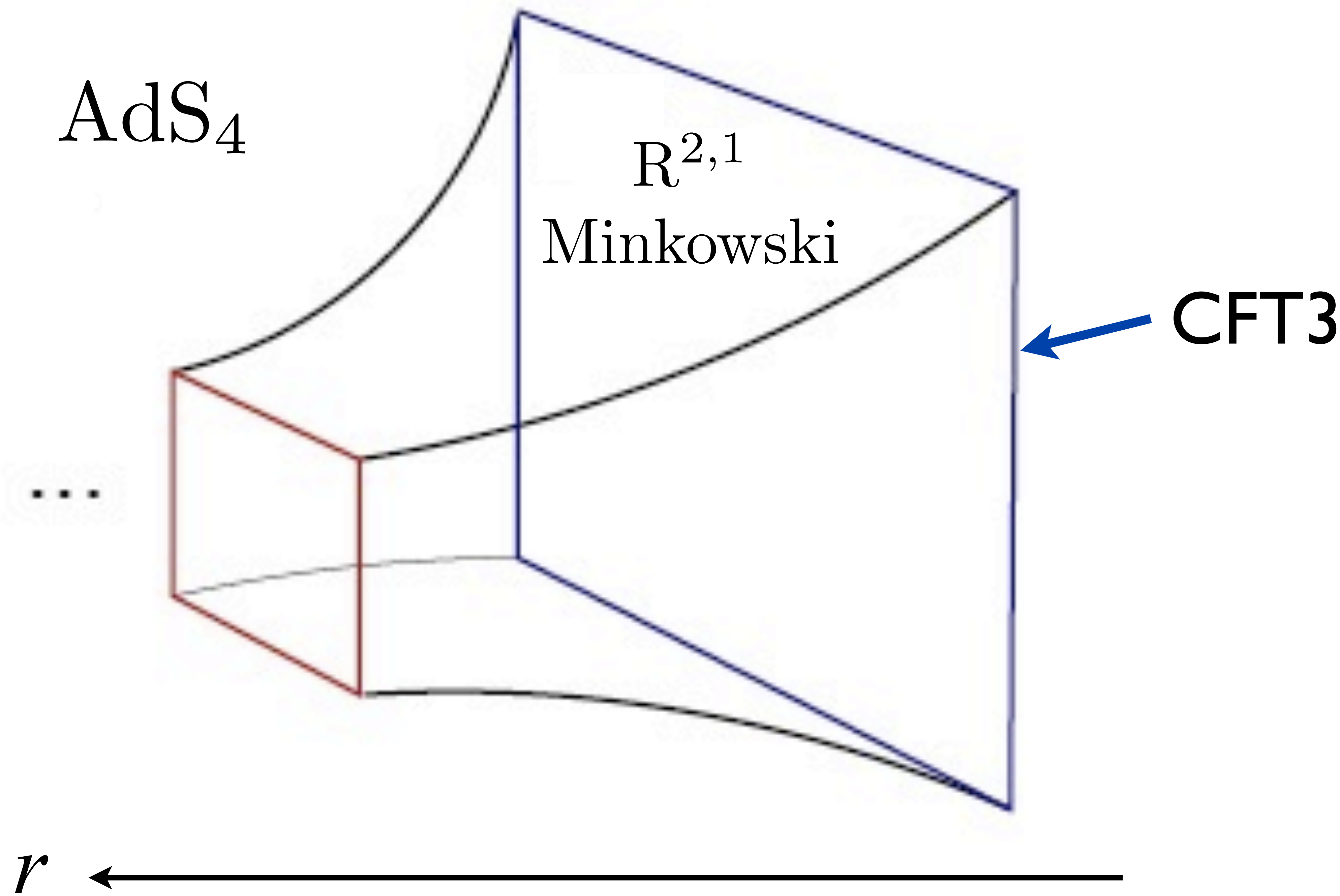
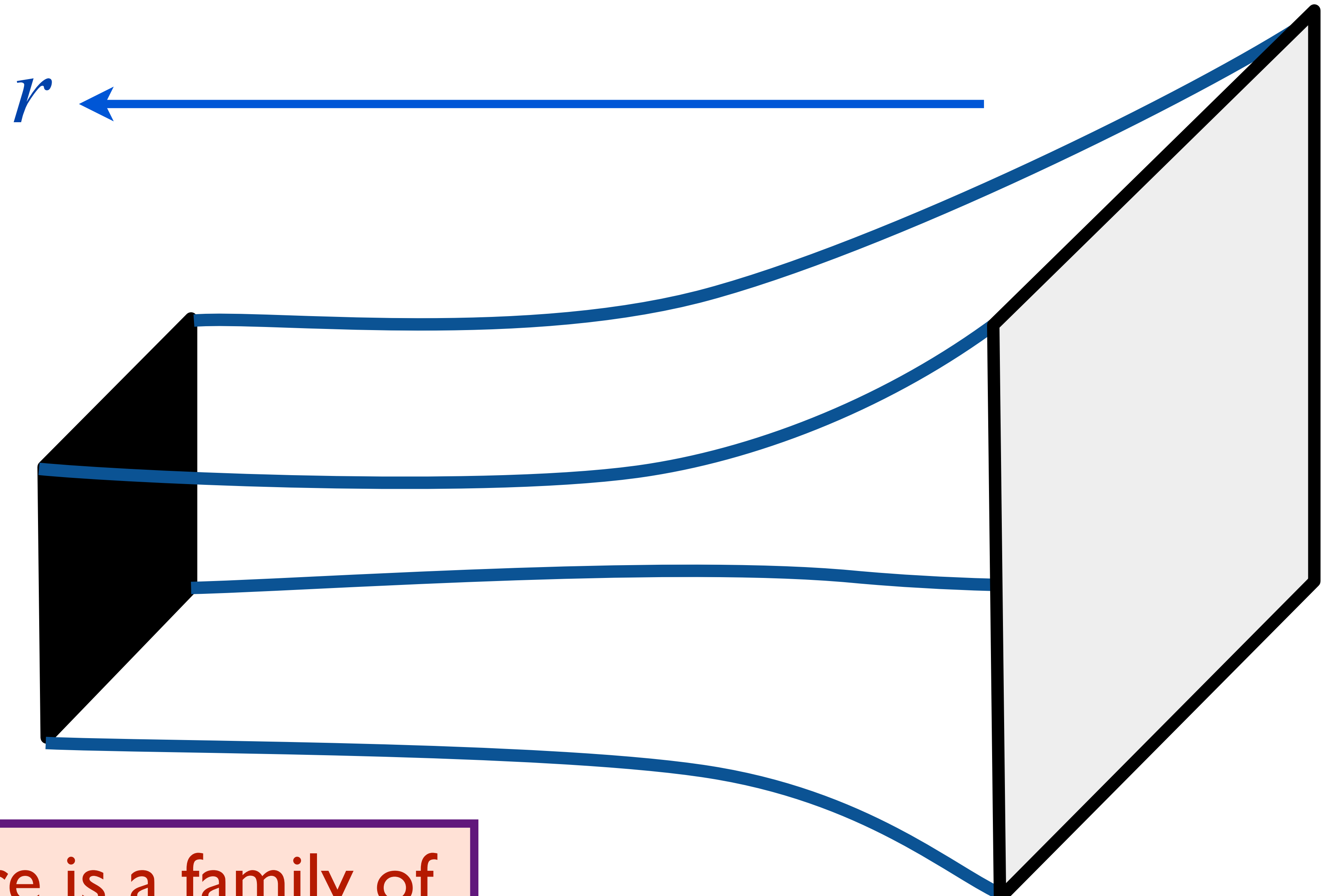


This emergent spacetime is a solution of Einstein gravity with a negative cosmological constant

$$\mathcal{S} = \int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R + \frac{6}{L^2}\right)\right]$$

AdS/CFT correspondence at non-zero temperatures

AdS$_4$-Schwarzschild black-brane



There is a family of solutions of Einstein gravity which describe non-zero temperatures

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AdS/CFT correspondence at non-zero temperatures

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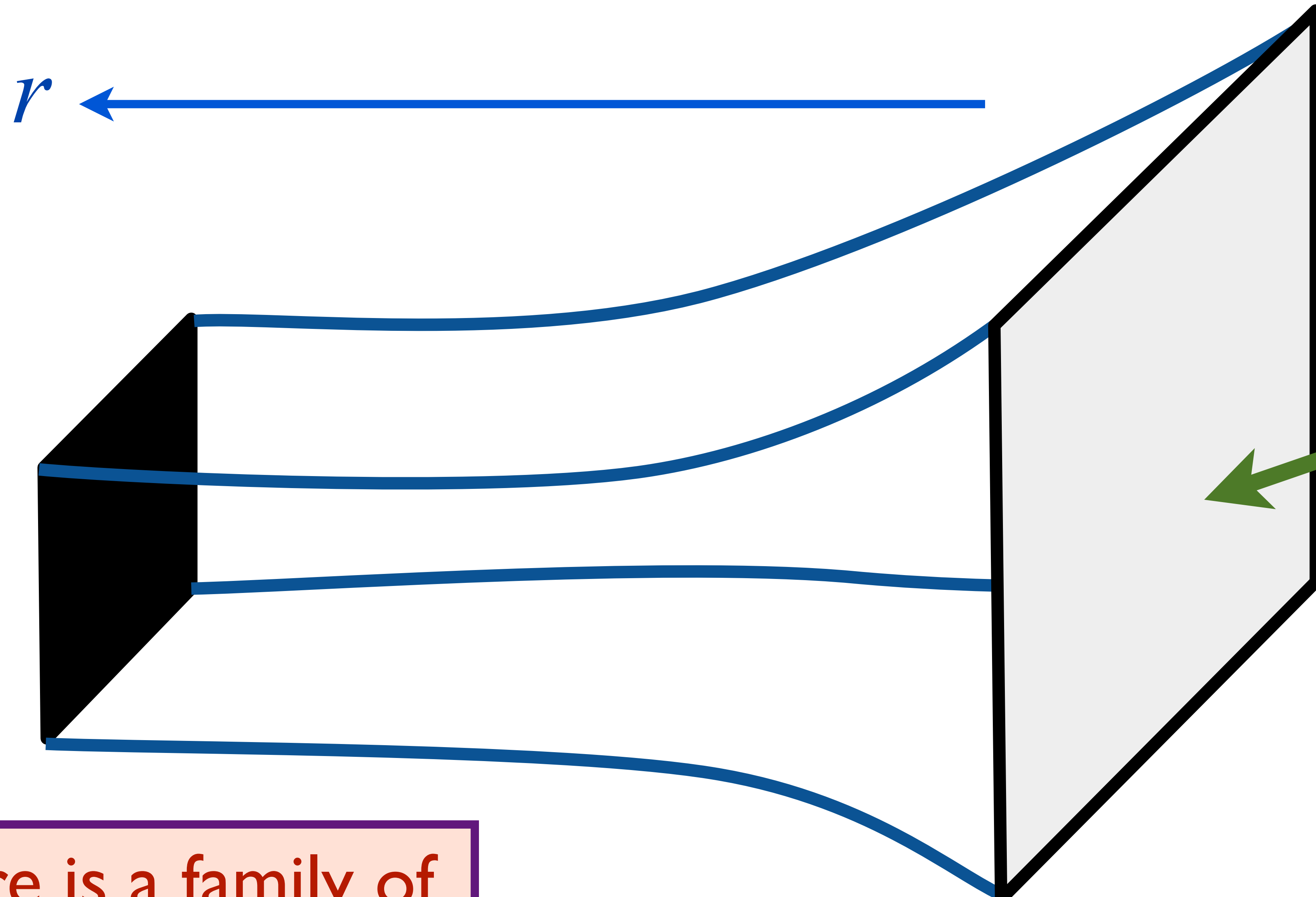


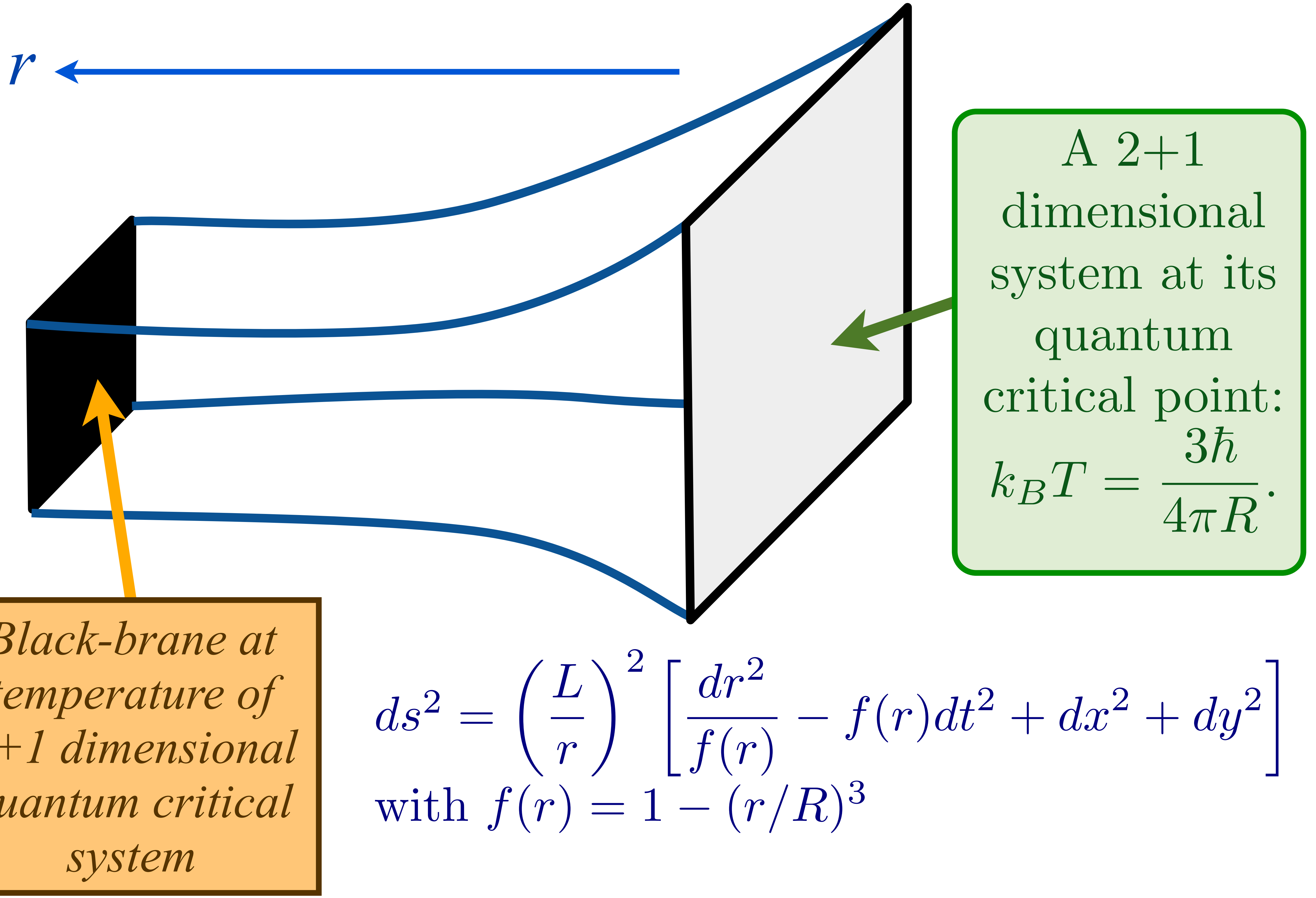
A 2+1 dimensional system at its quantum critical point: $k_B T = \frac{3\hbar}{4\pi R}$.

There is a family of solutions of Einstein gravity which describe non-zero temperatures

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = 1 - (r/R)^3$

AdS_4-Schwarzschild black-brane



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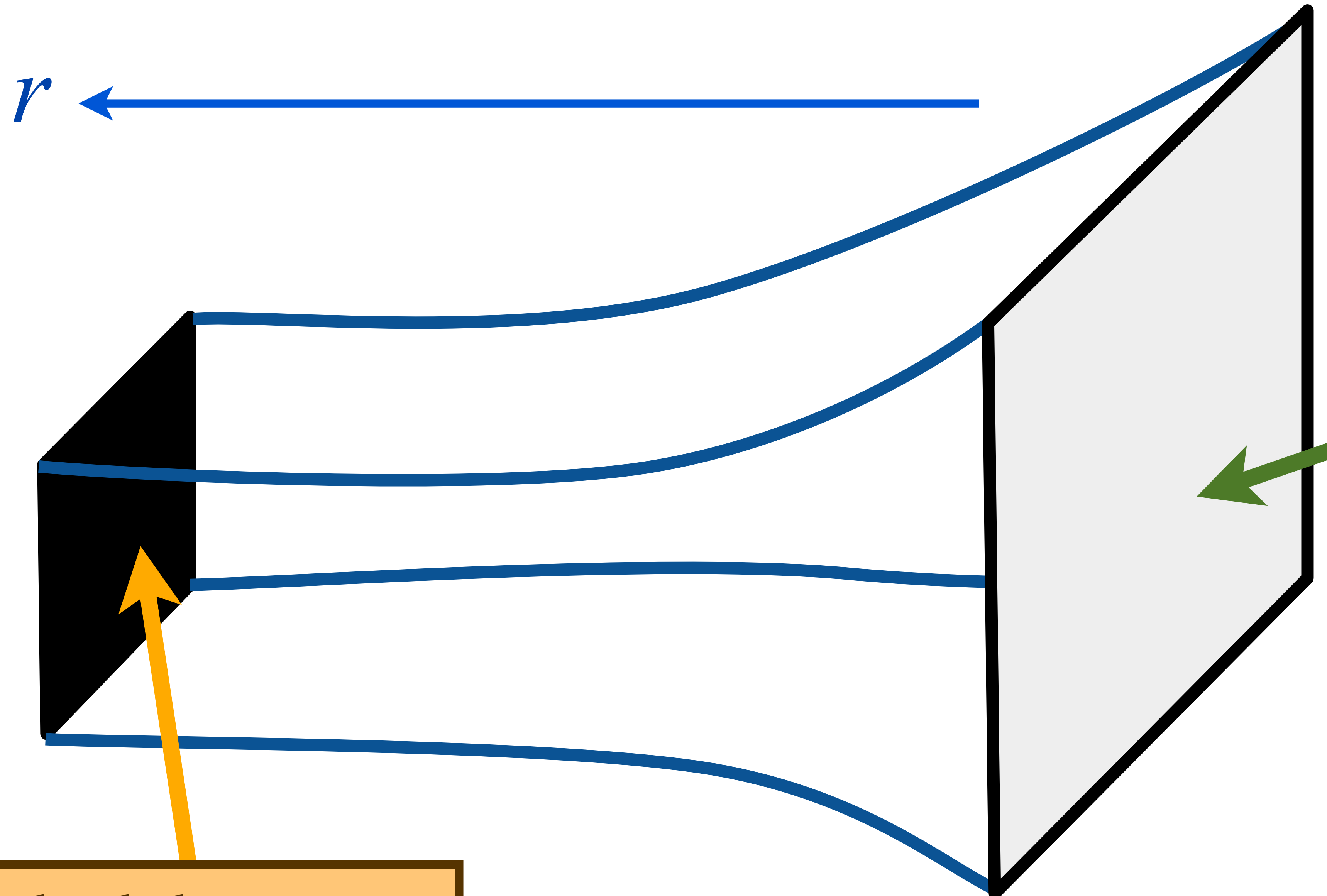


A 2+1 dimensional system at its quantum critical point:

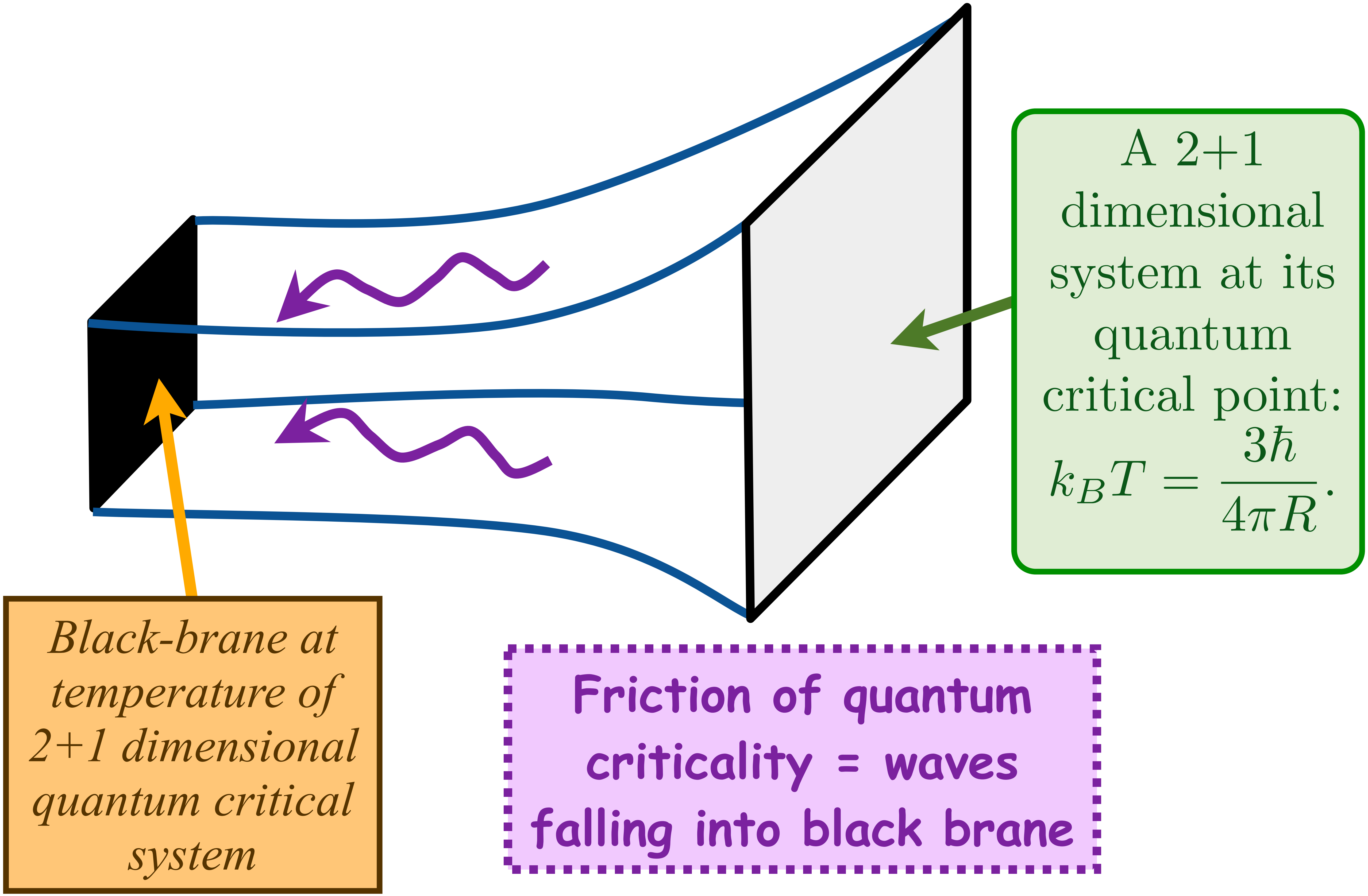
$$k_B T = \frac{3\hbar}{4\pi R}.$$

Black-brane at temperature of 2+1 dimensional quantum critical system

Beckenstein-Hawking entropy of black brane = entropy of CFT3

AdS/CFT correspondence at non-zero temperatures

AdS_4-Schwarzschild black-brane



AdS$_4$ theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS$_4$-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} \right] .$$

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

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$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} \right] .$$

This is to be solved subject to the constraint

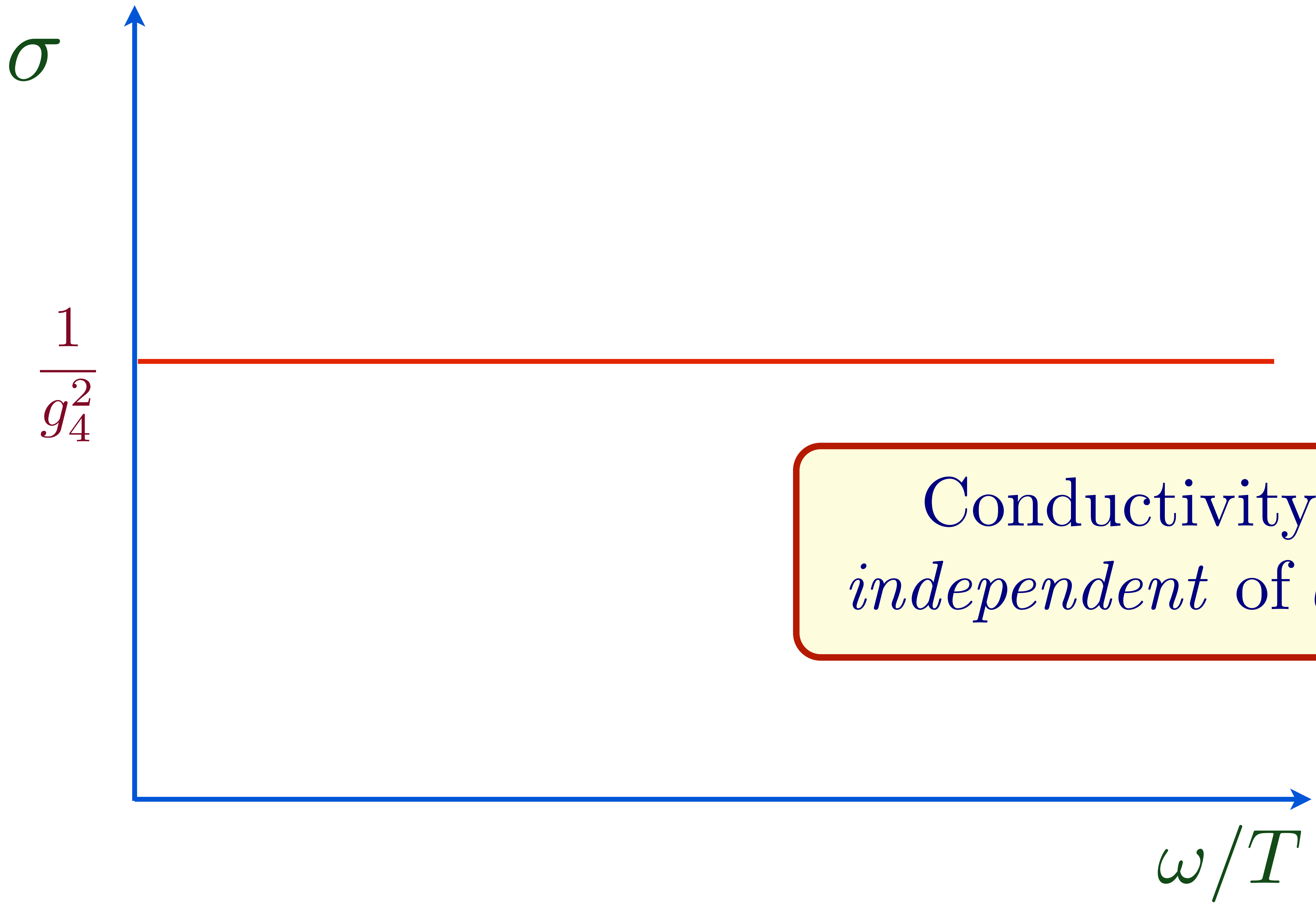
$$A_\mu(r \to \infty, x, y, t) = \mathcal{A}_\mu(x, y, t)$$

where $\mathcal{A}_\mu$ is a source coupling to a conserved U(1) current J_μ of the CFT3

$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

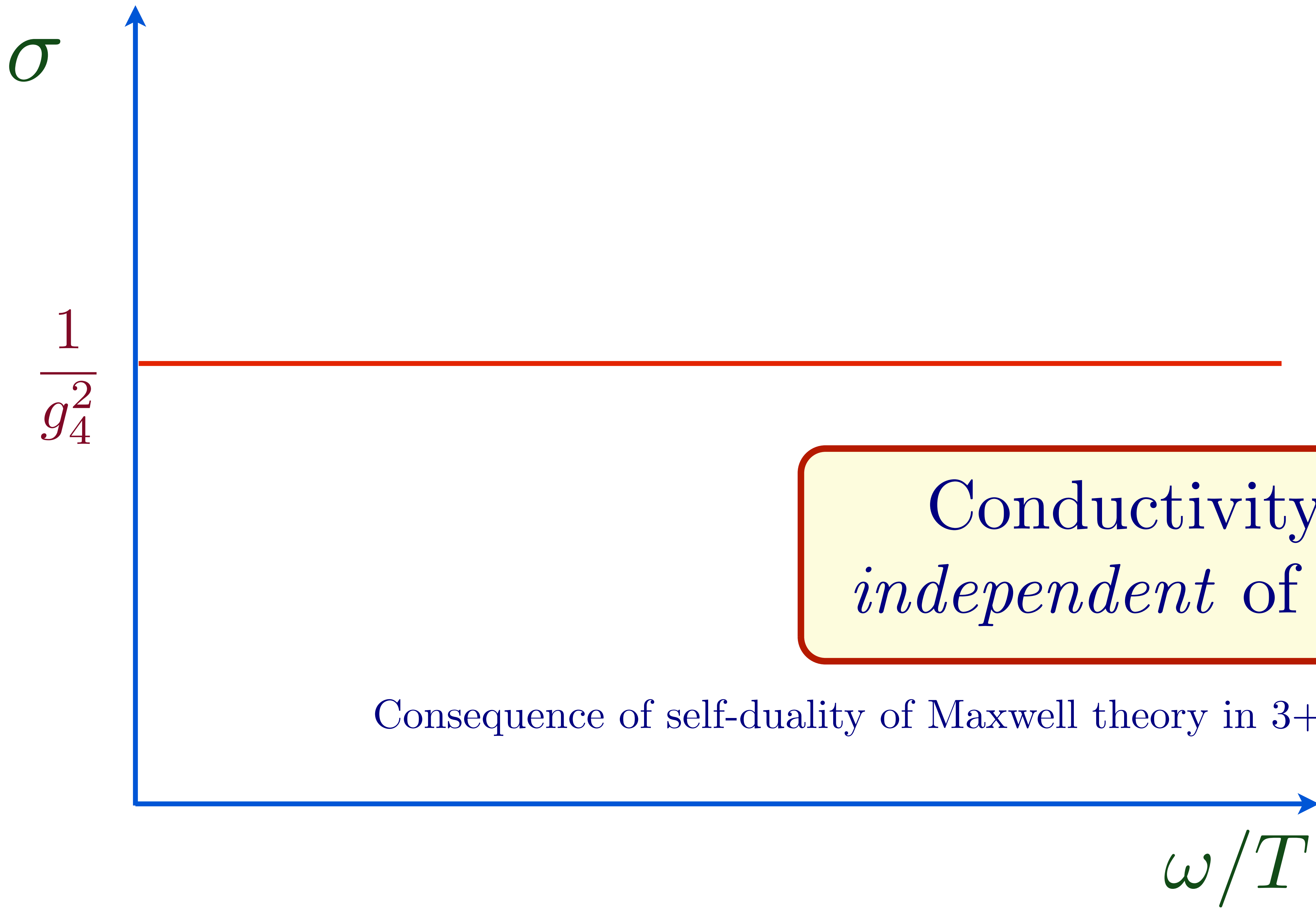
C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

AdS4 theory of electrical transport in a strongly interacting CFT3 for *T > 0*



Conductivity is $\textit{independent}$ of ω/T.

AdS4 theory of electrical transport in a strongly interacting CFT3 for *T > 0*

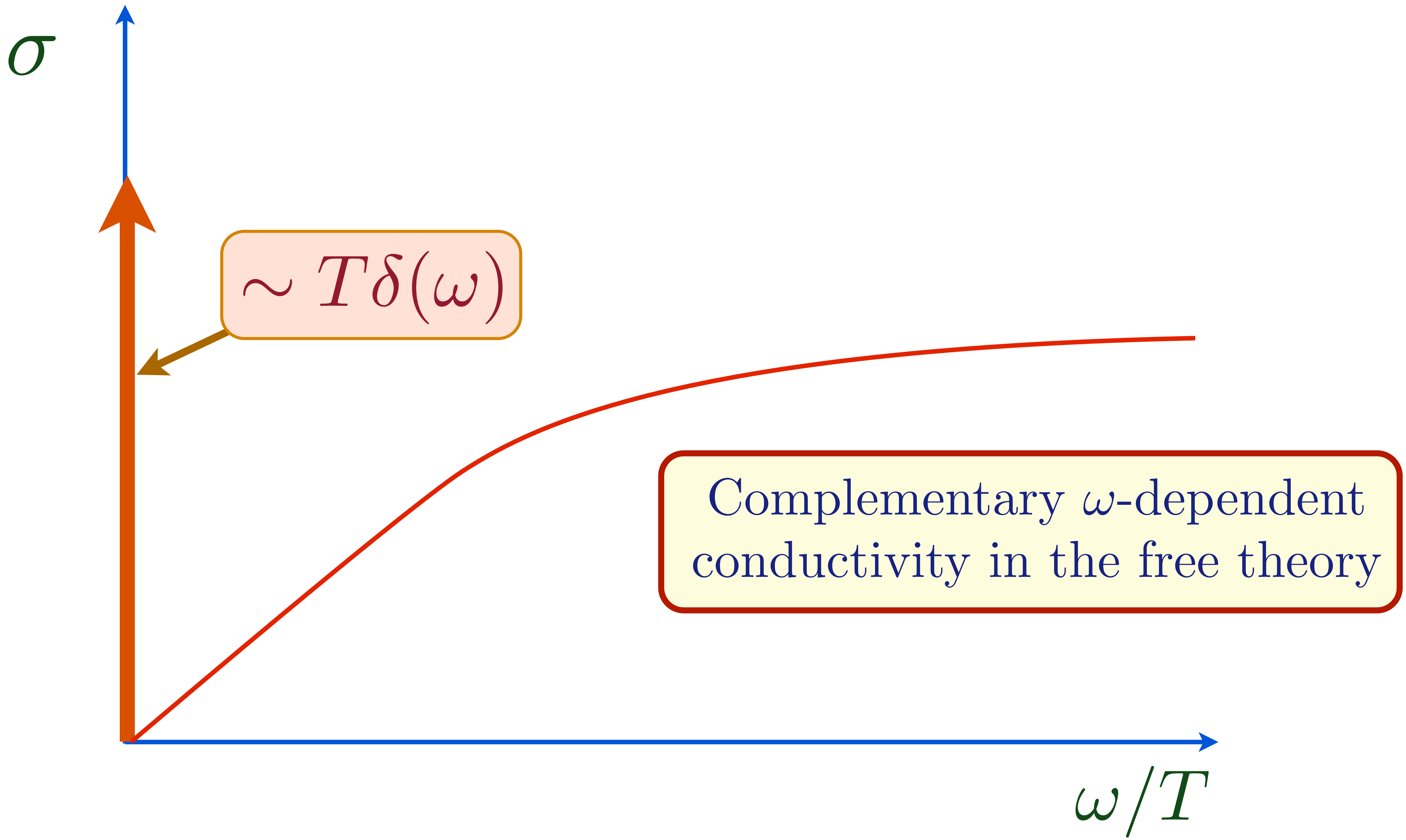


Conductivity is *independent* of ω/T.

Consequence of self-duality of Maxwell theory in 3+1 dimensions

C. P. Herzog, P. K. Kovtun, S. Sachdev, and D. T. Son, *Phys. Rev.* D **75**, 085020 (2007).

Electrical transport in a free CFT3 for *T* > 0



Improving the AdS_4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4-Schwarzschild

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS$_4$-Schwarzschild

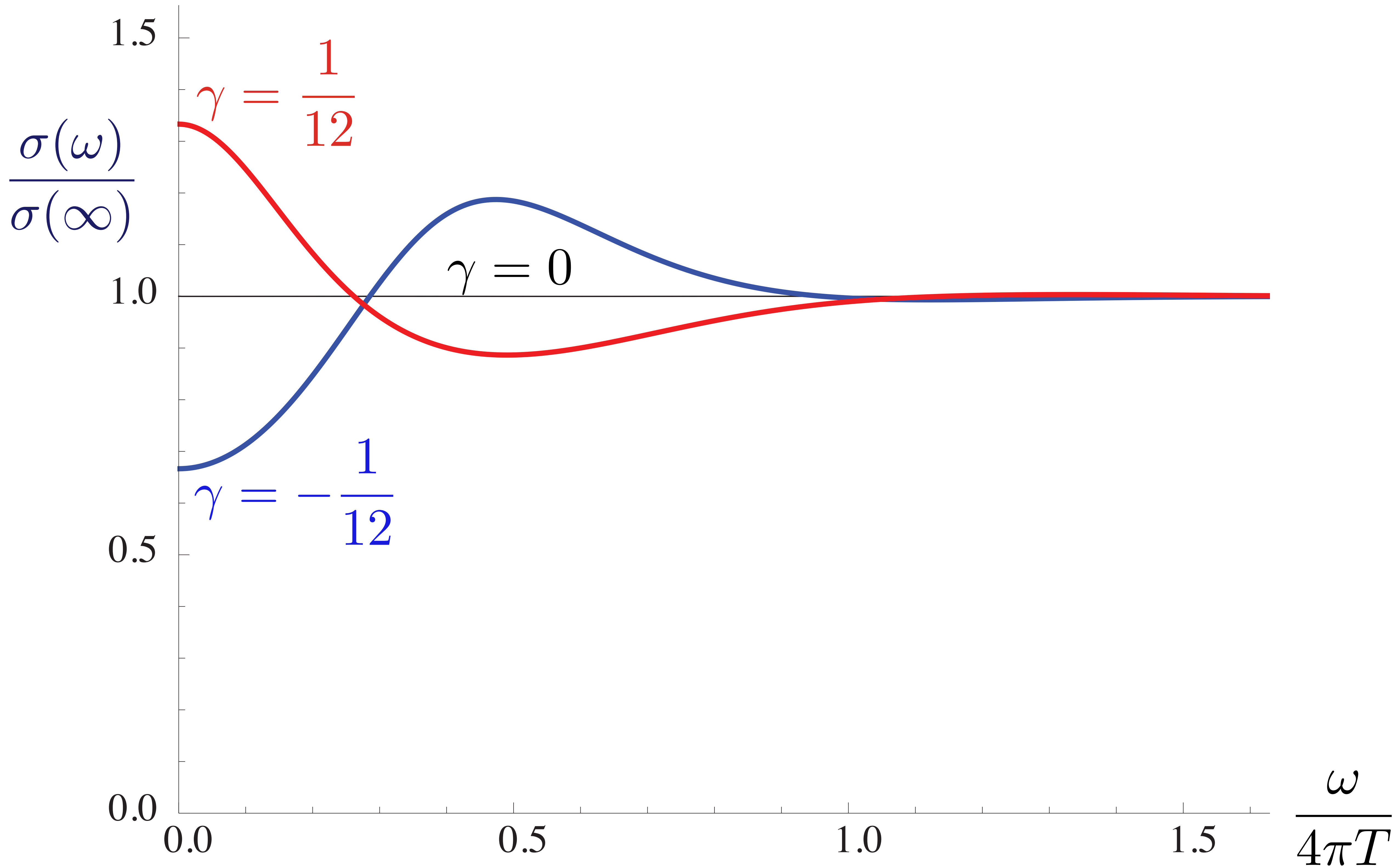
We include all possible 4-derivative terms: after suitable field redefinitions, the required theory has only *one* dimensionless constant γ (L is the radius of AdS$_4$):

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab}F^{ab} + \frac{\gamma L^2}{g_4^2} C_{abcd} F^{ab} F^{cd} \right] ,$$

where C_{abcd} is the Weyl curvature tensor.
Stability and causality constraints restrict $|\gamma| < 1/12$.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Improving the AdS_4 theory of "nearly perfect fluids"



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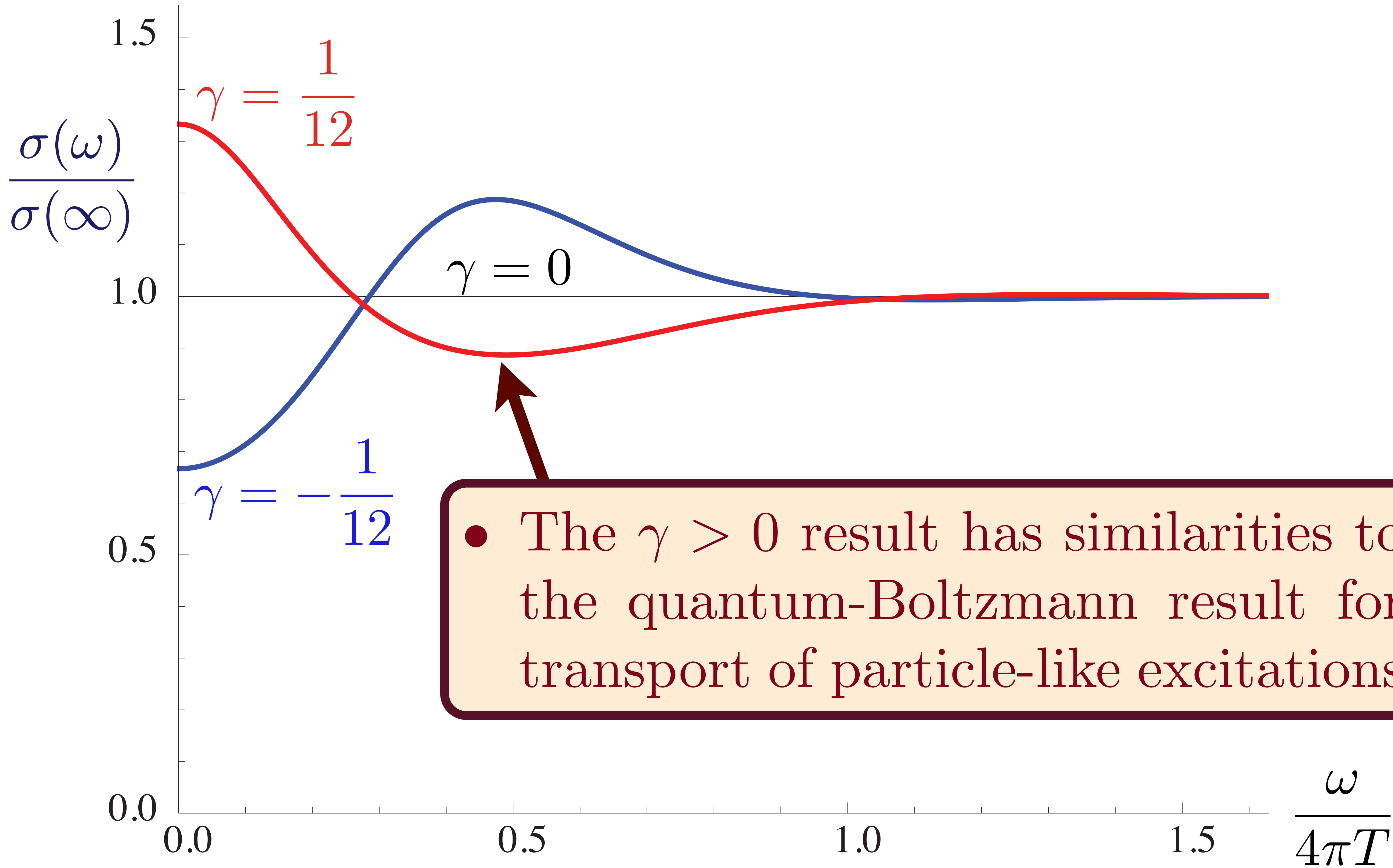

- The $\gamma > 0$ result has similarities to the quantum-Boltzmann result for transport of particle-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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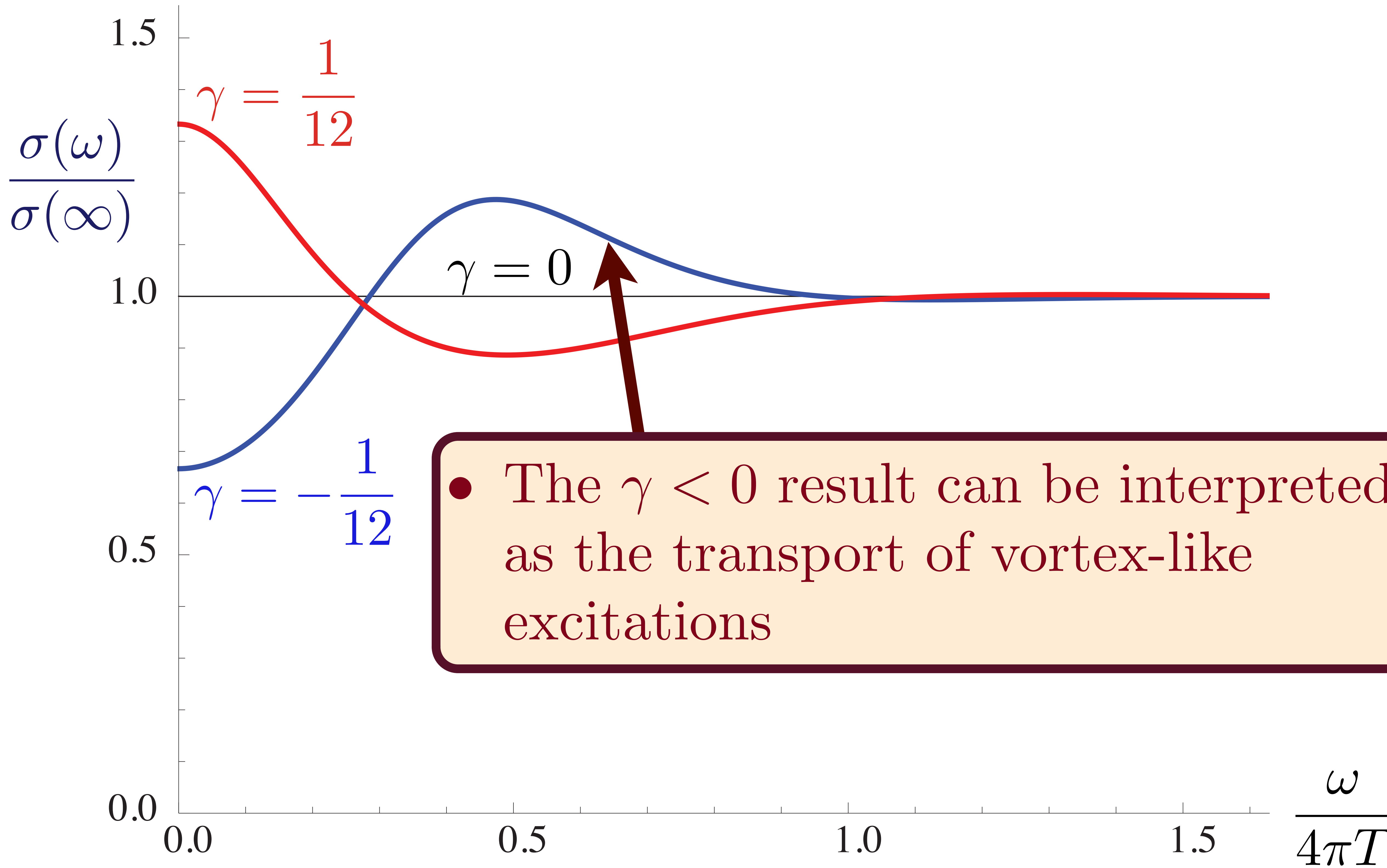
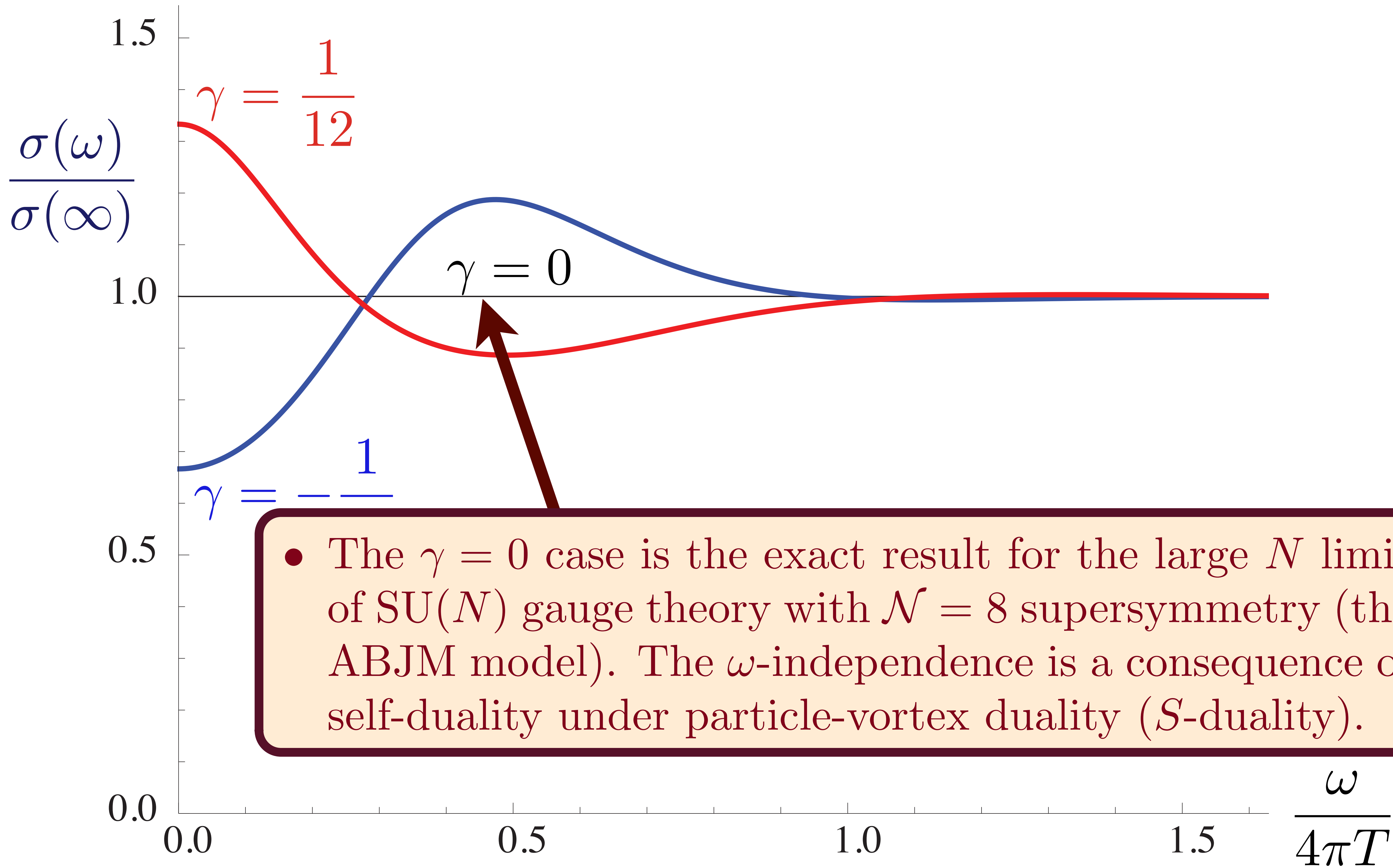


- The $\gamma < 0$ result can be interpreted as the transport of vortex-like excitations

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Improving the AdS$_4$ theory of "nearly perfect fluids"



- The $\gamma = 0$ case is the exact result for the large N limit of SU(N) gauge theory with $\mathcal{N} = 8$ supersymmetry (the ABJM model). The ω-independence is a consequence of self-duality under particle-vortex duality (S-duality).

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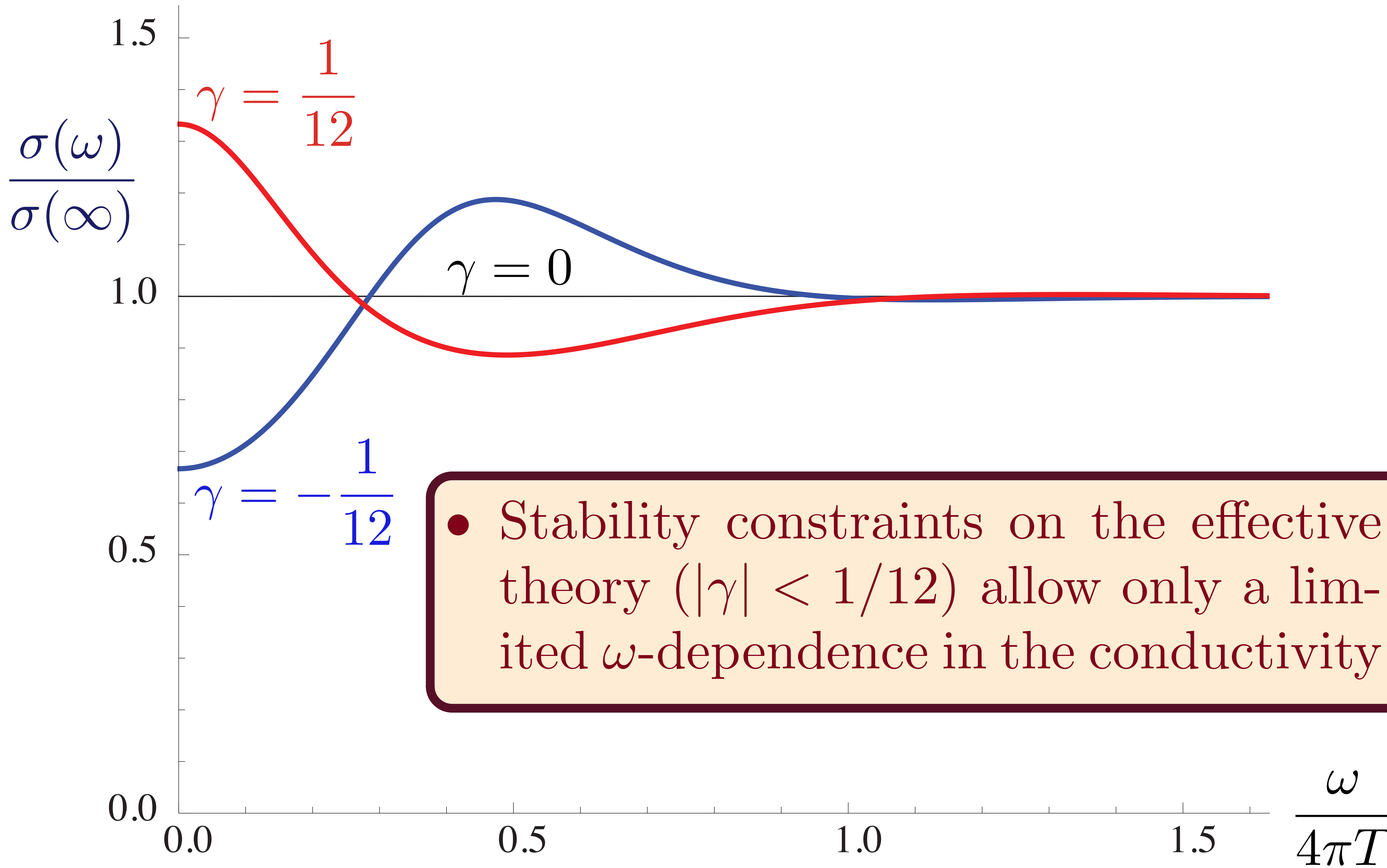

- Stability constraints on the effective theory ($|\gamma| < 1/12$) allow only a limited ω-dependence in the conductivity

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

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Theory for transport of conserved quantities in CFT3s:

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab}F^{ab} + \frac{\gamma L^2}{g_4^2} C_{abcd} F^{ab} F^{cd} \right] ,$$

where C_{abcd} is the Weyl curvature tensor.

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where C_{abcd} is the Weyl curvature tensor.

General approach:

- Theory has 2 free dimensionless parameters: g_4^2 and γ. We match these to correlators of the CFT3 of interest at $\omega \gg T$: g_4^2 is determines the current correlator $\langle J_\mu J_\nu \rangle$, while γ determines the 3-point function $\langle T_{\mu\nu} J_\rho J_\sigma \rangle$, where $T_{\mu\nu}$ is the stress-energy tensor.

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- We determine these $\omega \gg T$ correlators of the CFT3 by other methods (*e.g.* vector large N expansion), and so obtain values of g_4^2 and γ.
- We use $\mathcal{S}_{EM}$ to extrapolate to transport properties for $\omega \ll T$. This step is traditionally carried out by descendants of the Boltzmann equation.

R. C. Myers, S. Sachdev, and A. Singh, *Physical Review D* **83**, 066017 (2011)

Conclusions

Conformal quantum matter

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- New insights and solvable models for diffusion and transport of strongly interacting systems near quantum critical points

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- Prospects for experimental tests of frequency-dependent, non-linear, and non-equilibrium transport

States of quantum matter with long-range entanglement in *d* spatial dimensions

Gapped quantum matter

Spin liquids, quantum Hall states

Conformal quantum matter

Graphene, ultracold atoms, antiferromagnets

Compressible quantum matter

Graphene, strange metals in high temperature superconductors, spin liquids

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Liza Huijse

Max Metlitski

Brian Swingle

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C. Non-Fermi liquids: nematic critical point (and U(1) spin liquids)

D. Holography: scaling arguments for entropy and entanglement entropy

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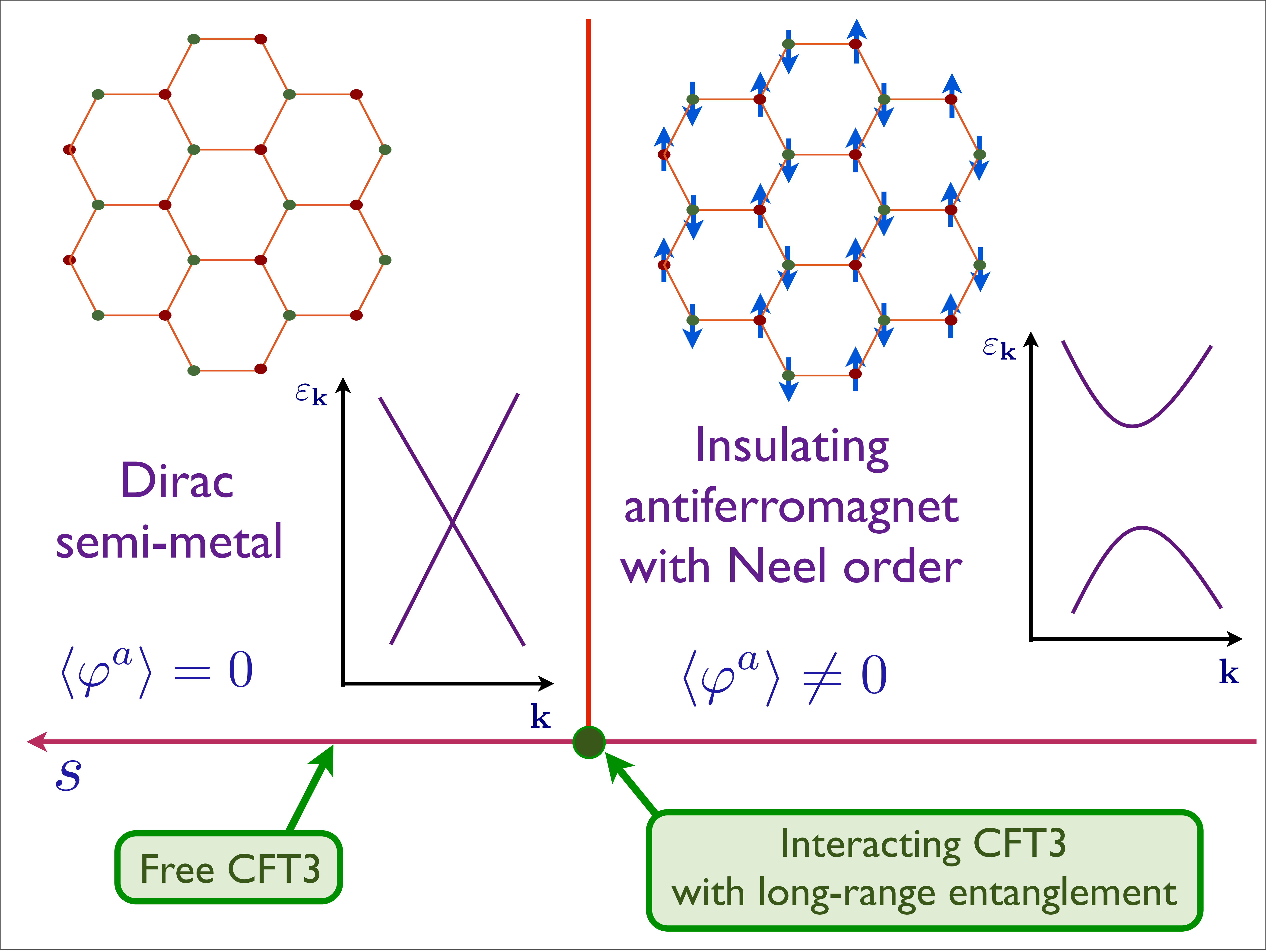
Dirac
semi-metal
$\langle \varphi^a \rangle = 0$
$\varepsilon_{\mathbf{k}}$
$\mathbf{k}$
Insulating
antiferromagnet
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s
Free CFT3
Interacting CFT3
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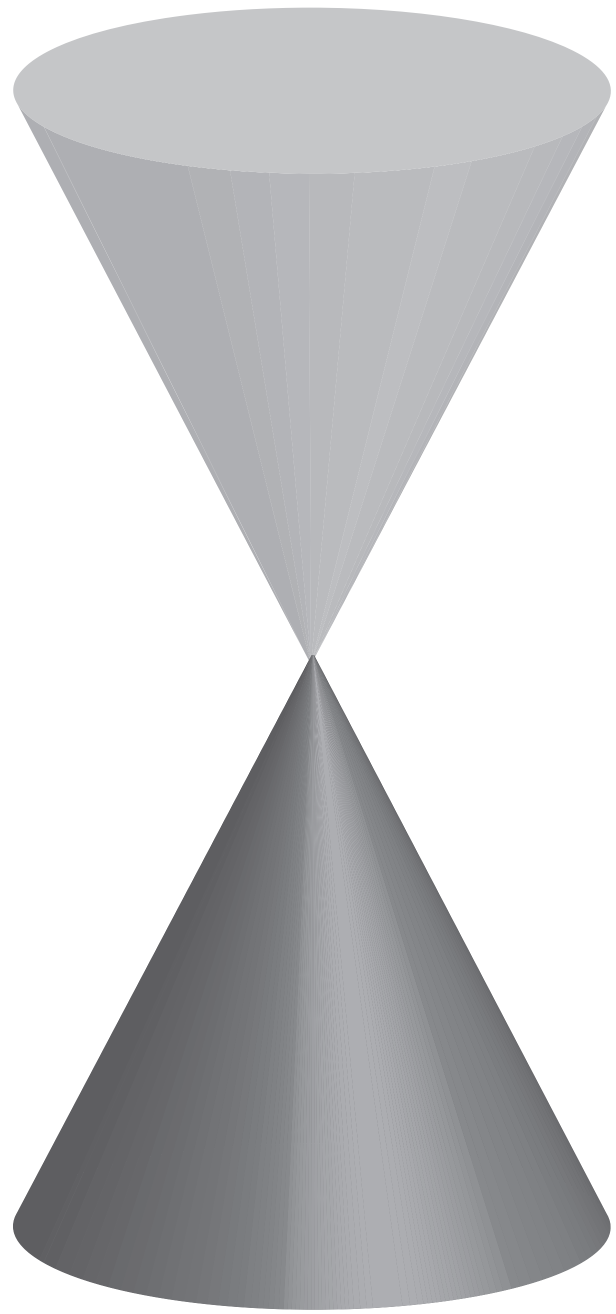
Quantum phase transition in graphene tuned by a chemical potential (gate voltage)



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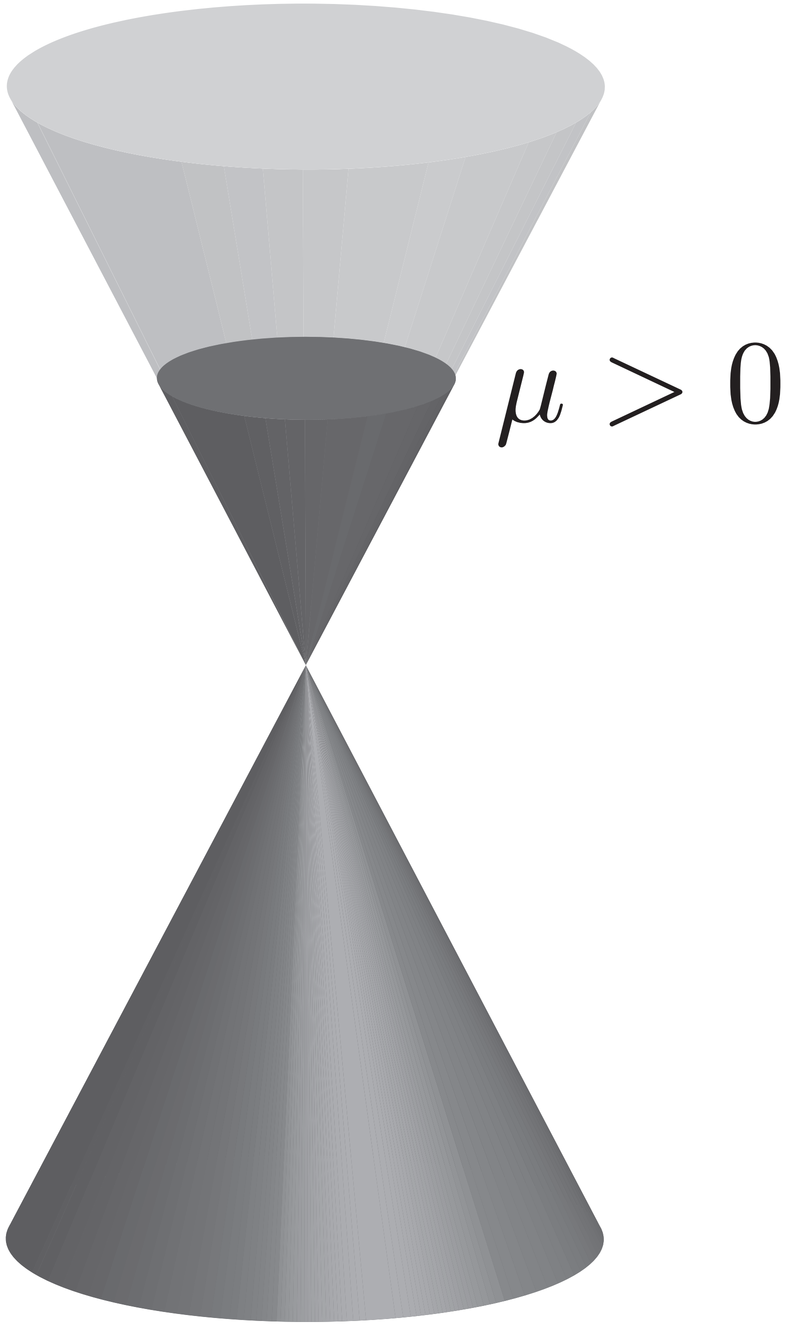

Electron
Fermi surface

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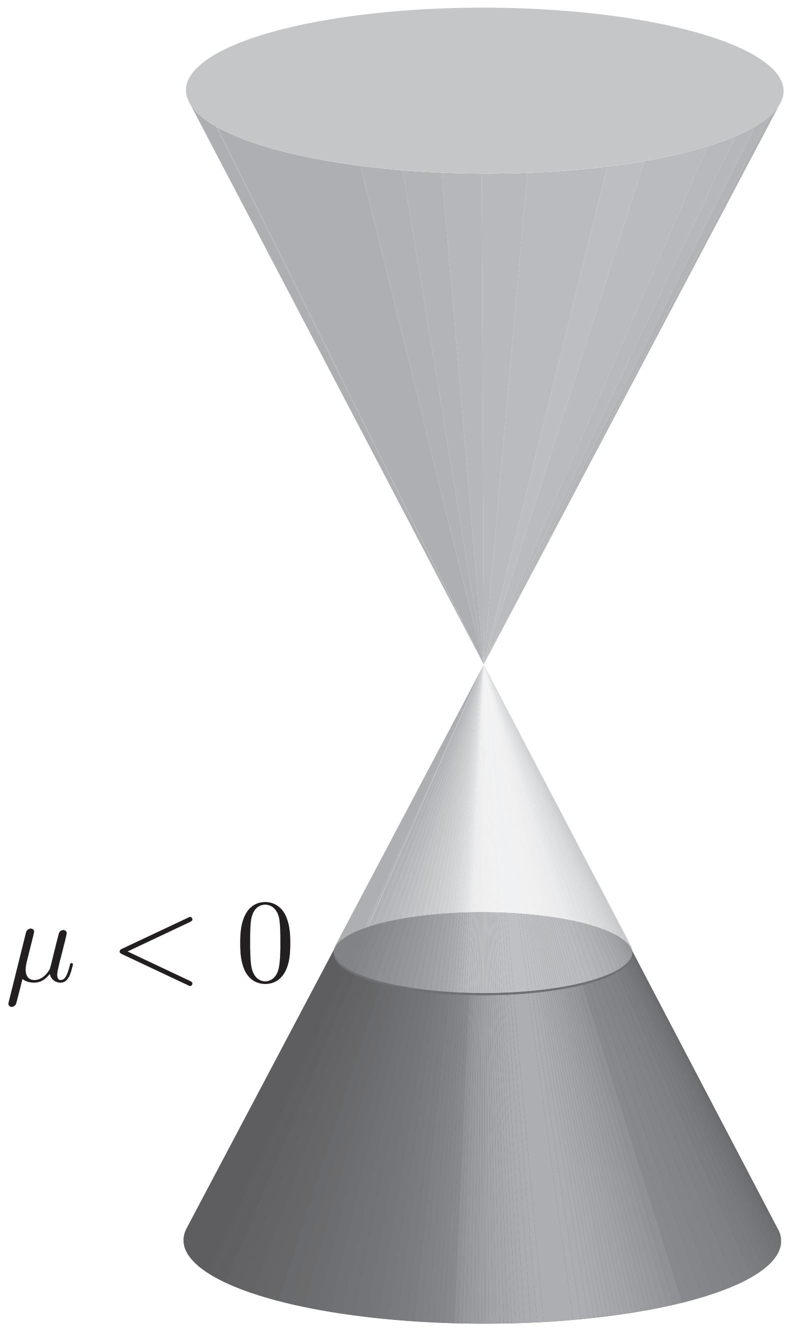
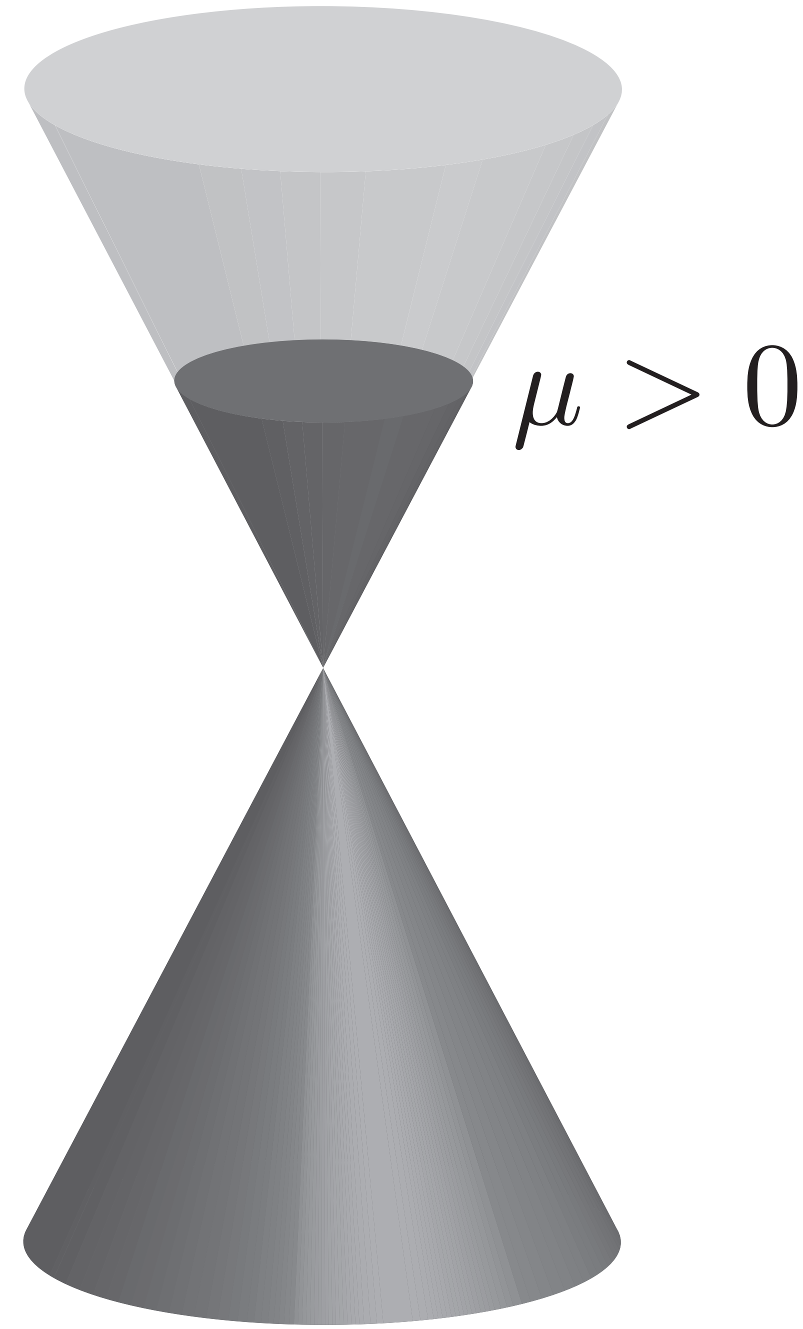


Hole
Fermi surface



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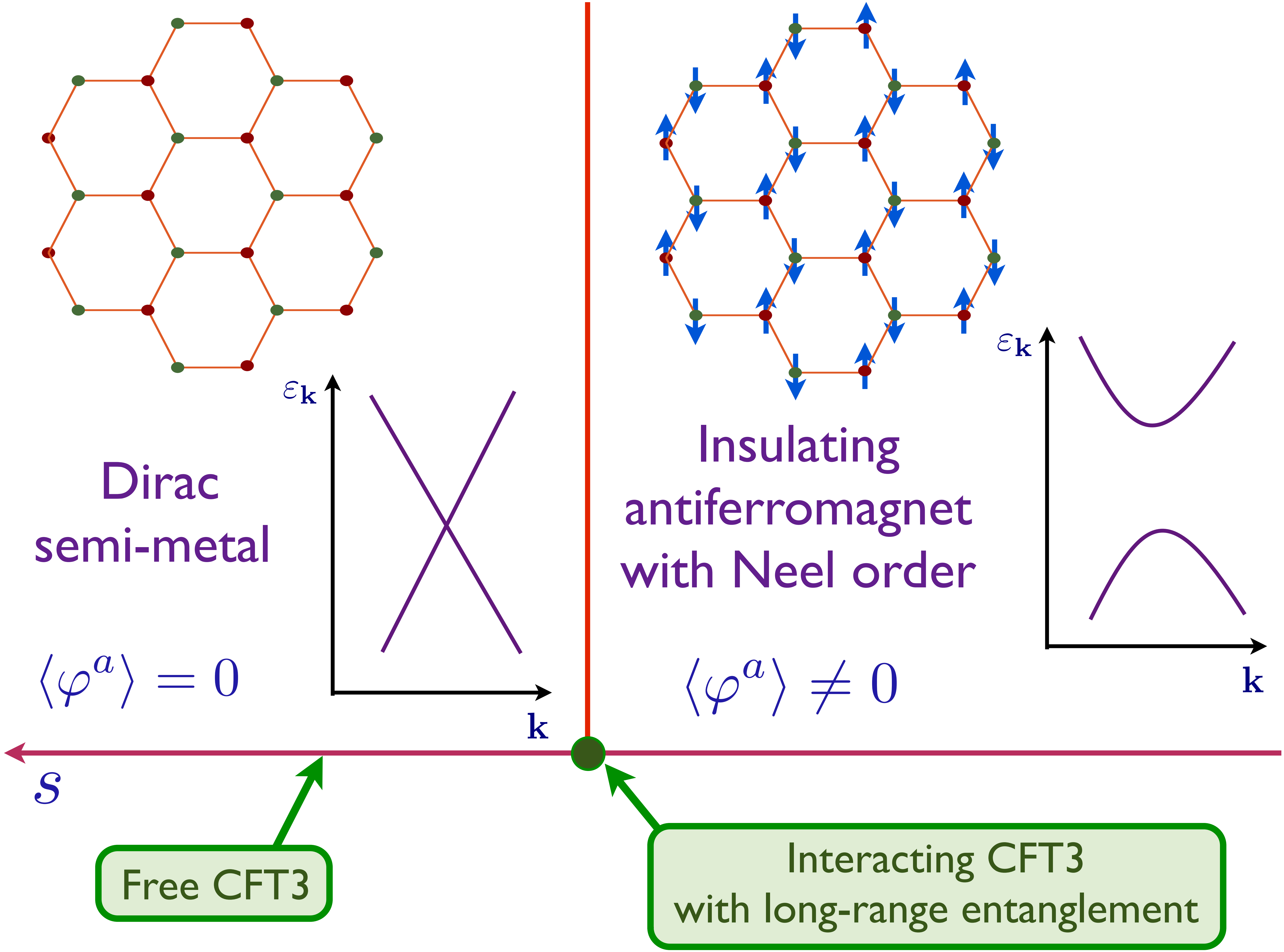

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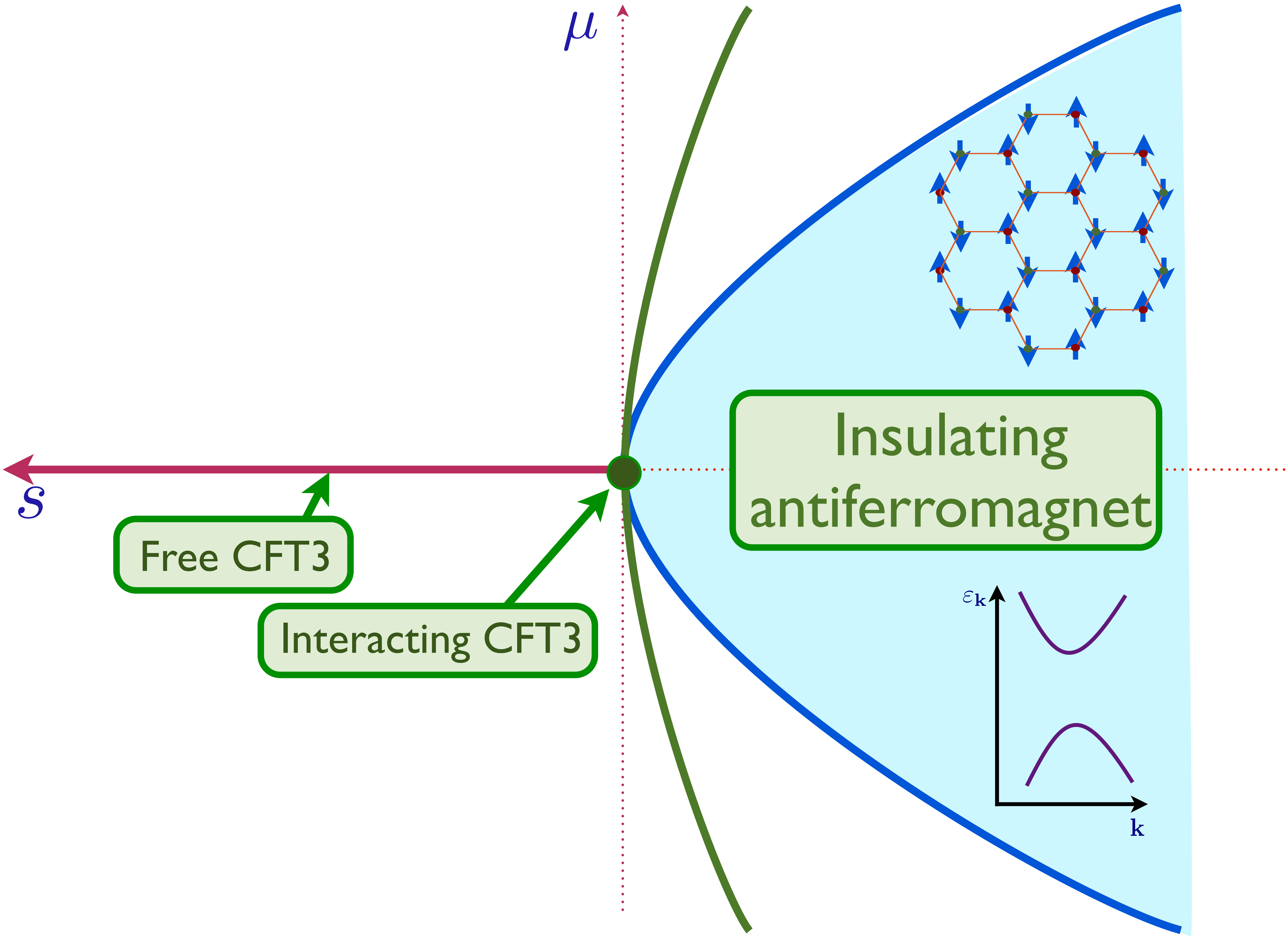
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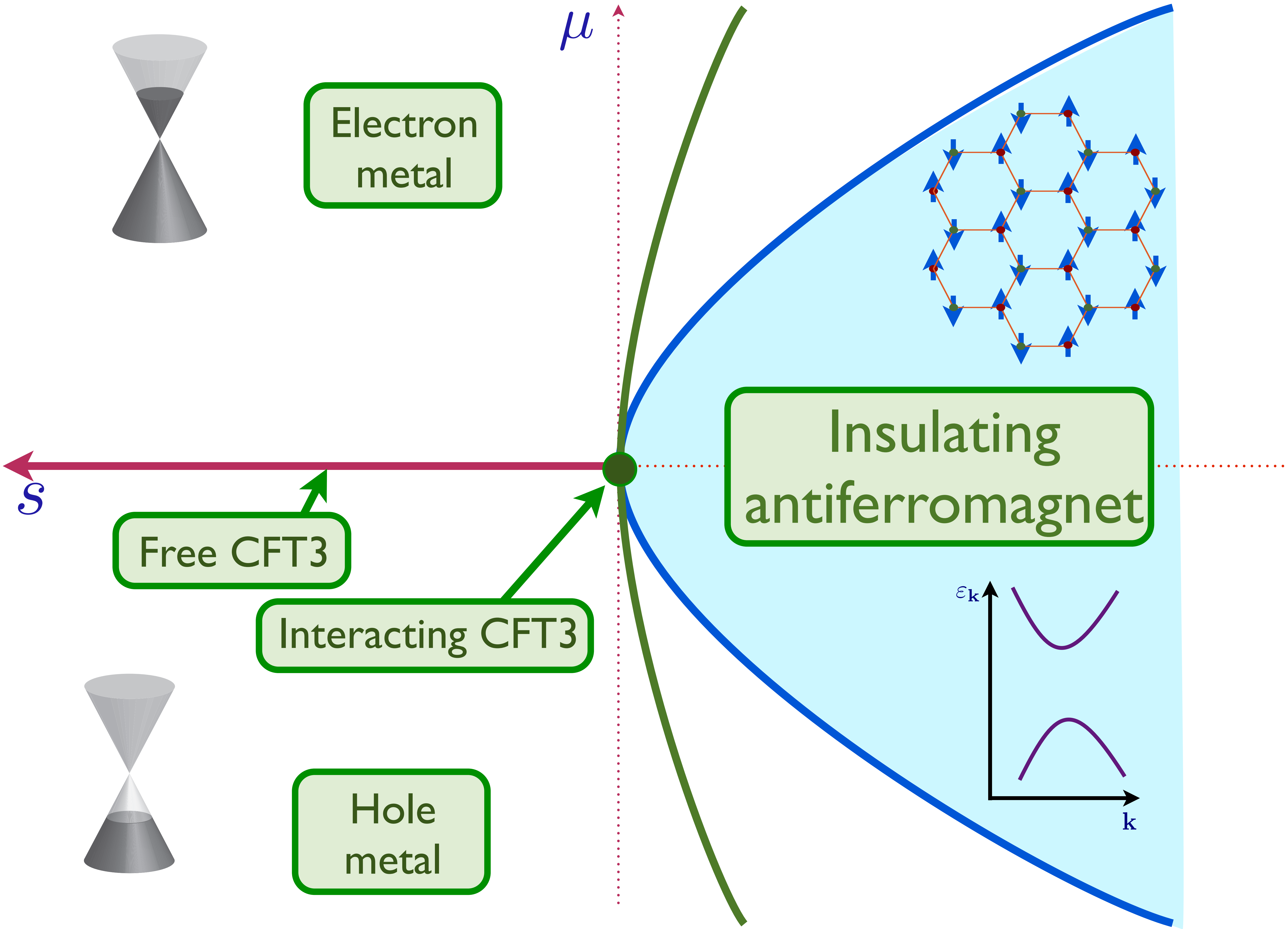

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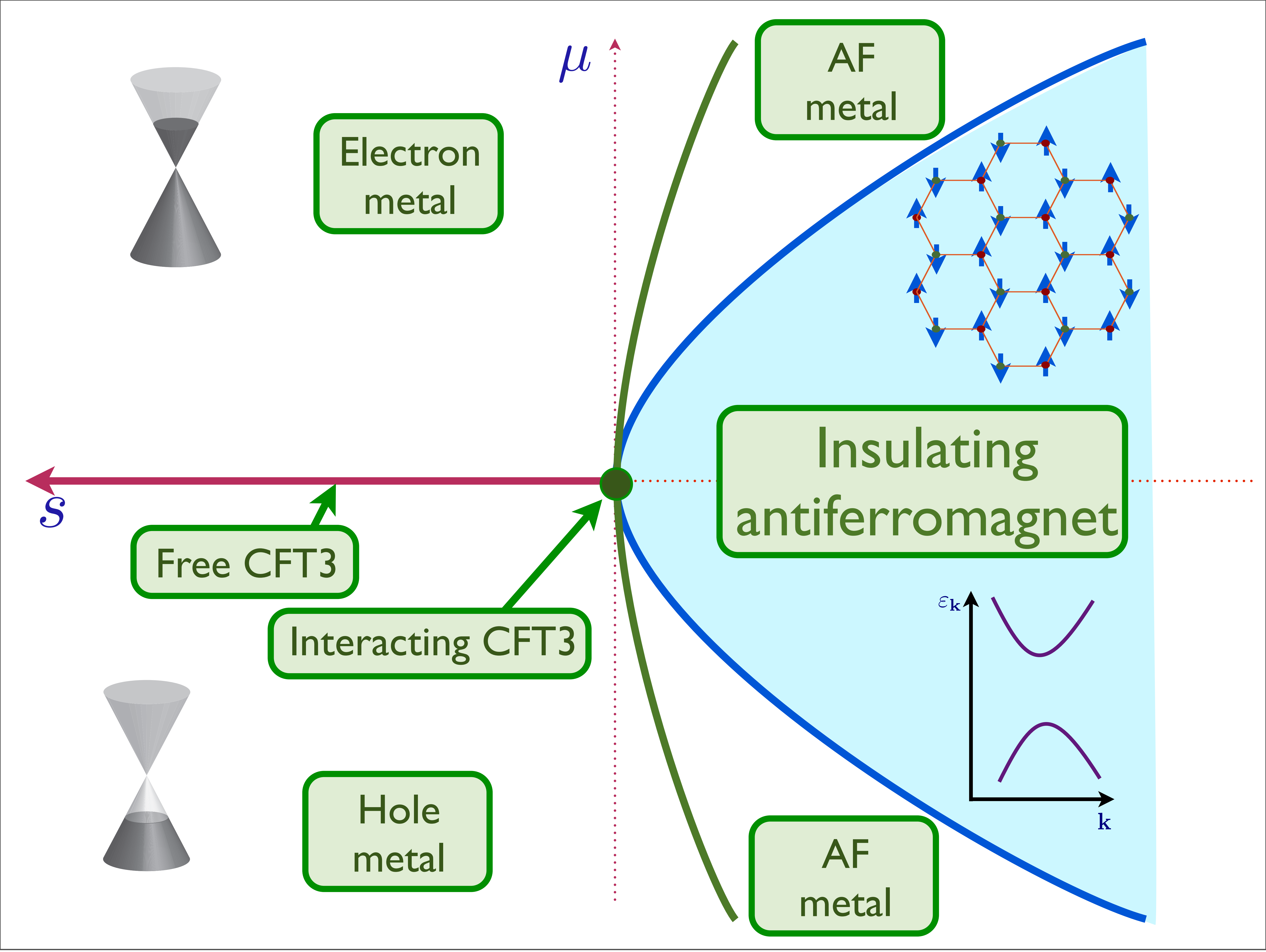

μ
AF
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k
Hole
metal
AF
metal

Transport in graphene at non-zero μ

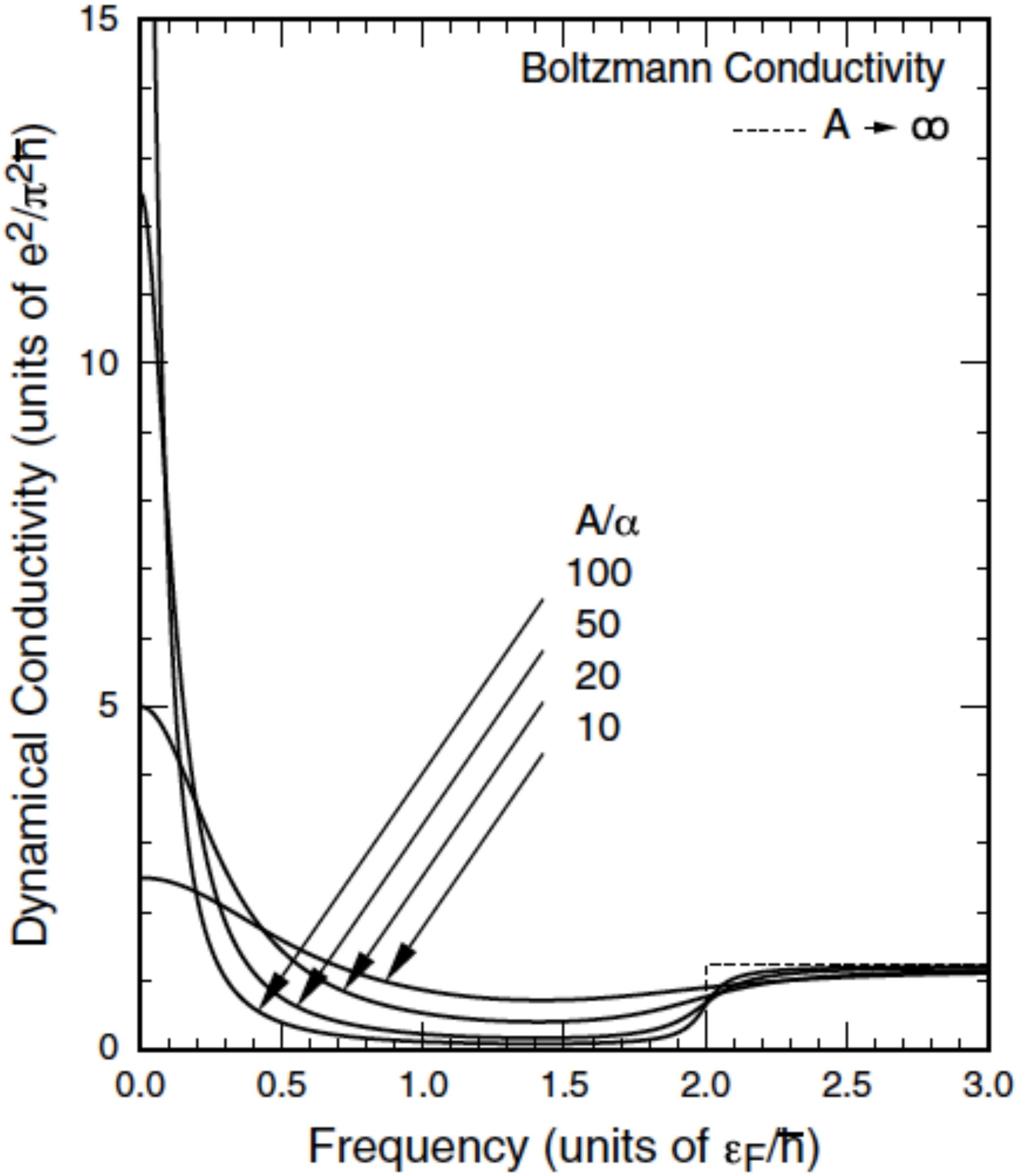
From the Kubo formula

$$\sigma(\omega) = 2\,(ev_F)^2\,\frac{\hbar}{i}\sum_{ss'}\int\frac{d^2k}{4\pi^2}\,\frac{f(\varepsilon_s(\mathbf{k})) - f(\varepsilon_{s'}(\mathbf{k}))}{(\varepsilon_s(\mathbf{k}) - \varepsilon_{s'}(\mathbf{k}))(\varepsilon_s(\mathbf{k}) - \varepsilon_{s'}(\mathbf{k}) + \hbar\omega + i\eta)}$$

where $\varepsilon_s(\mathbf{k}) = s\hbar v_F|\mathbf{k}|$ and $s, s' = \pm 1$ for the valence and conduction bands.

T. Ando, Y. Zheng and H. Suzuura, J. Phys. Soc. Jpn. **71** (2002) pp. 1318-1324

Transport in graphene at non-zero μ



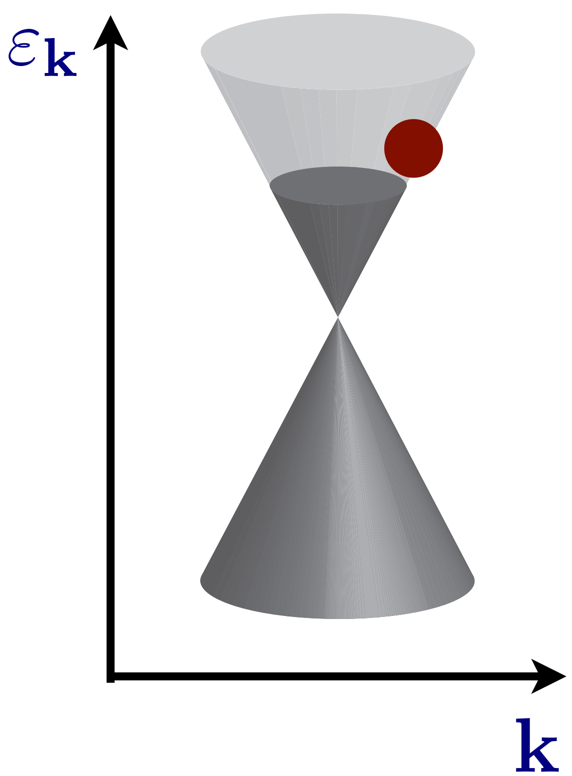
A is inversely proportional to disorder. In the clean limit $A \to \infty$, at $T = 0$

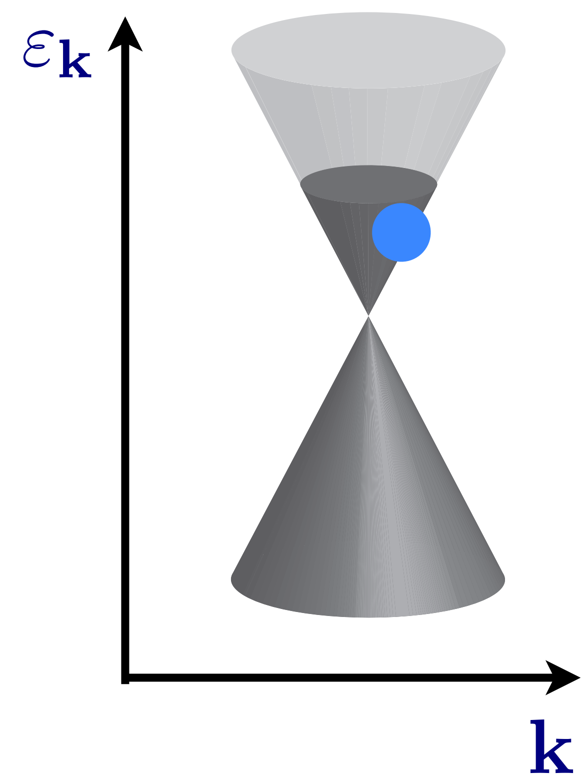
$$\text{Re}[\sigma(\omega)] = \frac{e^2}{\hbar}\left[\frac{\varepsilon_F}{\hbar}\delta(\omega) + \frac{1}{4}\theta(|\omega| - 2\varepsilon_F)\right]$$

Notice delta function is present even at $T = 0$ at non-zero density: this is a generic consequence of the conservation of momentum in any clean interacting Fermi liquid. Only "umklapp" scattering can broaden this delta function.

T. Ando, Y. Zheng and H. Suzuura, J. Phys. Soc. Jpn. **71** (2002) pp. 1318-1324

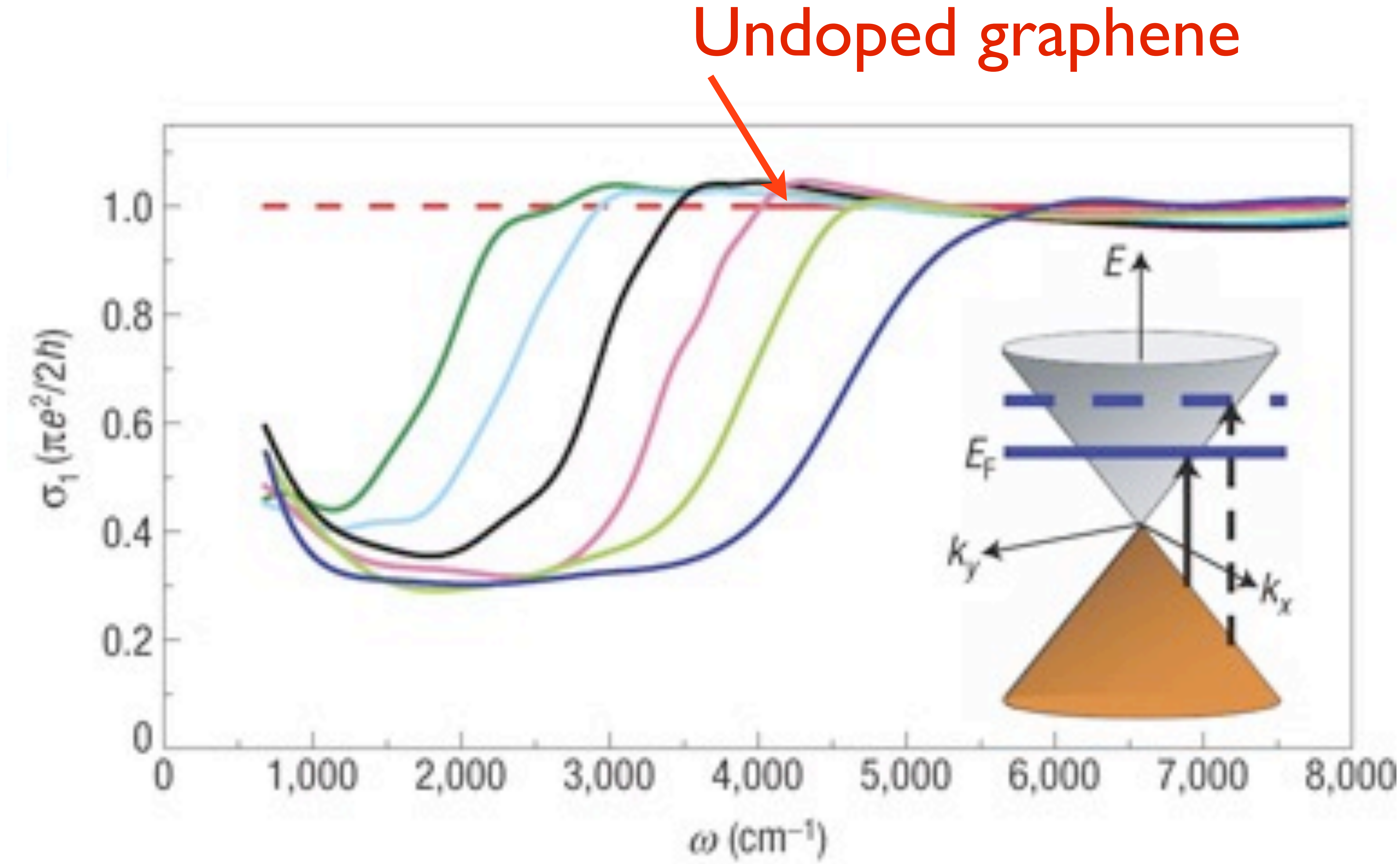
Particles	Holes
Momentum (→)	Momentum (←)
Current (→)	Current (←)





Current carrying state has non-zero momentum, and collisions cannot relax current to zero

Optical conductivity of graphene



Z. Q. Li, E. A. Henriksen, Z. Jiang, Z. Hao, M. C. Martin, P. Kim, H. L. Stormer, and D. N. Basov, *Nature Physics* **4**, 532 (2008).

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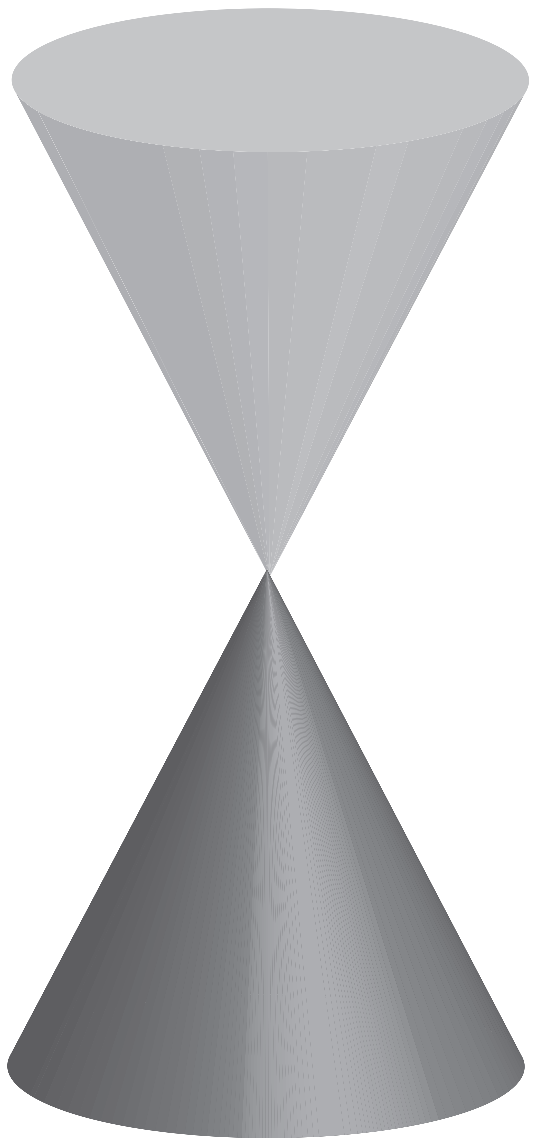
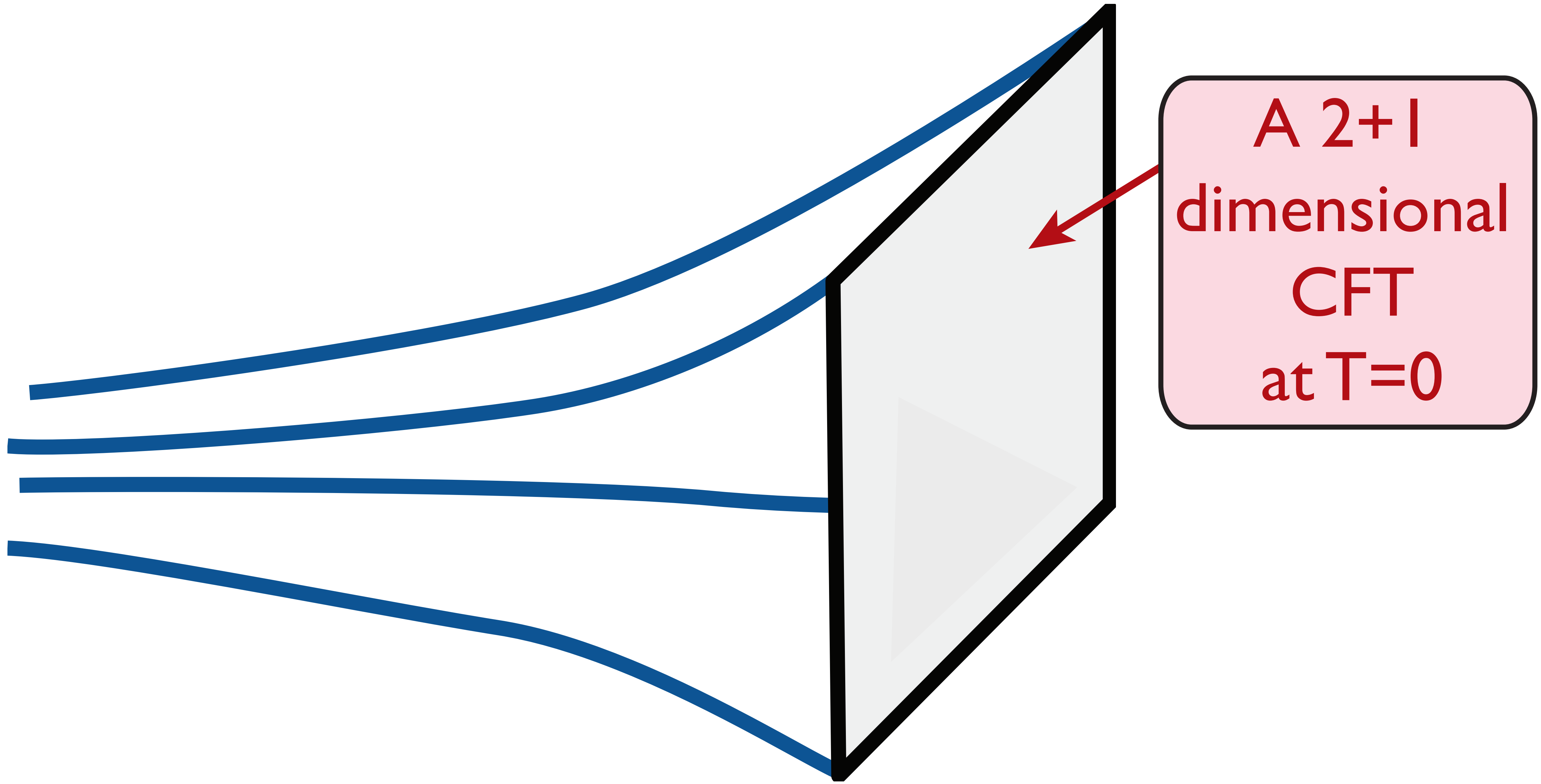
Begin with a CFT

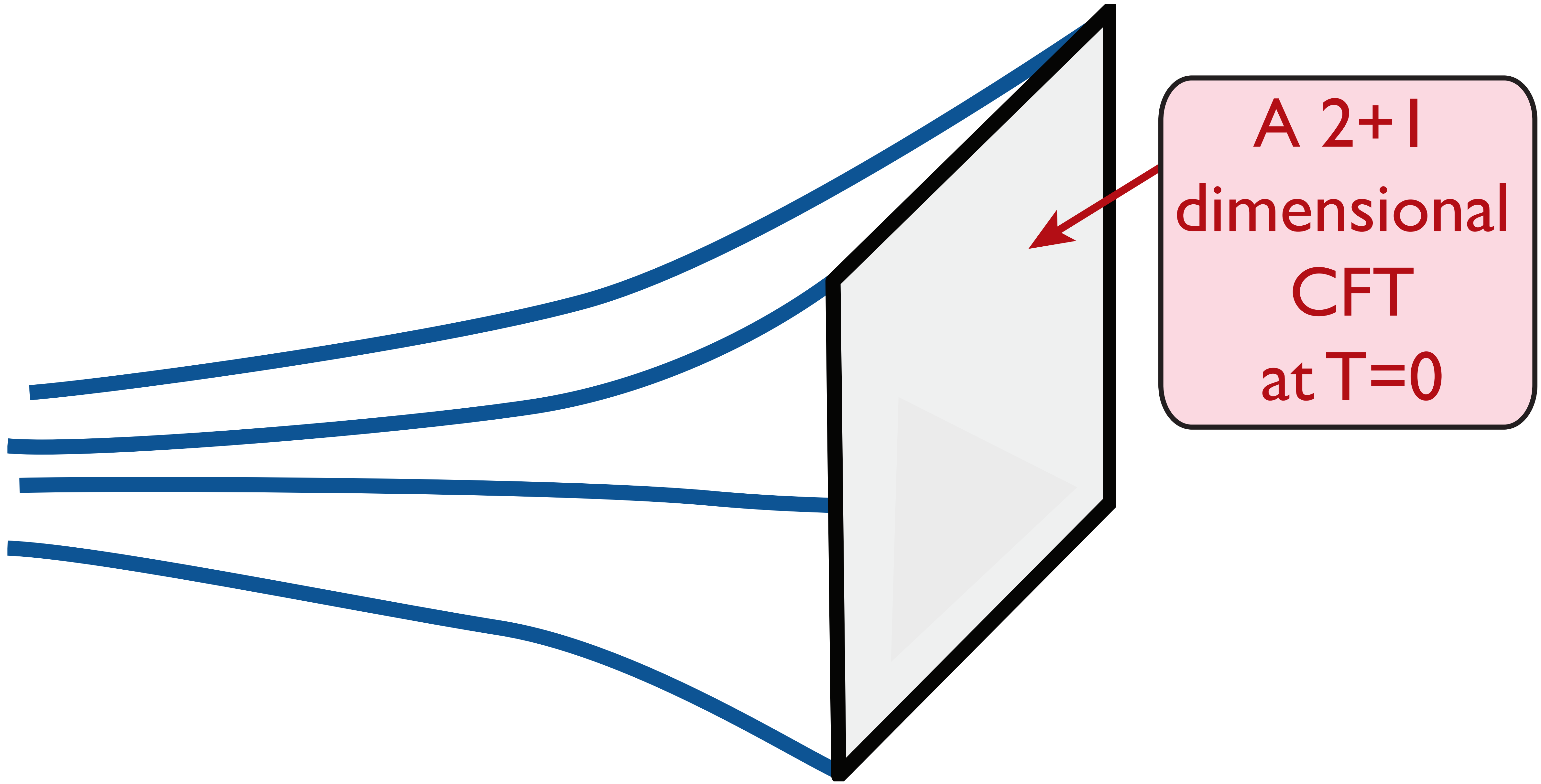


Holographic representation: AdS_4



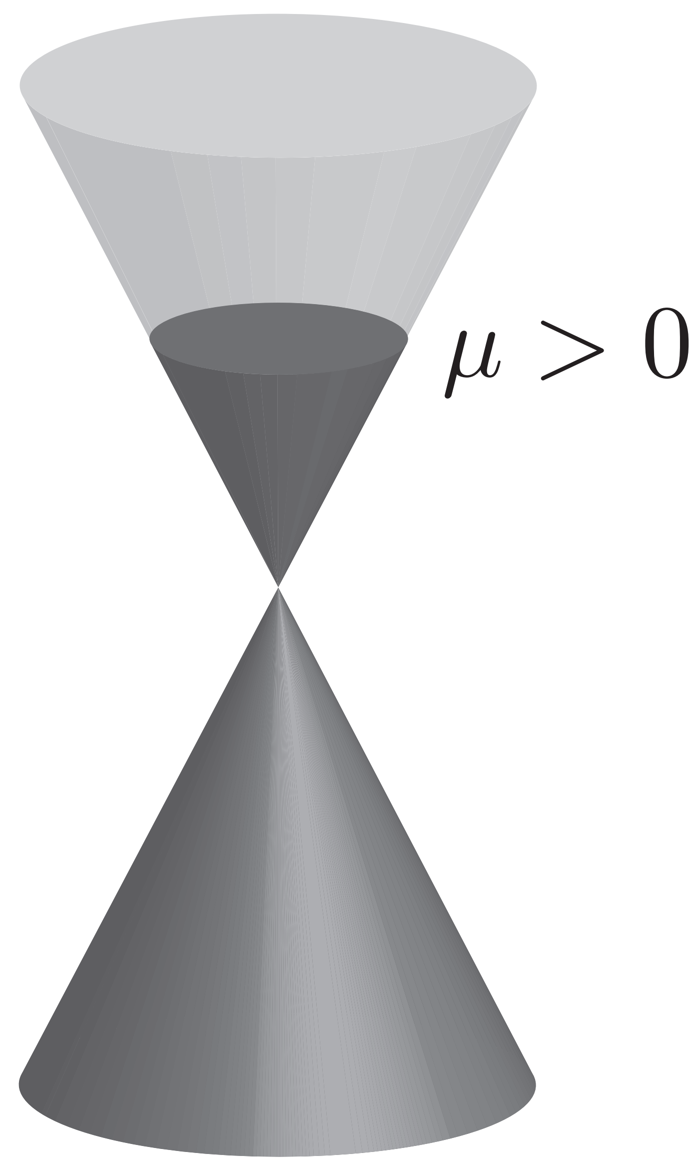
$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) \right]$$

Holographic representation: AdS_4



$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$
with $f(r) = 1$

Apply a chemical potential



AdS_4 theory of "nearly perfect fluids"

To leading order in a gradient expansion, charge transport in an infinite set of strongly-interacting CFT3s can be described by Einstein-Maxwell gravity/electrodynamics on AdS_4-Schwarzschild

$$\mathcal{S}_{EM} = \int d^4x \sqrt{-g} \left[-\frac{1}{4g_4^2} F_{ab} F^{ab} \right] .$$

This is to be solved subject to the constraint

$$A_\mu(r \to \infty, x, y, t) = \mathcal{A}_\mu(x, y, t)$$

where $\mathcal{A}_\mu$ is a source coupling to a conserved U(1) current J_μ of the CFT3

$$\mathcal{S} = \mathcal{S}_{CFT} + i \int dx dy dt \mathcal{A}_\mu J_\mu$$

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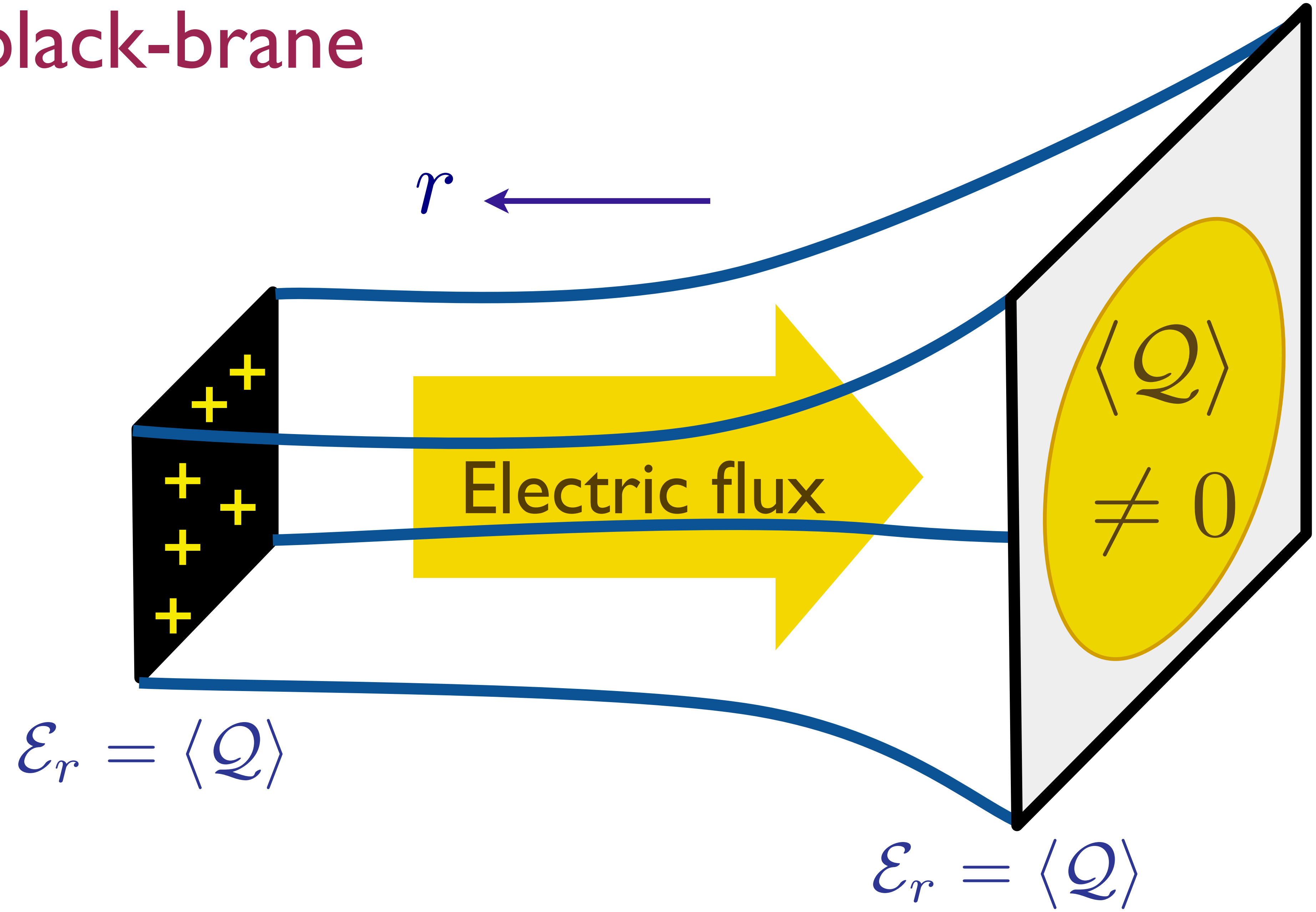
At non-zero chemical potential we simply require $\mathcal{A}_\tau = \mu$.

The Maxwell-Einstein theory of the applied chemical potential yields a AdS4-Reissner-Nordtröm black-brane



$$\mathcal{S} = \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa^2} \left(R + \frac{6}{L^2} \right) - \frac{1}{4g_4^2} F_{ab} F^{ab} \right]$$

S.A. Hartnoll, P. K. Kovtun, M. Müller, and S. Sachdev, Physical Review B **76**, 144502 (2007)

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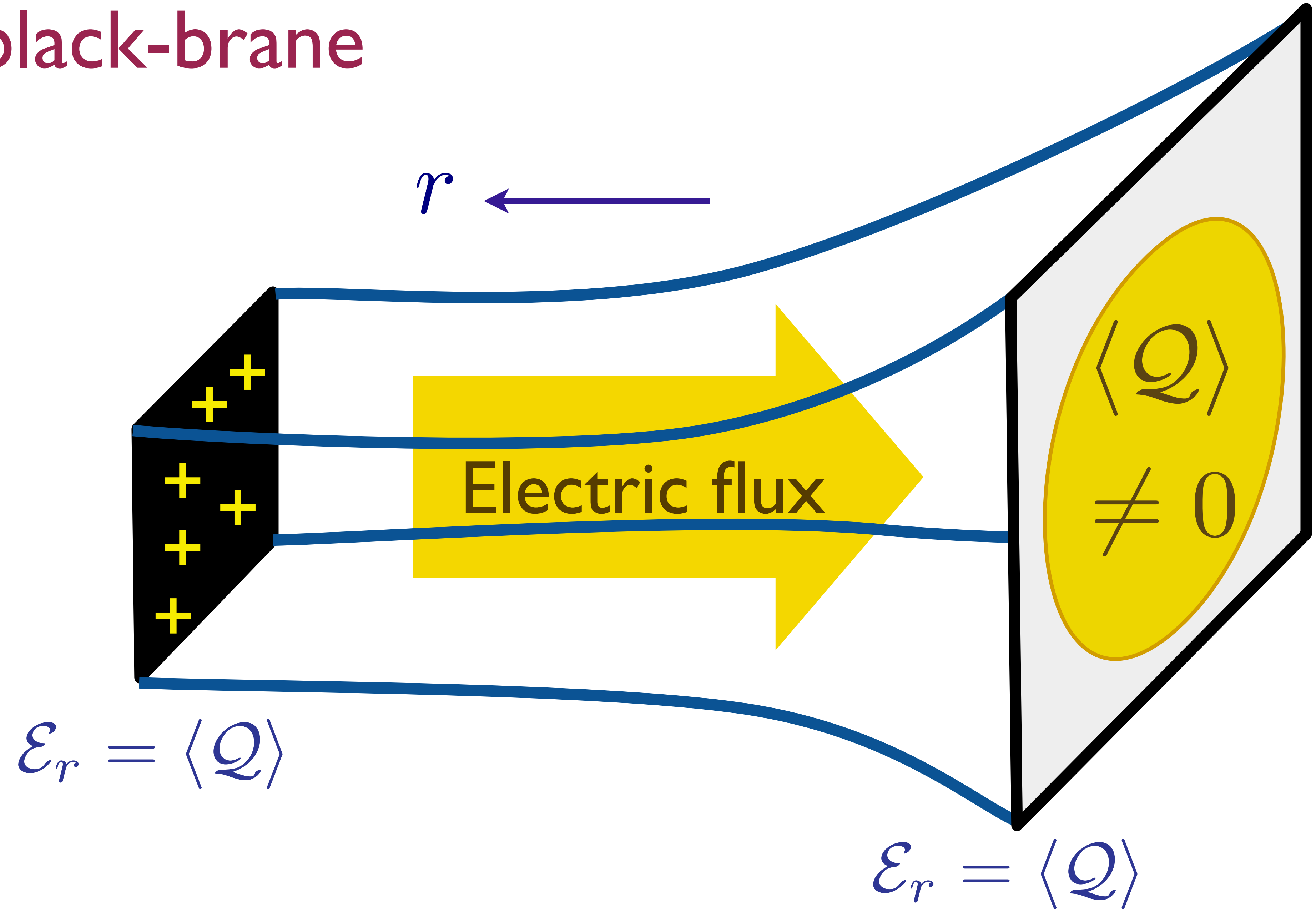

$$ds^2 = \left(\frac{L}{r}\right)^2 \left[\frac{dr^2}{f(r)} - f(r)dt^2 + dx^2 + dy^2\right]$$

with $f(r) = \left(1 - \frac{r}{R}\right)^2 \left(1 + \frac{2r}{R} + \frac{3r^2}{R^2}\right)$ and $R = \frac{\sqrt{6}Lg_4}{\kappa\mu}$, and $A_\tau = \mu\left(1 - \frac{r}{R}\right)$

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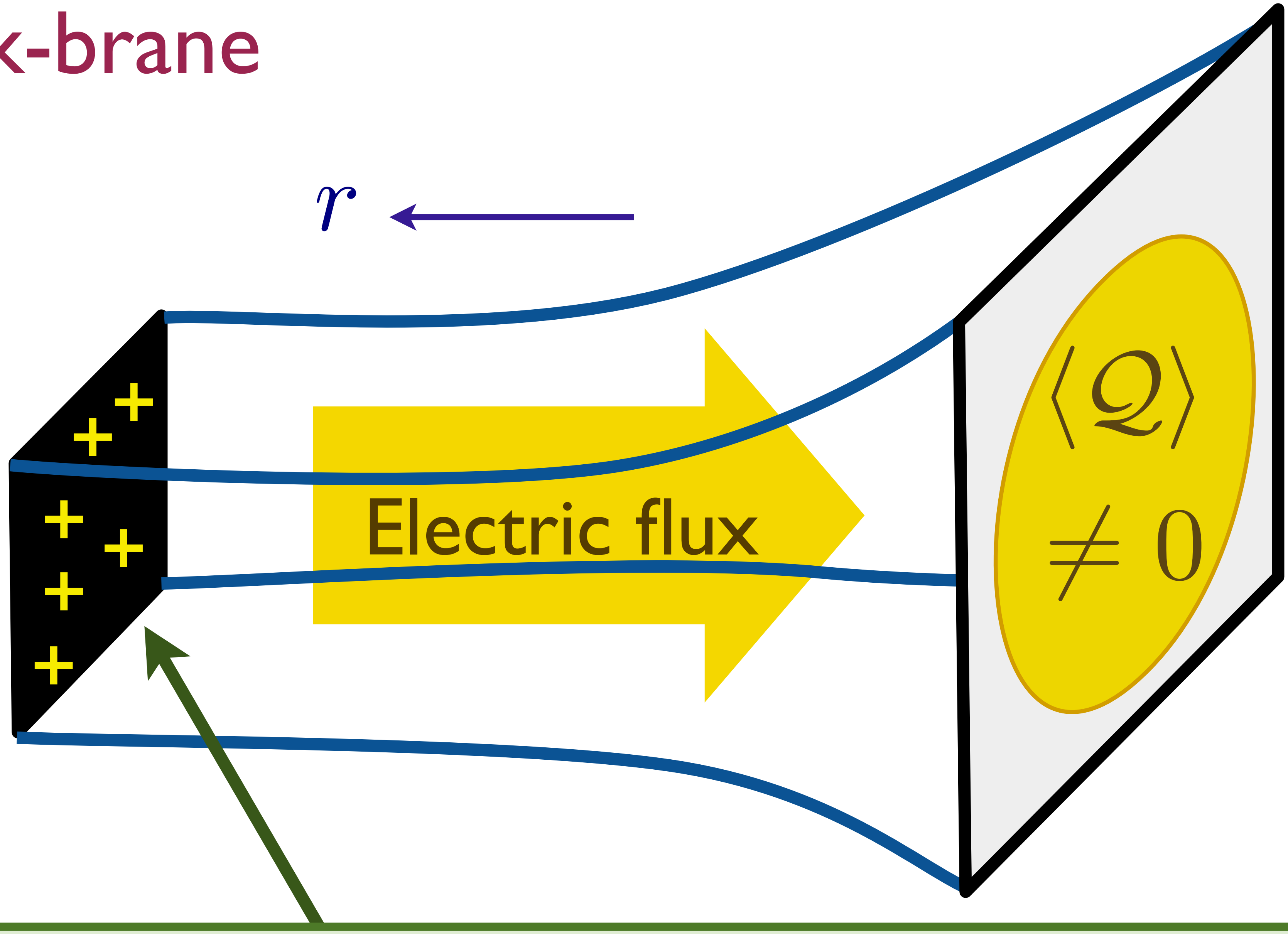
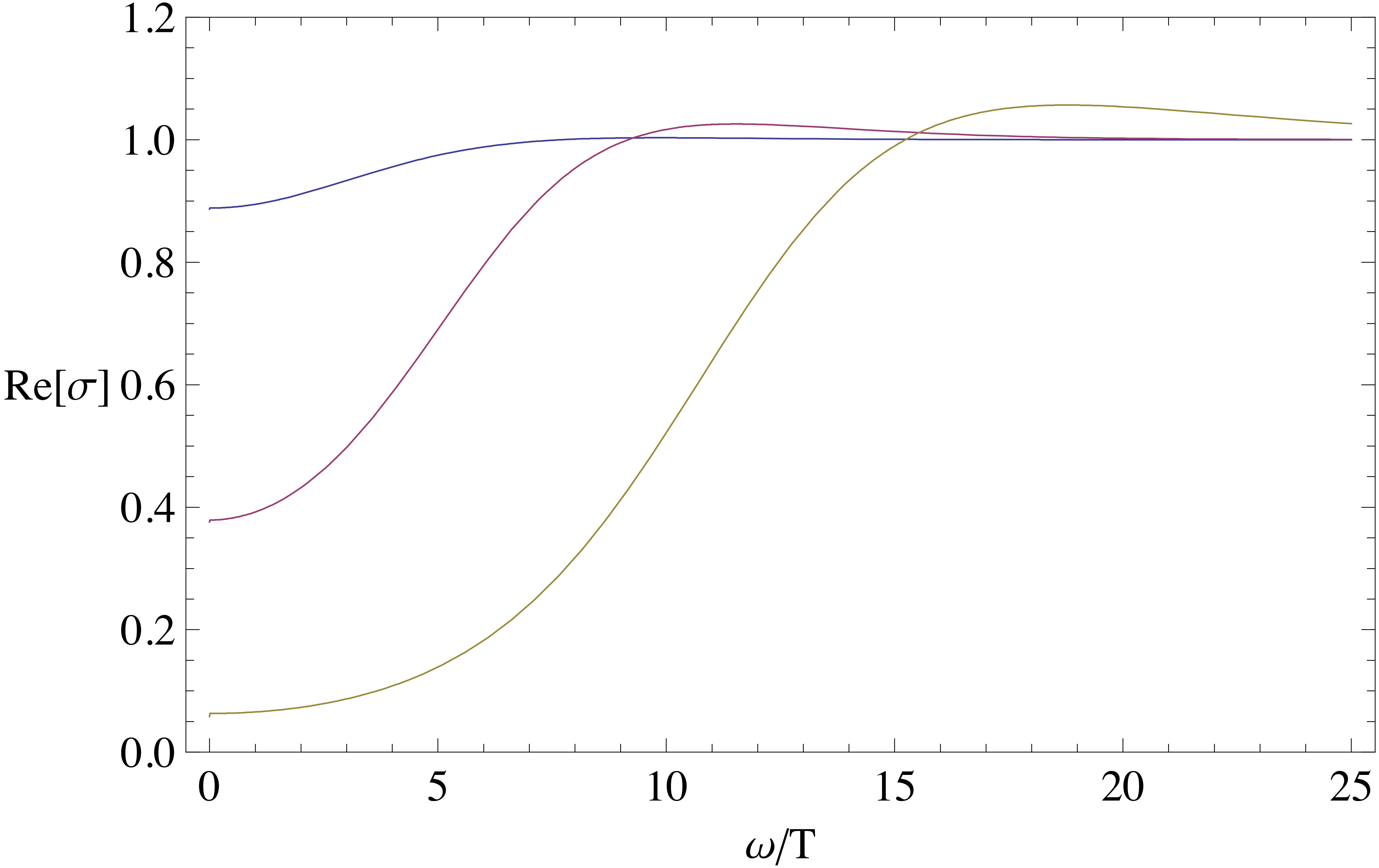


At $T = 0$, we obtain an extremal black-brane, with a near-horizon (IR) metric of $AdS_2 \times R^2$

$$ds^2 = \frac{L^2}{6}\left(\frac{-dt^2 + dr^2}{r^2}\right) + dx^2 + dy^2$$

T. Faulkner, H. Liu, J. McGreevy, and D. Vegh, arXiv:0907.2694

Compute conductivity using response to a time-dependent vector potential as a function of ω/T and μ/T



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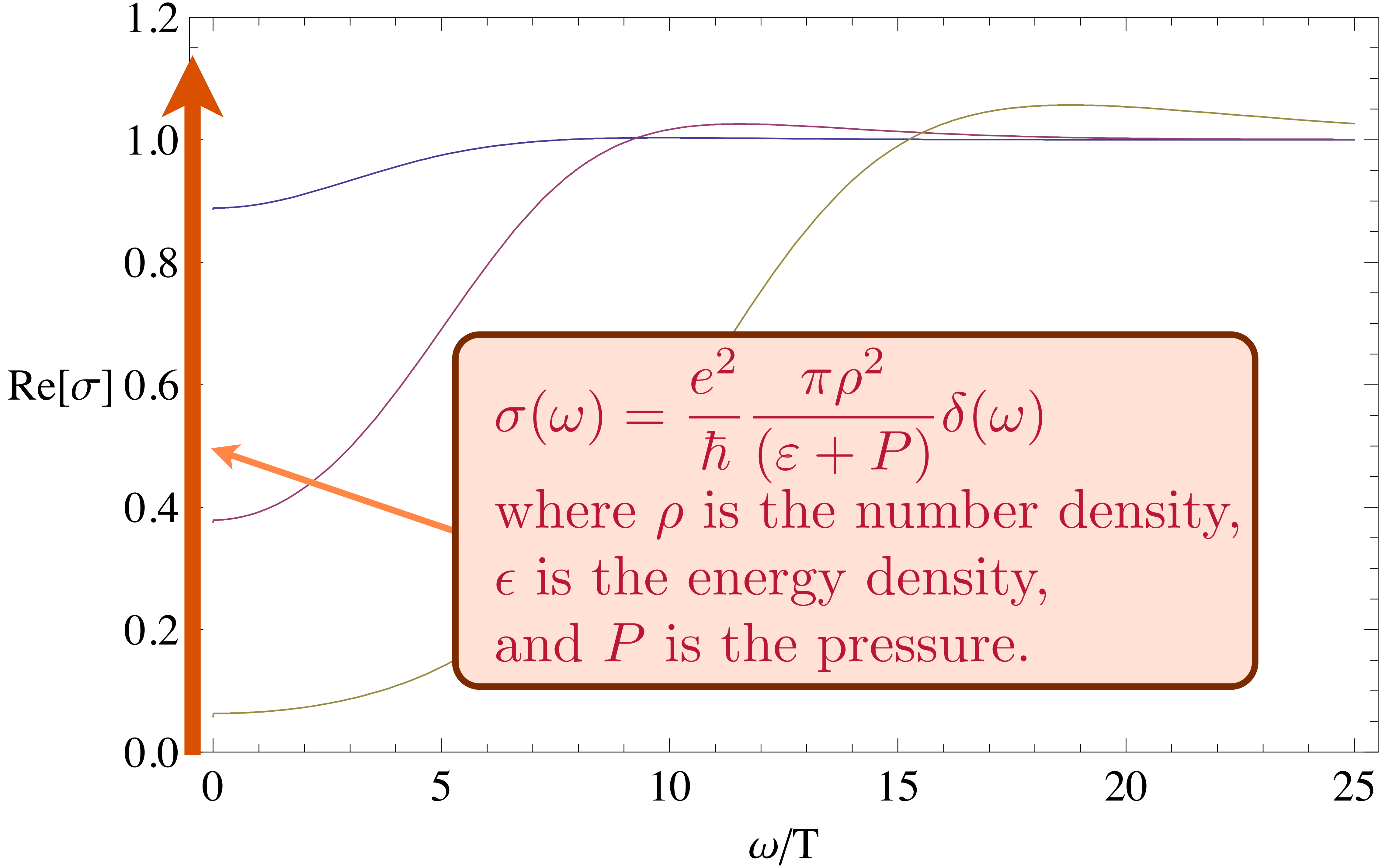

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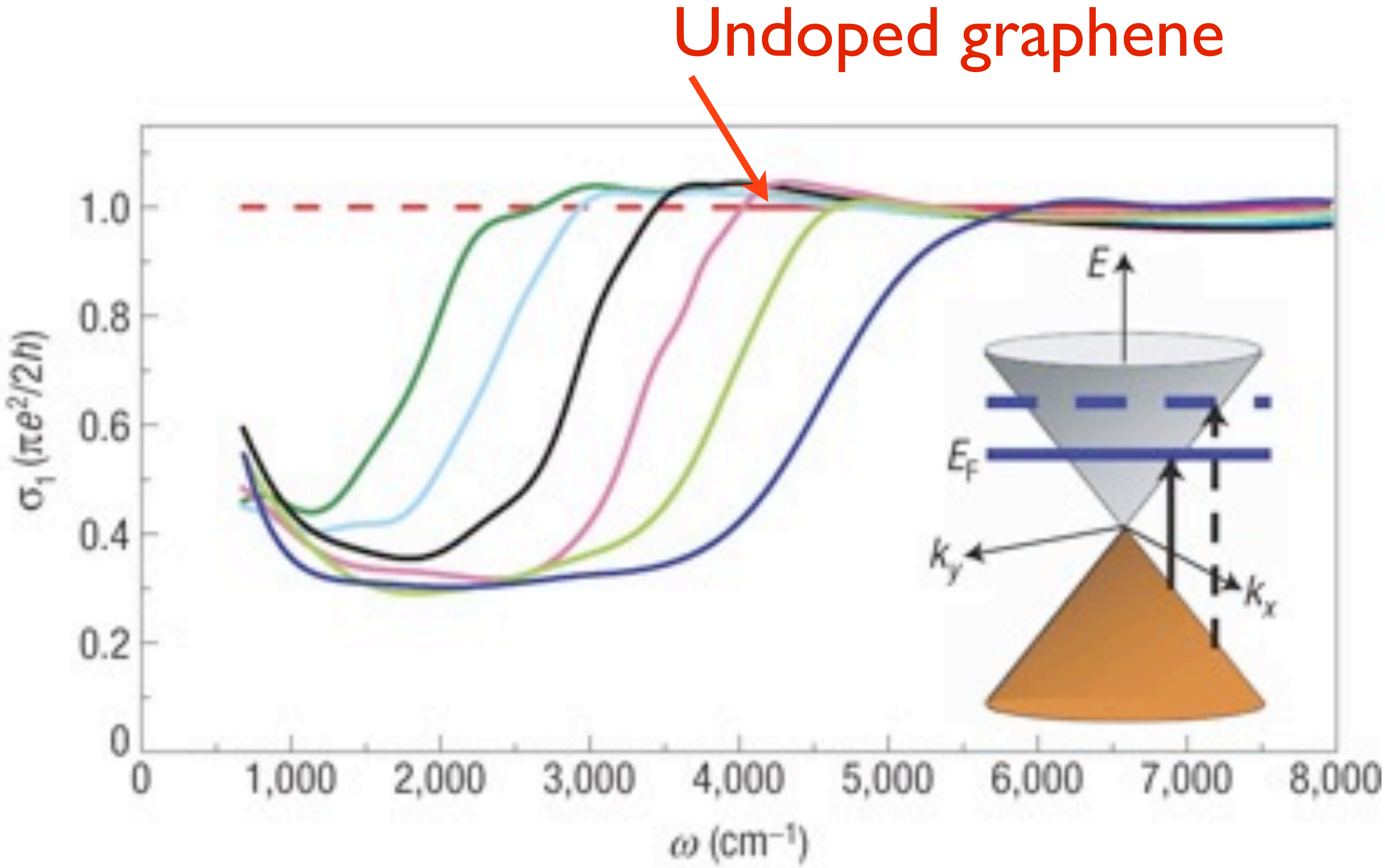

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Features of $AdS_2 \times R^2$

- Has non-zero entropy density at $T = 0$, and "volume" law for entanglement entropy.

- Green's function of a probe fermion (a *mesino*) can have a Fermi surface, but self energies are momentum independent, and the singular behavior is the same on and off the Fermi surface

- Deficit of order $\sim N^2$ in the volume enclosed by the mesino Fermi surfaces: presumably associated with "hidden Fermi surfaces" of gauge-charged particles (the *quarks*).

S.-S. Lee, *Phys. Rev. D* **79**, 086006 (2009);
M. Cubrovic, J. Zaanen, and K. Schalm, *Science* **325**, 439 (2009);
T. Faulkner, H. Liu, J. McGreevy, and D.Vegh, arXiv:0907.2694
S. Sachdev, *Phys. Rev. Lett.* **105**, 151602 (2010).

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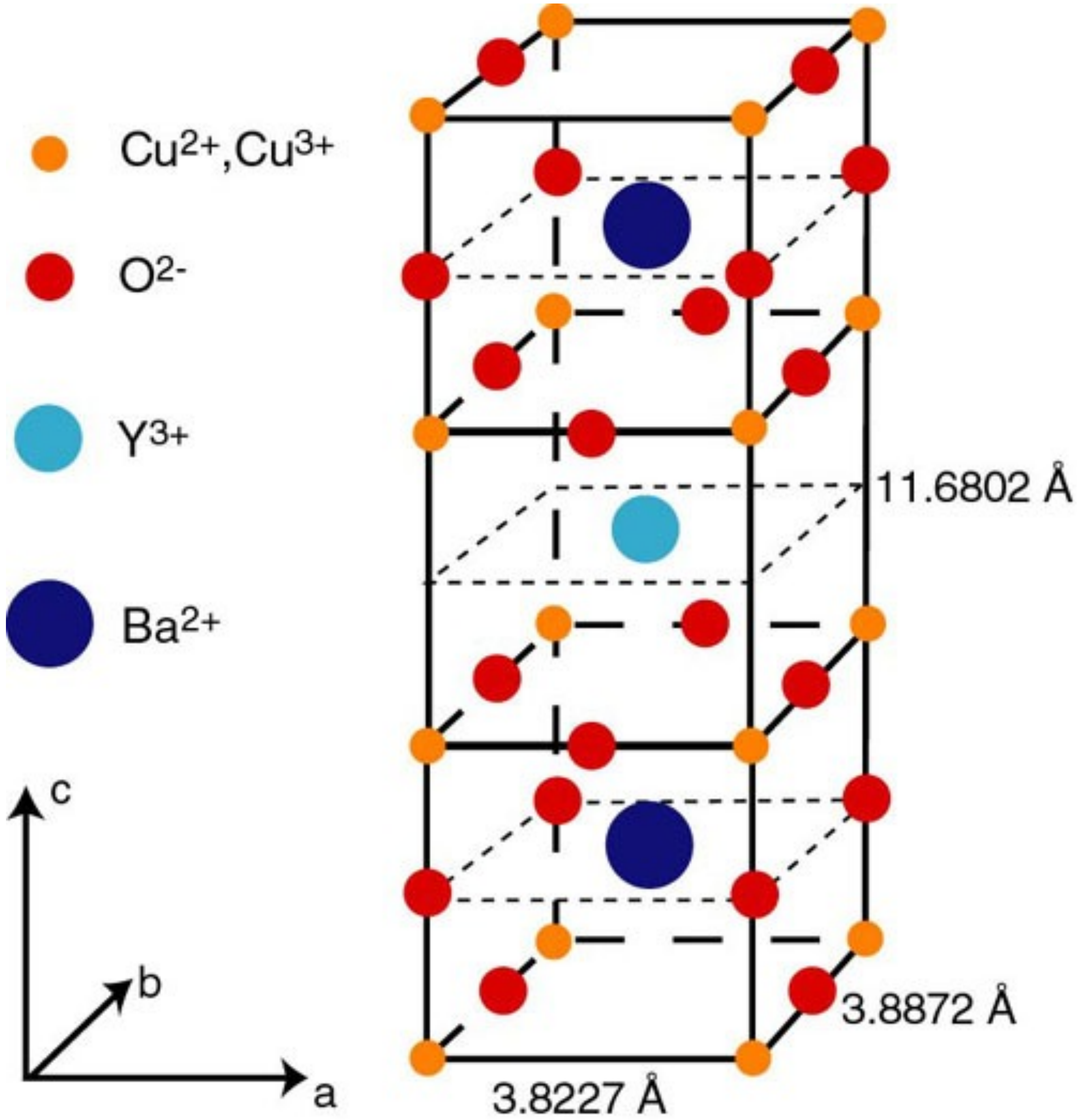
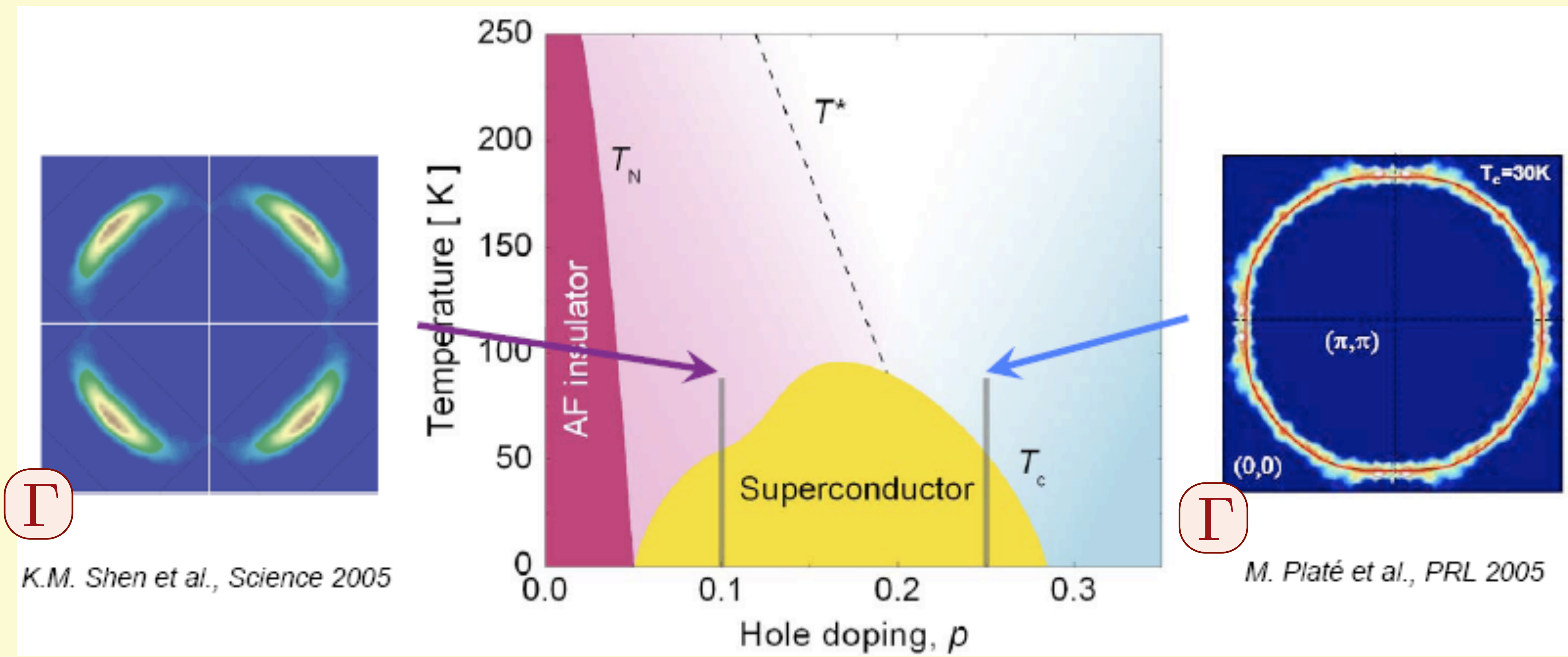
High temperature superconductors



$$YBa_2Cu_3O_{6+x}$$

Evolution of the (ARPES) Fermi surface on the cuprate phase diagram



Smaller hole Fermi-pockets

Large hole Fermi surface

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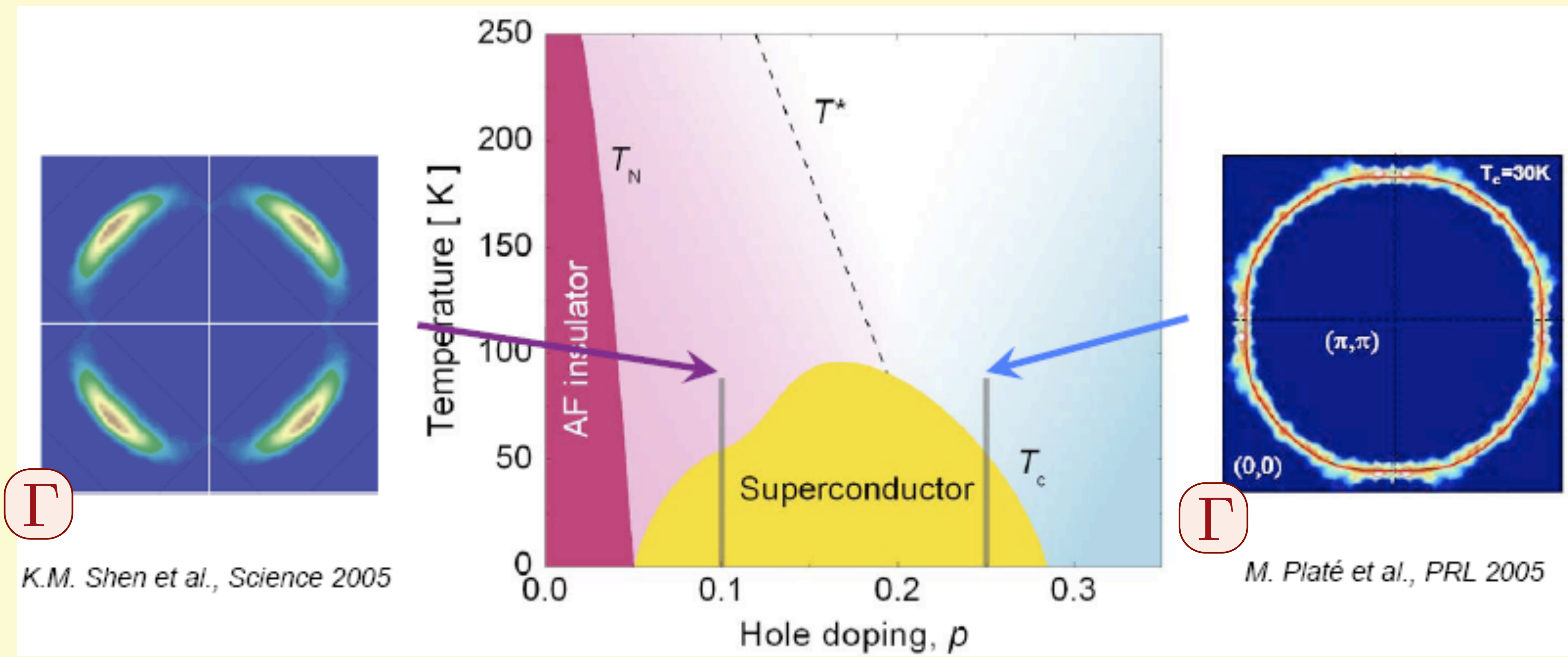

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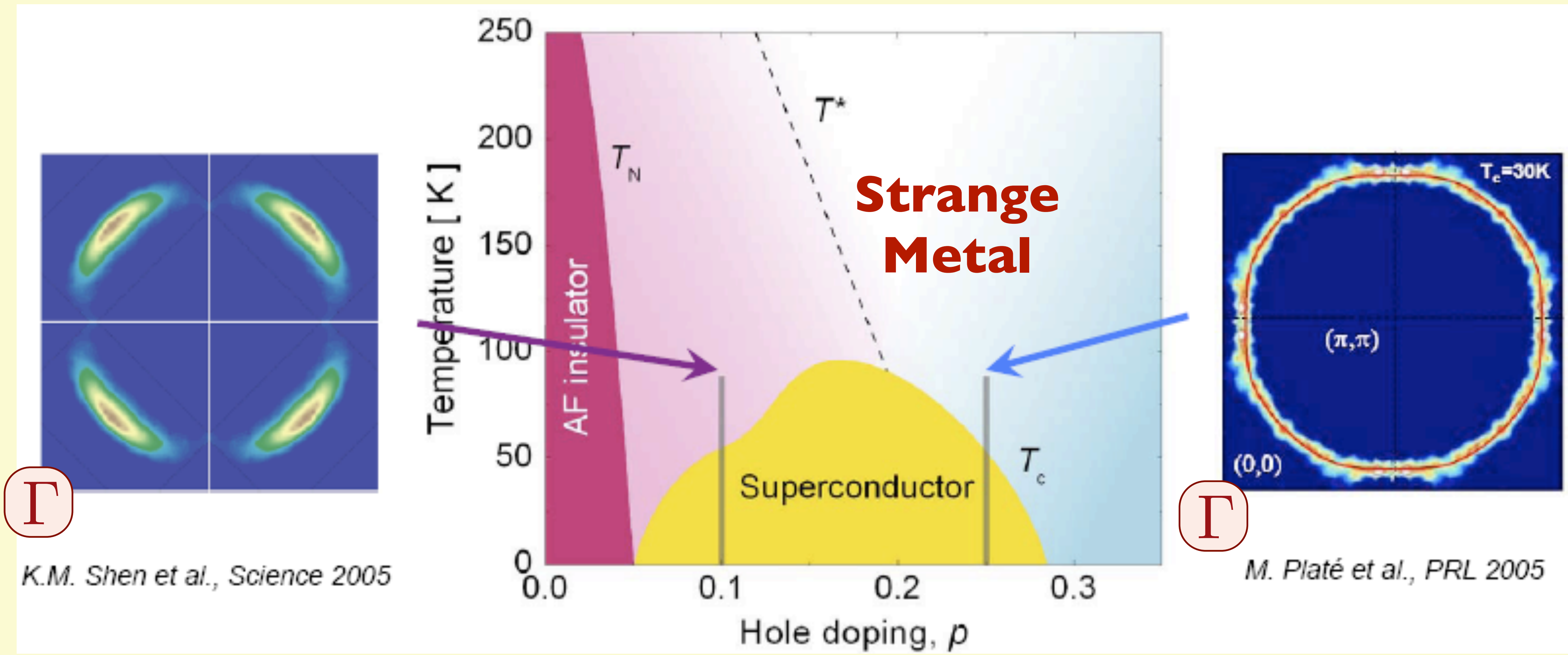

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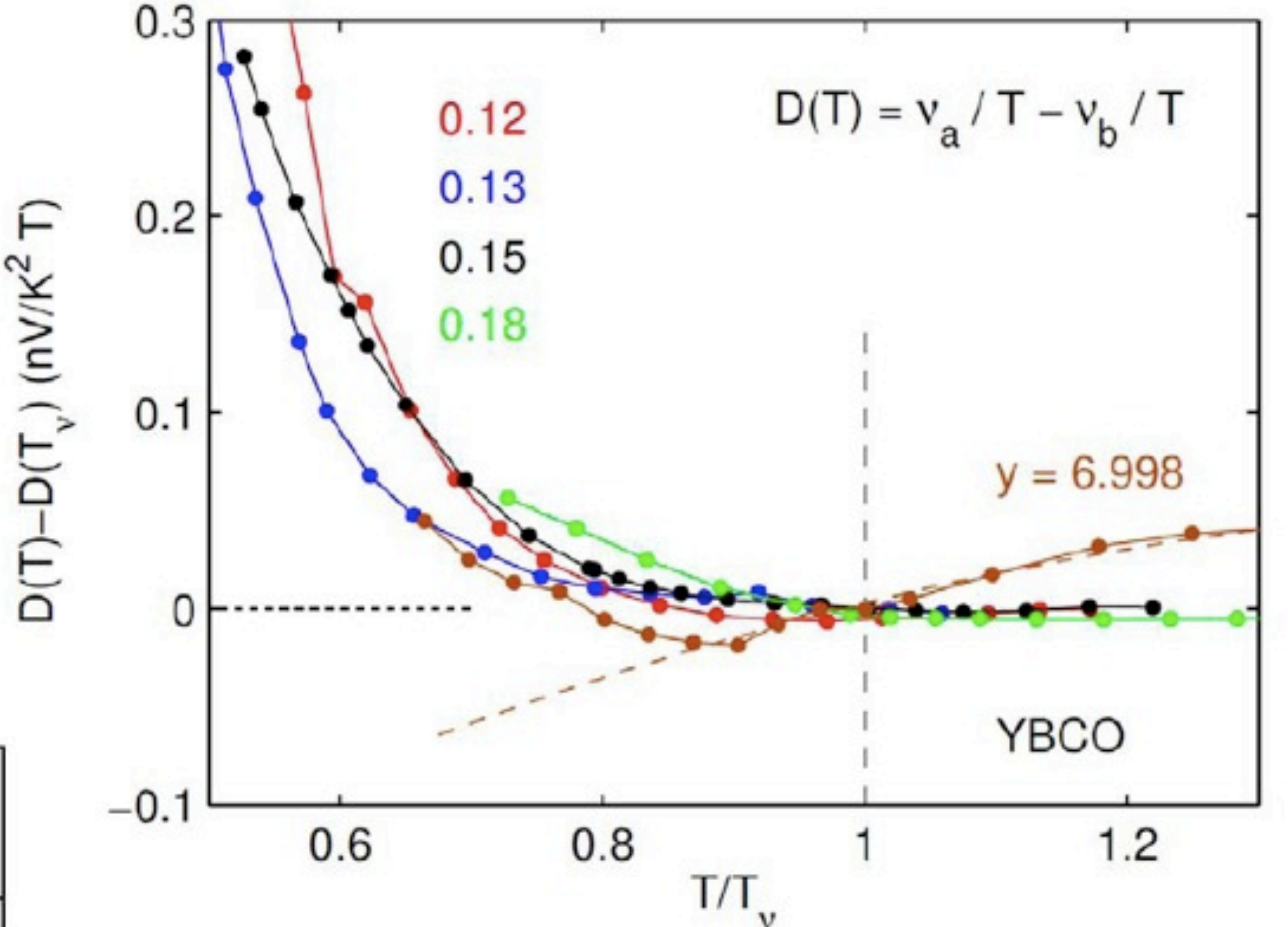
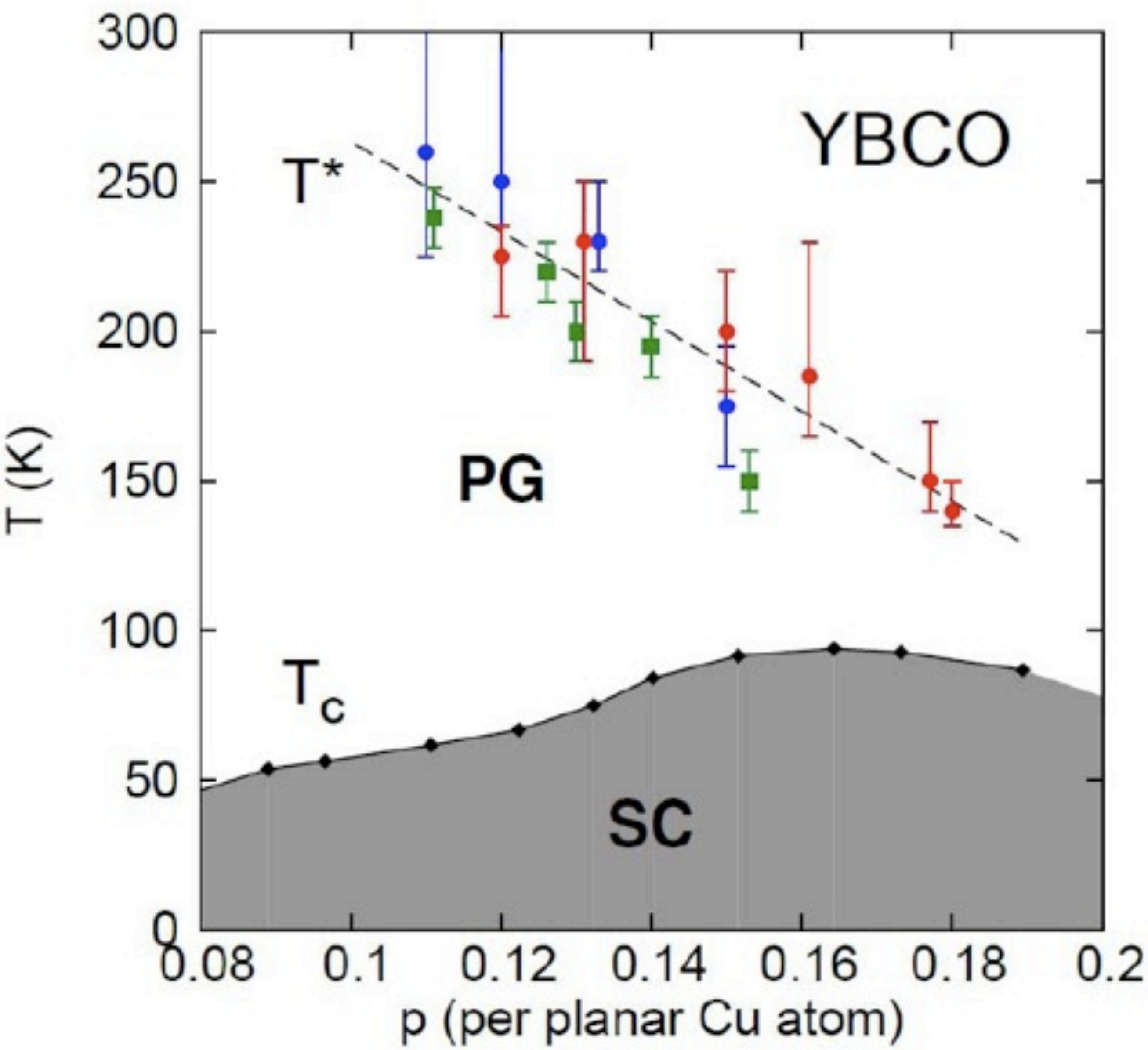
Large hole Fermi surface

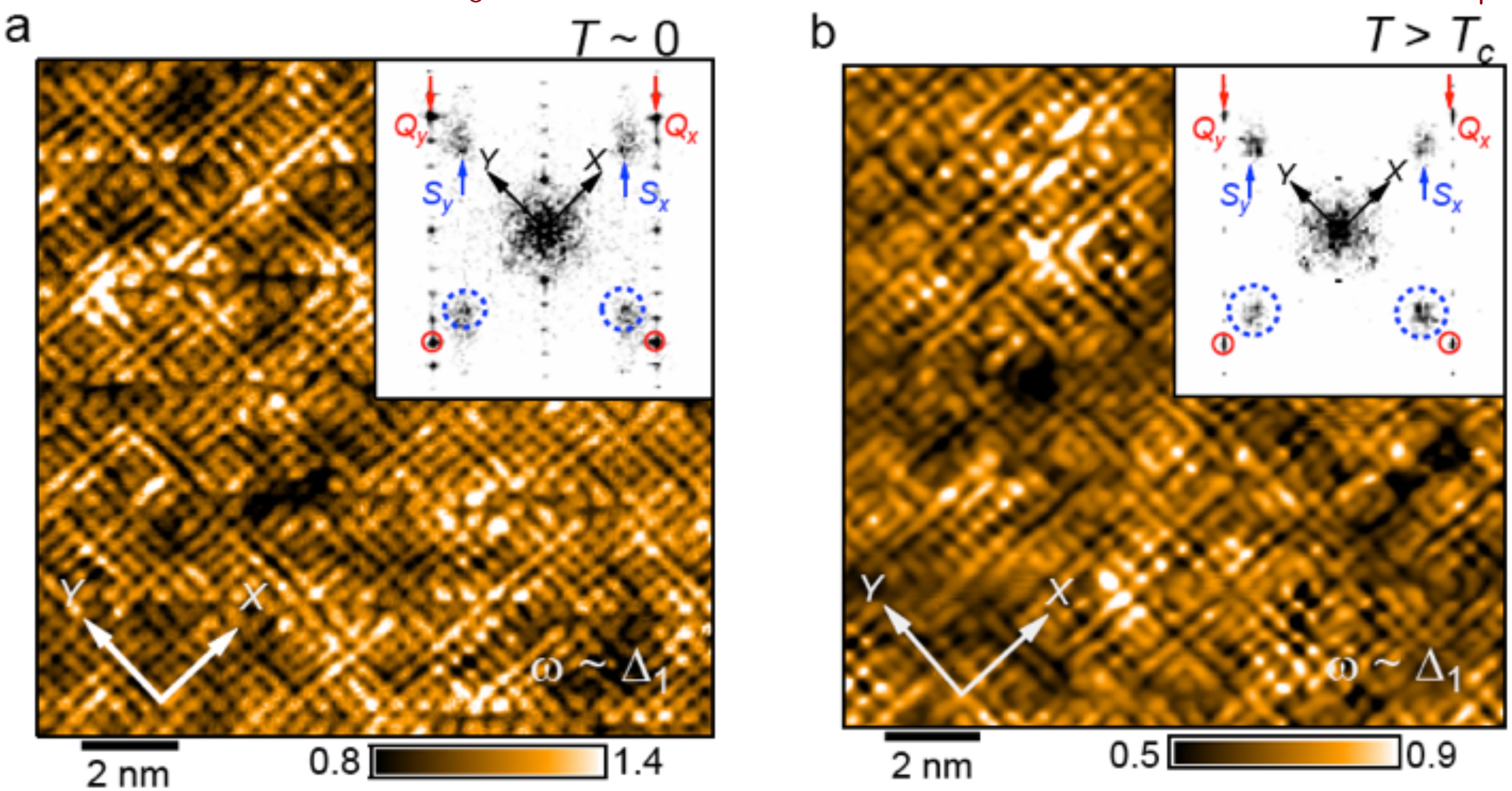
Broken rotational symmetry in the pseudogap phase of a high-Tc superconductor

R. Daou, J. Chang, David LeBoeuf, Olivier Cyr-Choiniere, Francis Laliberte, Nicolas Doiron-Leyraud, B. J. Ramshaw, Ruixing Liang, D. A. Bonn, W. N. Hardy, and Louis Taillefer
Nature, **463**, 519 (2010).





STM measurements of $Z(r)$, the energy asymmetry in density of states in $Bi_2Sr_2CaCu_2O_{8+\delta}$.



M. J. Lawler, K. Fujita, Jhinhwan Lee, A. R. Schmidt, Y. Kohsaka, Chung Koo Kim, H. Eisaki, S. Uchida, J. C. Davis, J. P. Sethna, and Eun-Ah Kim, Nature **466**, 347 (2010)

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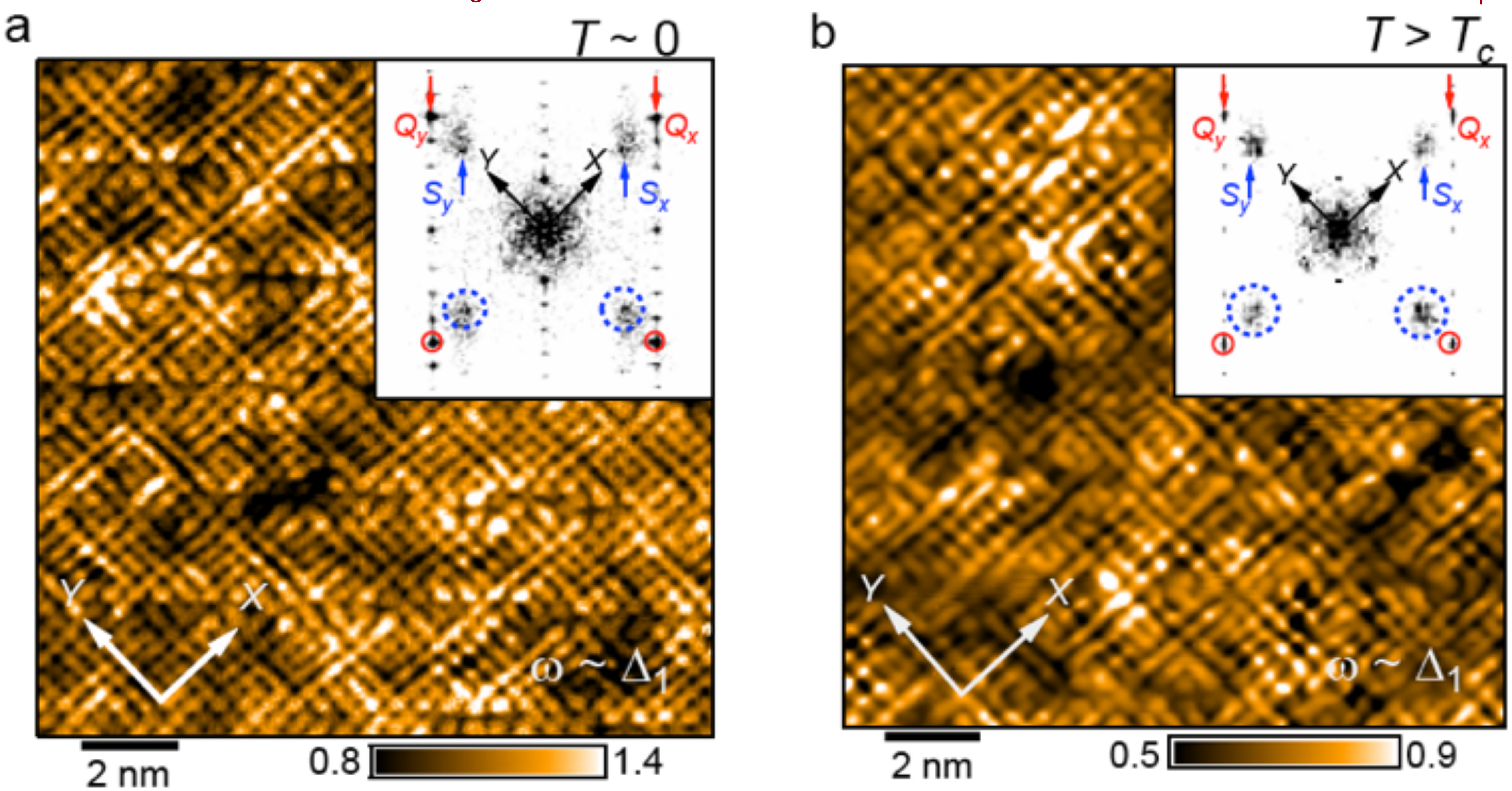

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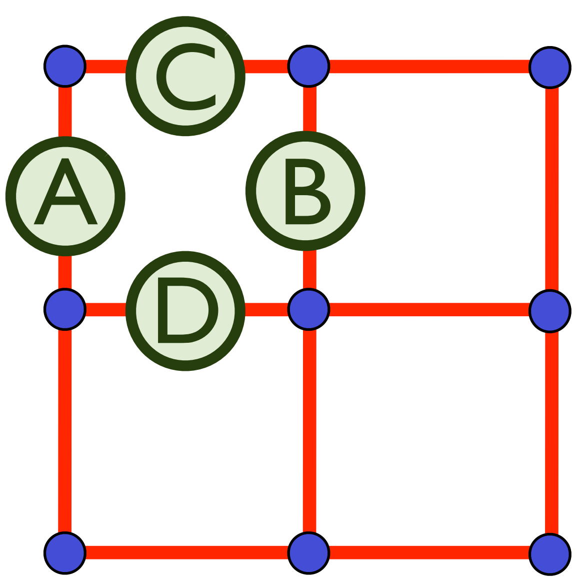

$$O_N = Z_A + Z_B - Z_C - Z_D$$

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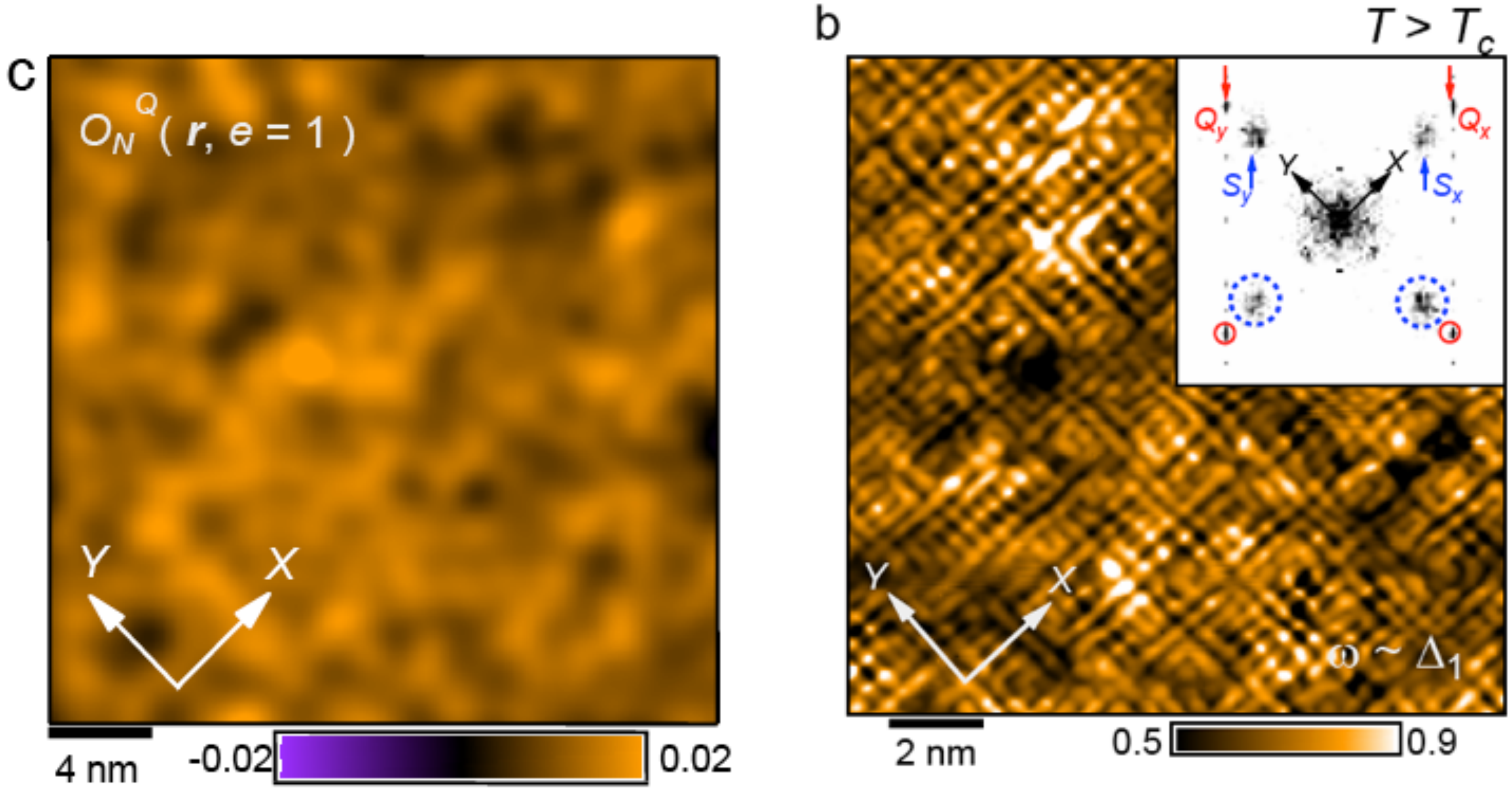

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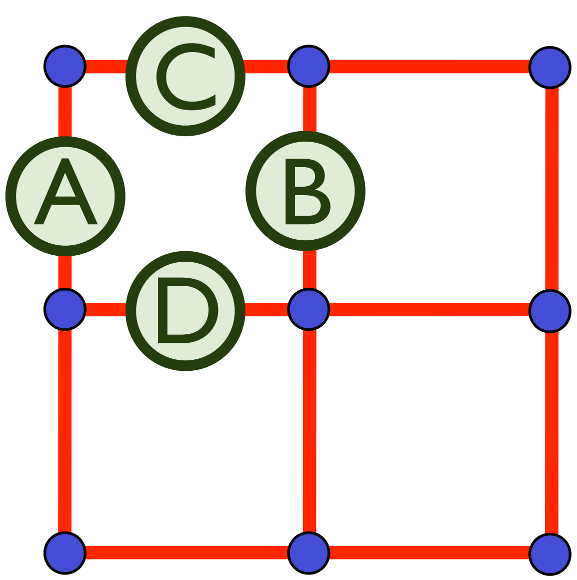
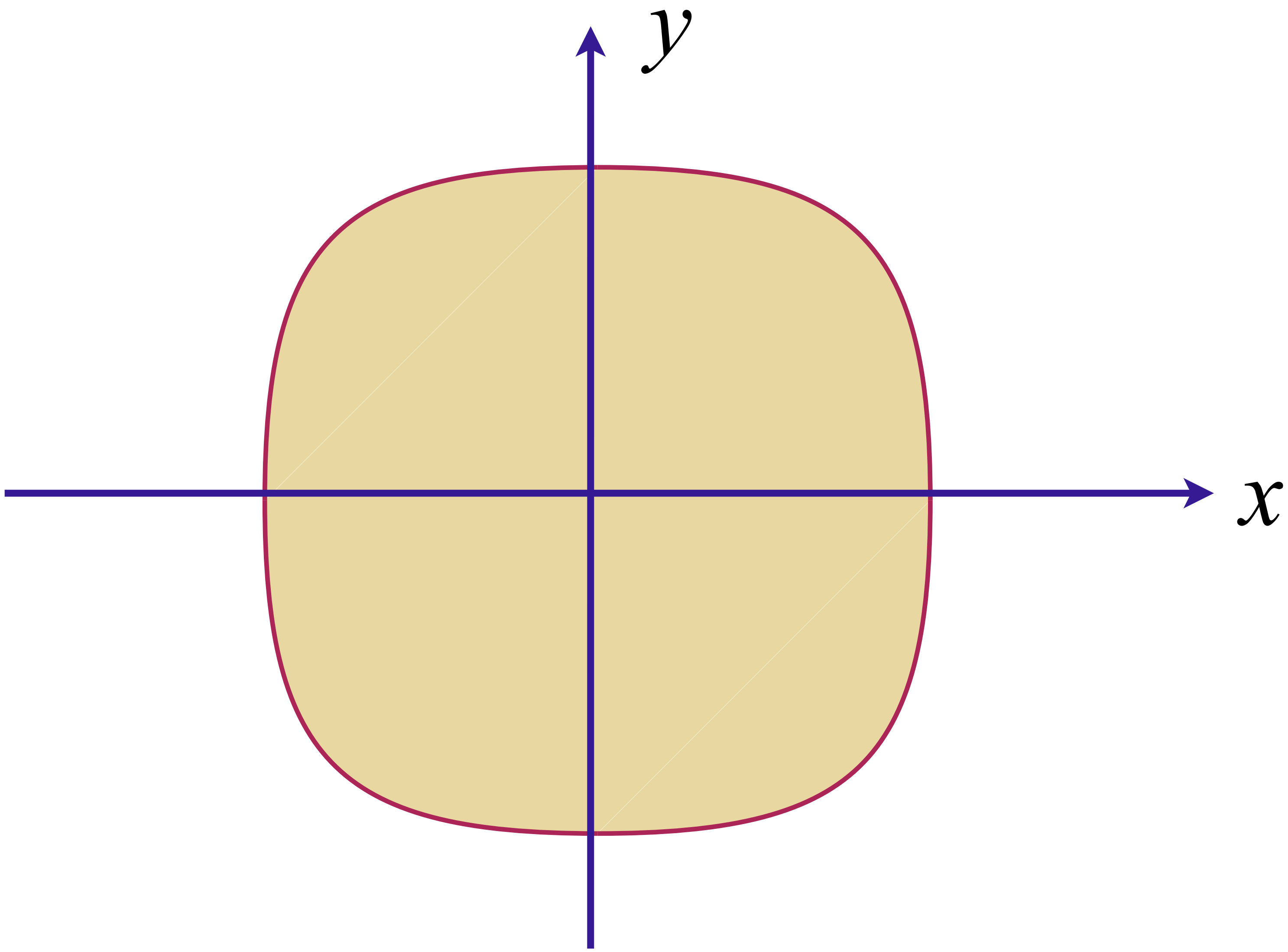


$$O_N = Z_A + Z_B - Z_C - Z_D$$

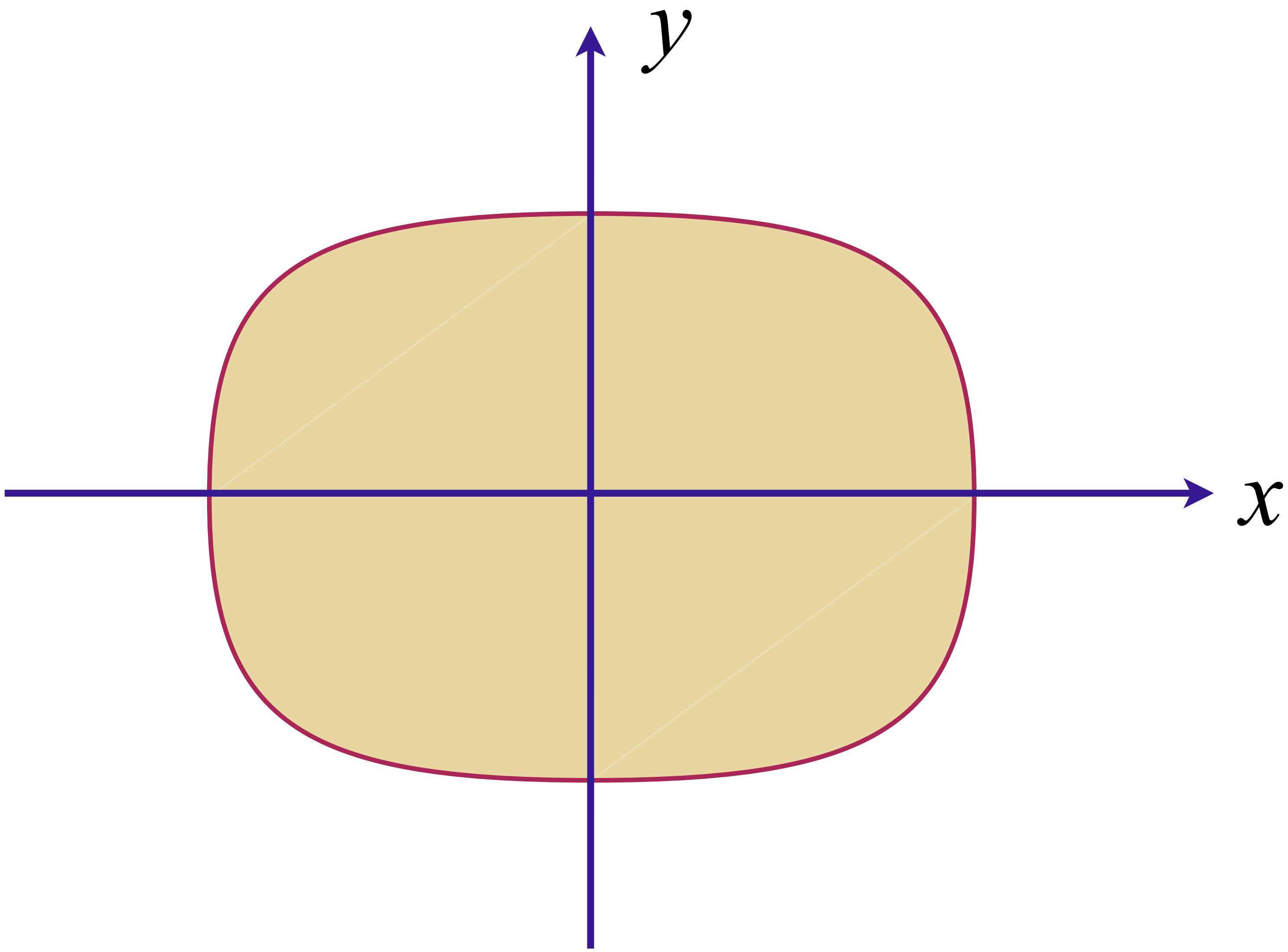
Strong anisotropy of electronic states between *x* and *y* directions: Electronic "Ising-nematic" order

Quantum criticality of Ising-nematic ordering



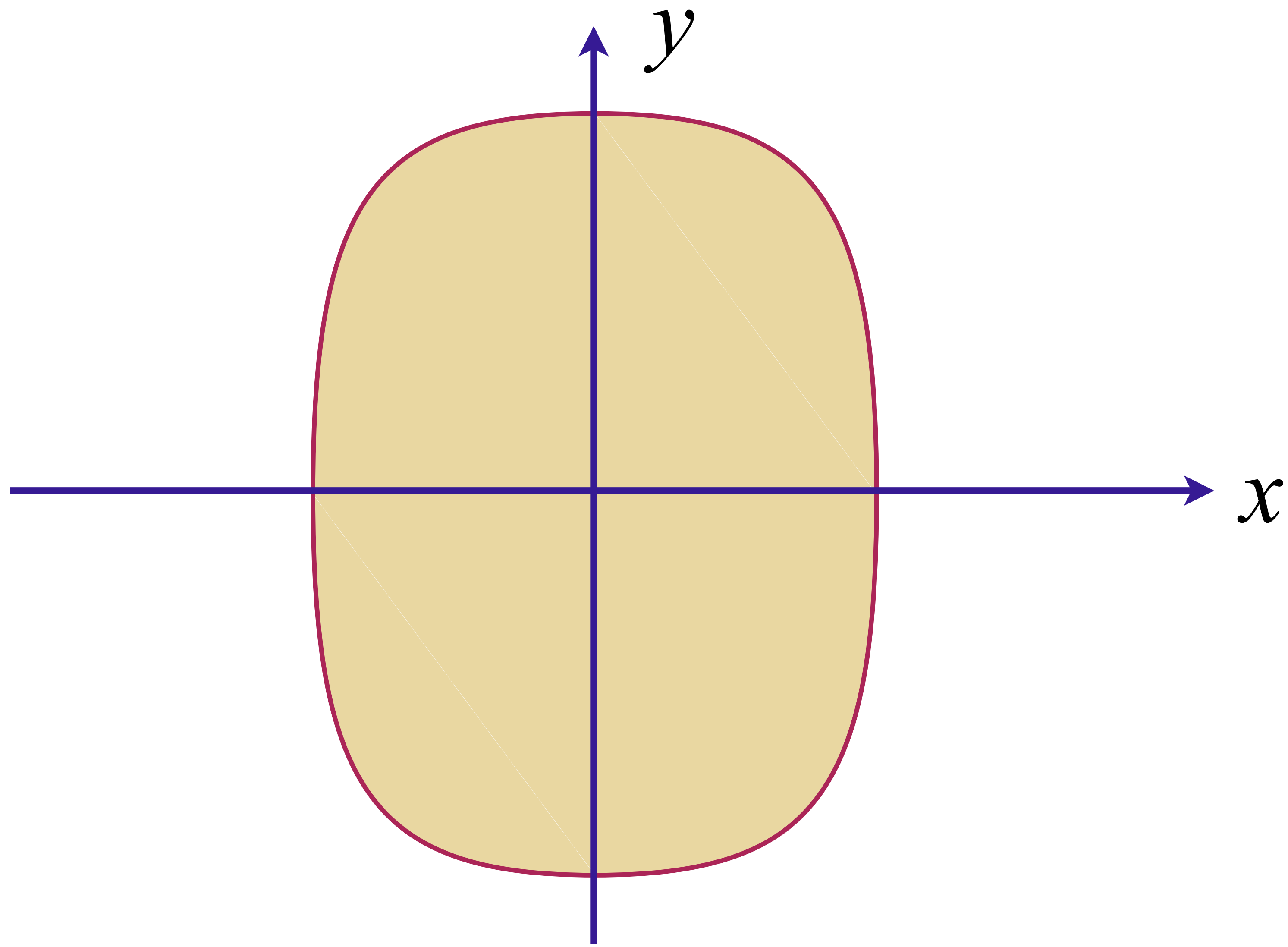
Fermi surface with full square lattice symmetry

Quantum criticality of Ising-nematic ordering



Spontaneous elongation along x direction:

Quantum criticality of Ising-nematic ordering



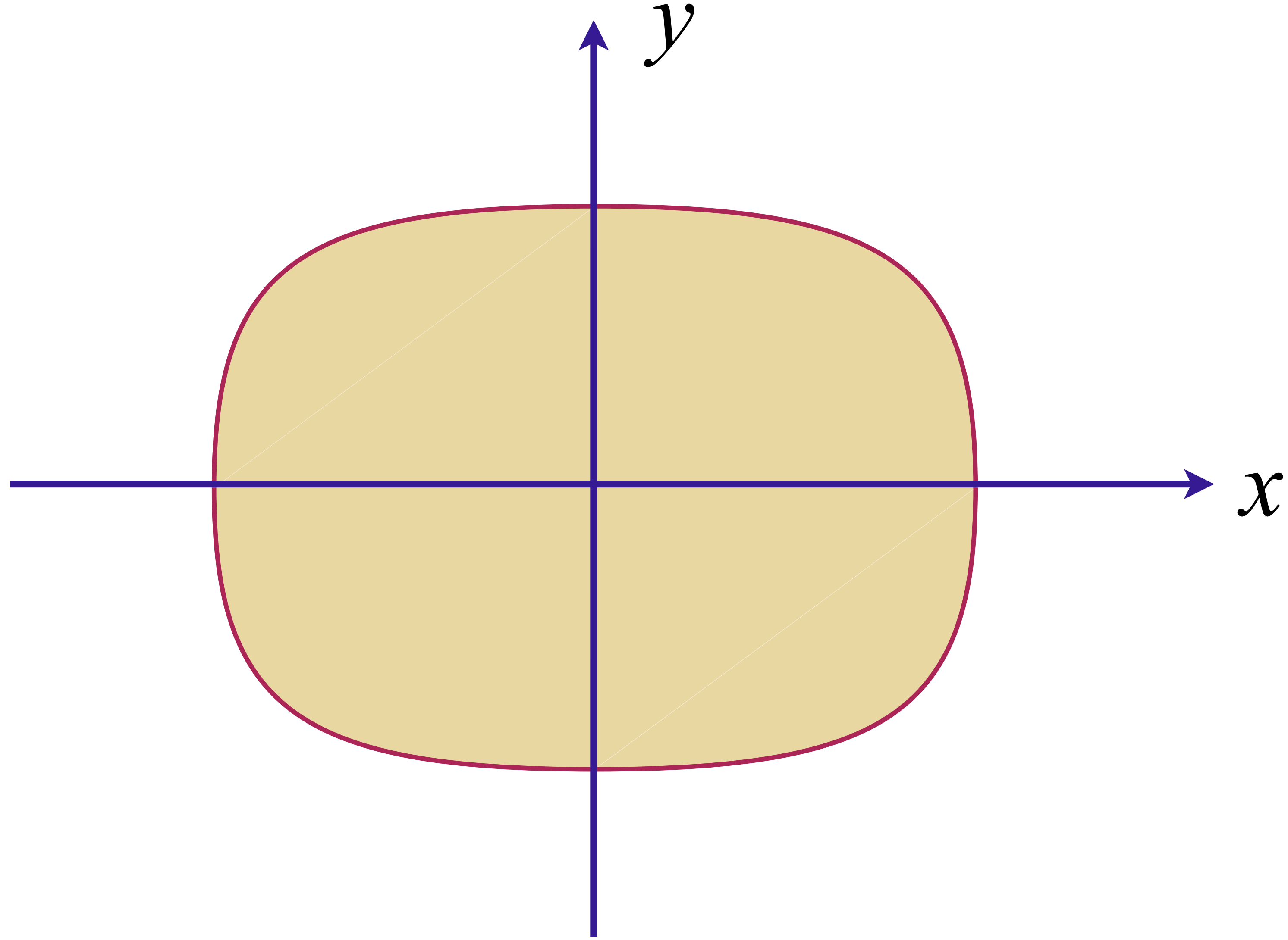
Spontaneous elongation along y direction:

Ising-nematic order parameter

$$\phi \sim \int d^2 k \, (\cos k_x - \cos k_y) \, c^\dagger_{\mathbf{k}\sigma} c_{\mathbf{k}\sigma}$$

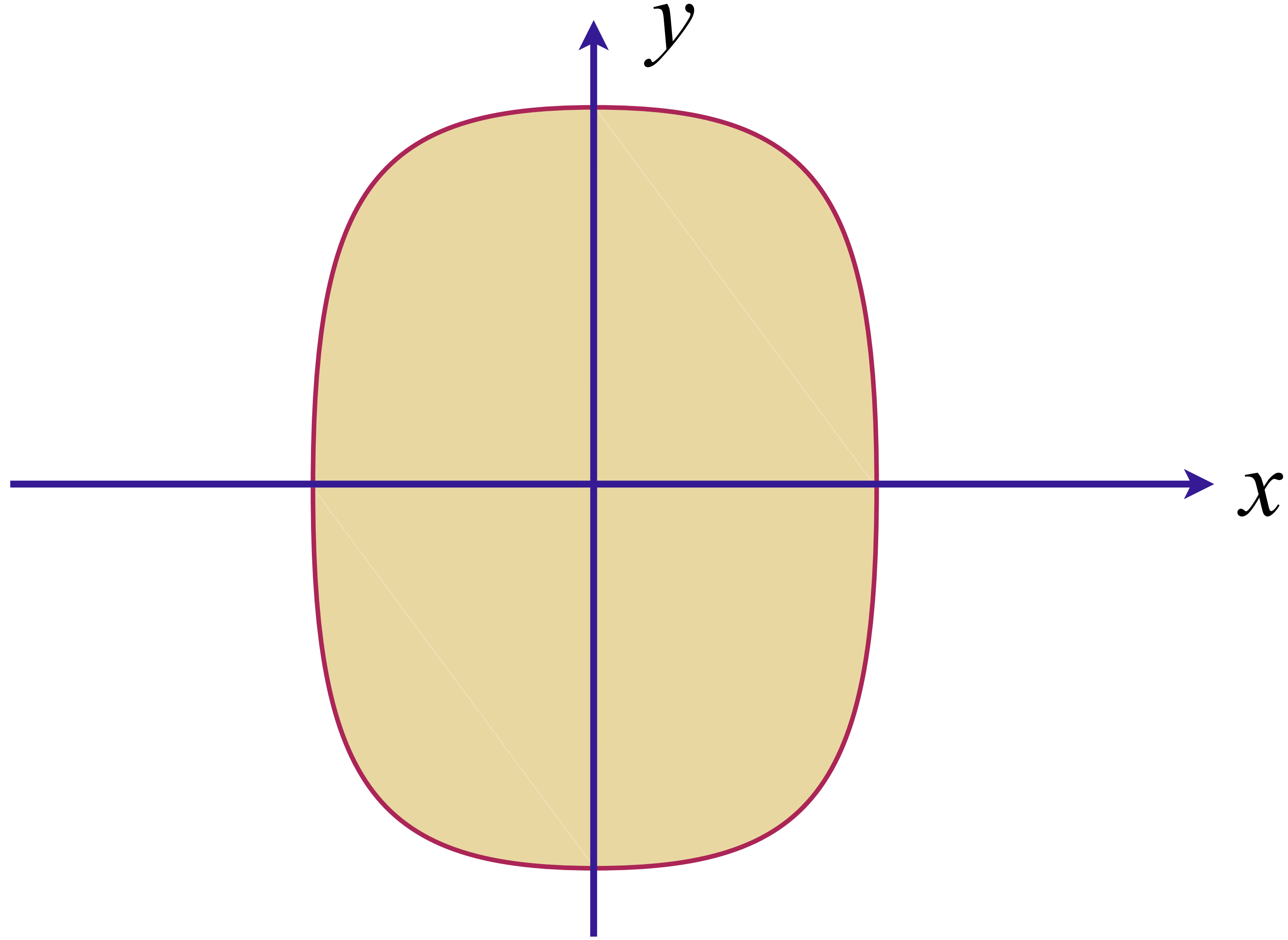
Measures spontaneous breaking of square lattice point-group symmetry of underlying Hamiltonian

Quantum criticality of Ising-nematic ordering



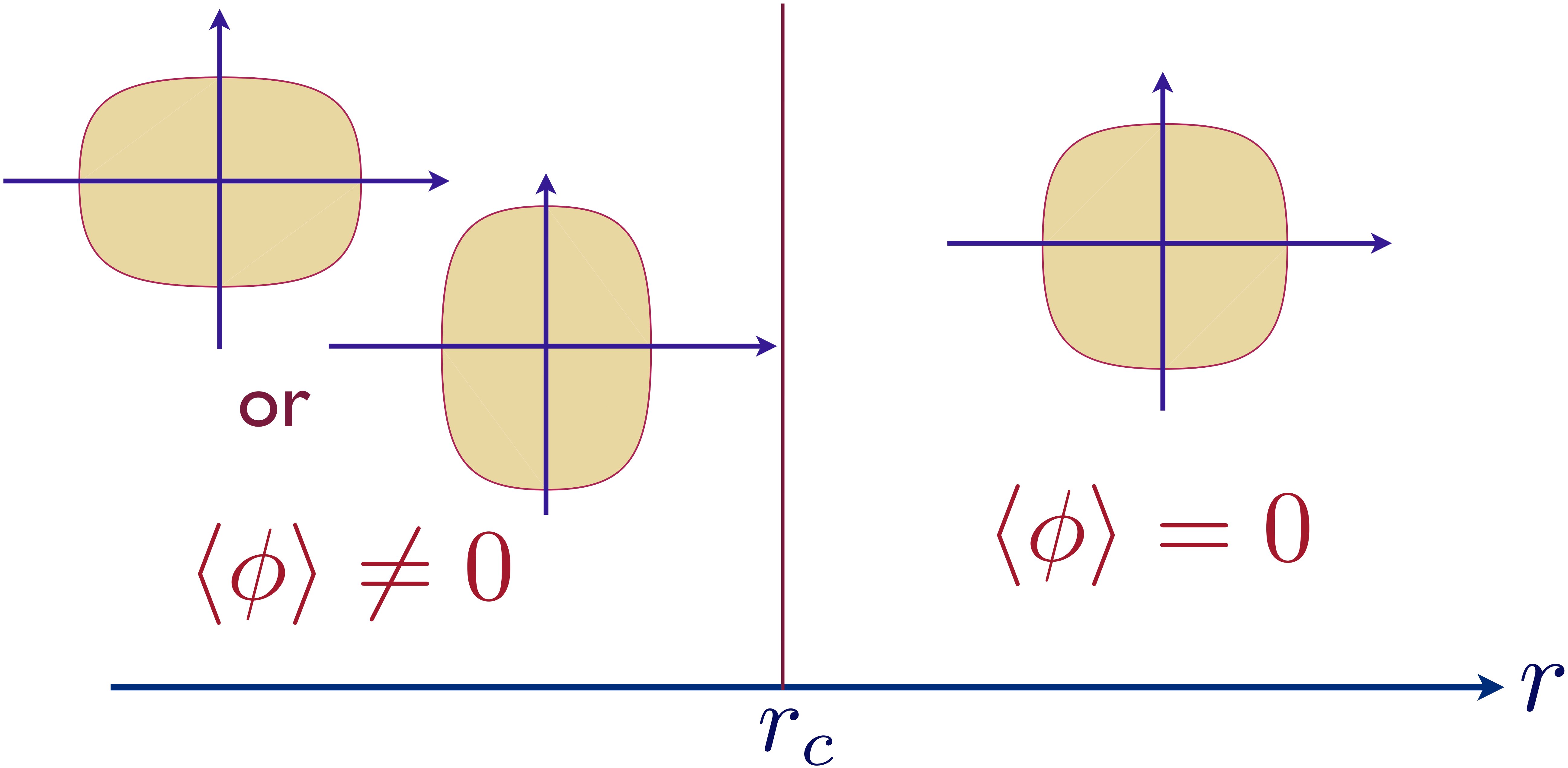
Spontaneous elongation along x direction:
Ising order parameter $\phi > 0$.

Quantum criticality of Ising-nematic ordering



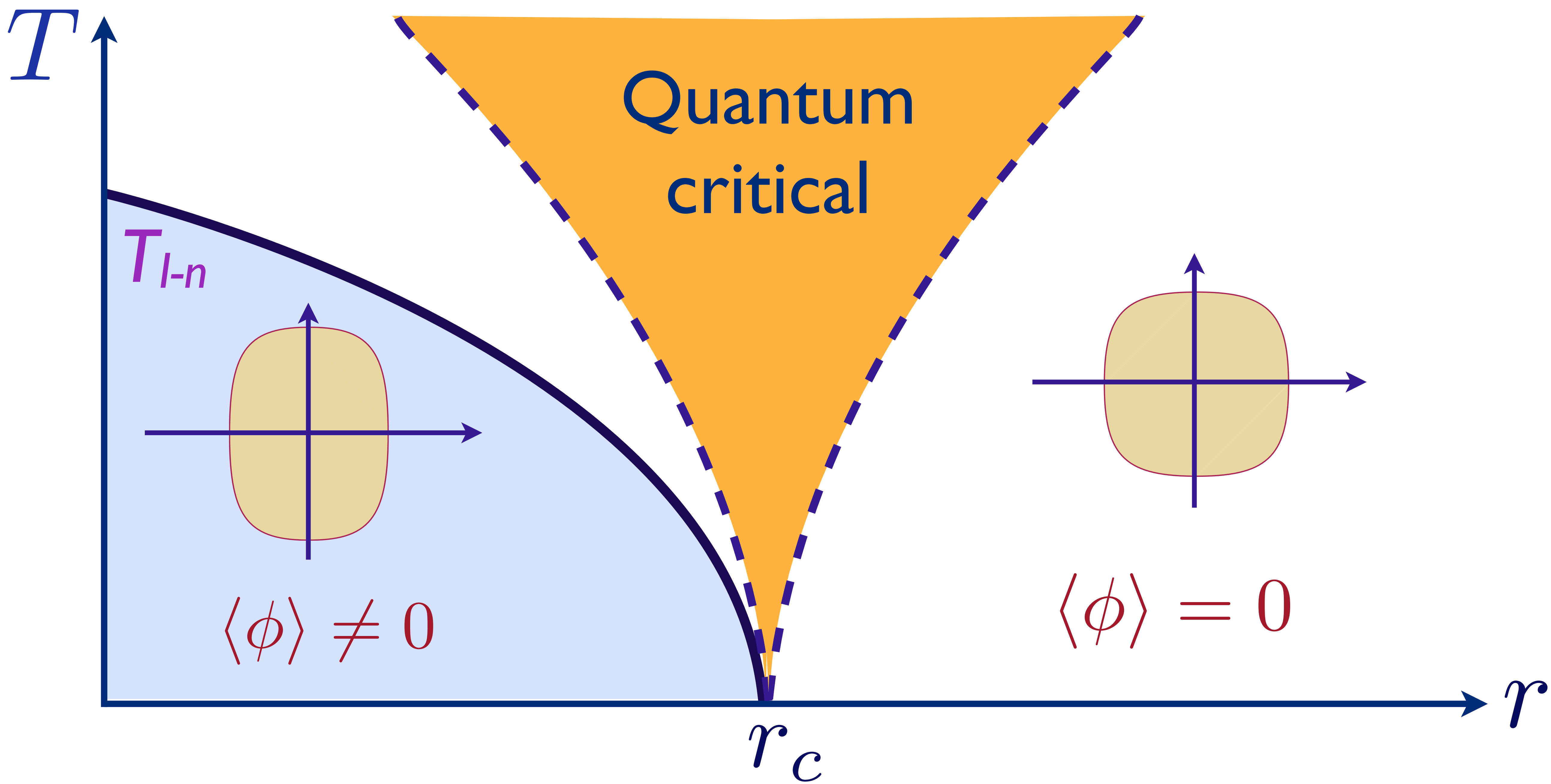
Spontaneous elongation along y direction:
Ising order parameter $\phi < 0$.

Quantum criticality of Ising-nematic ordering



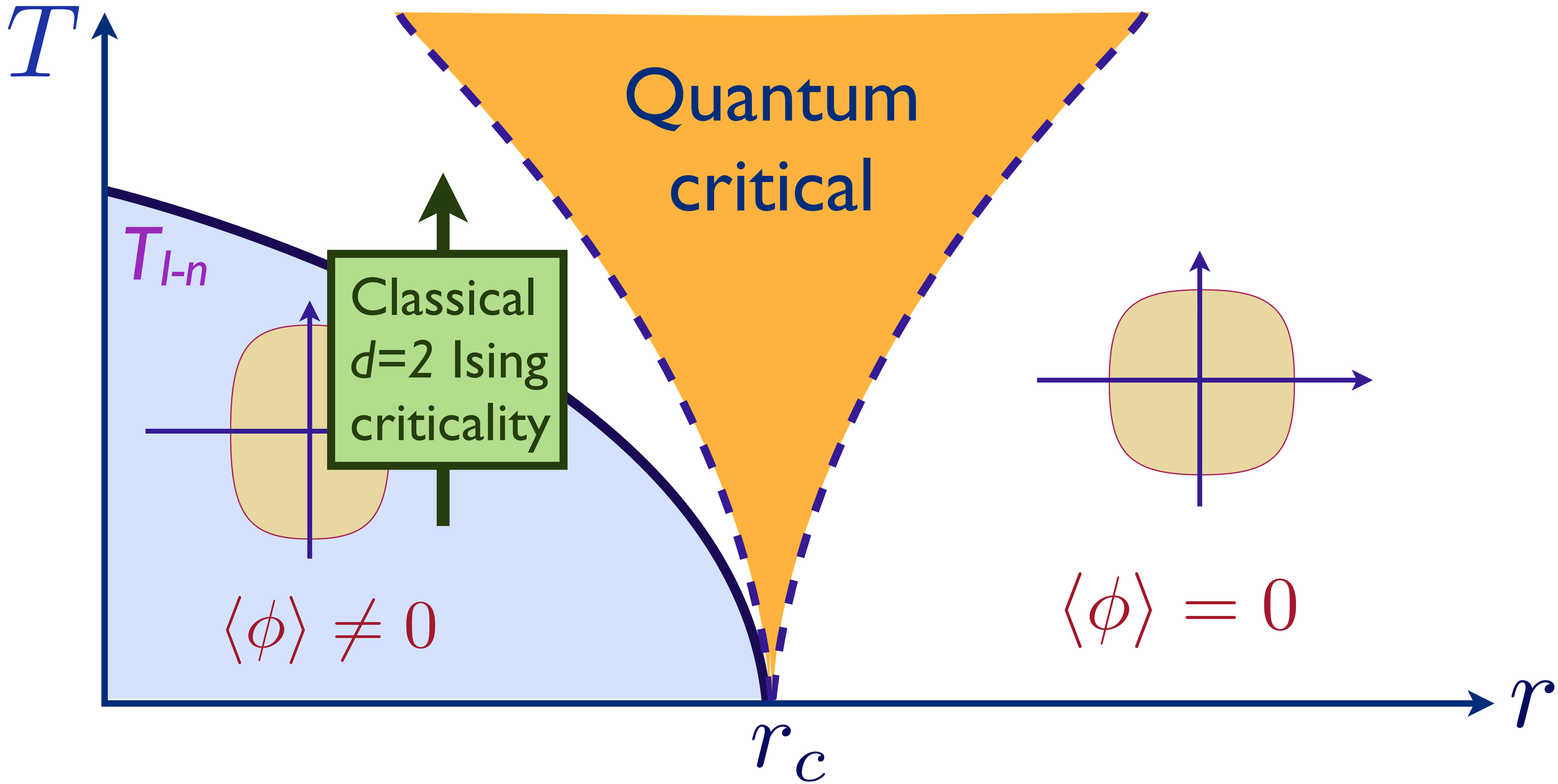
Pomeranchuk instability as a function of coupling r

Quantum criticality of Ising-nematic ordering



Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering



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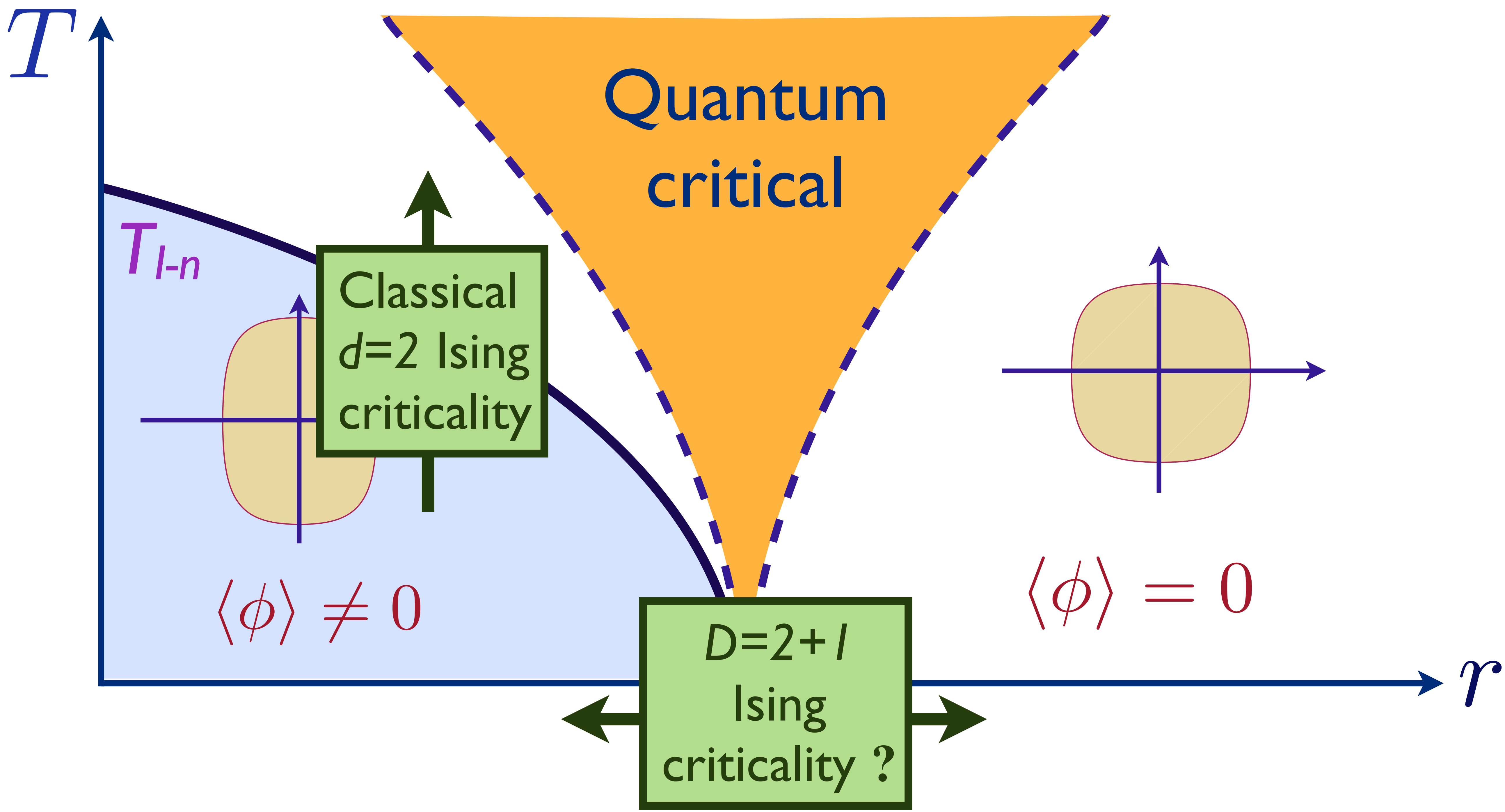

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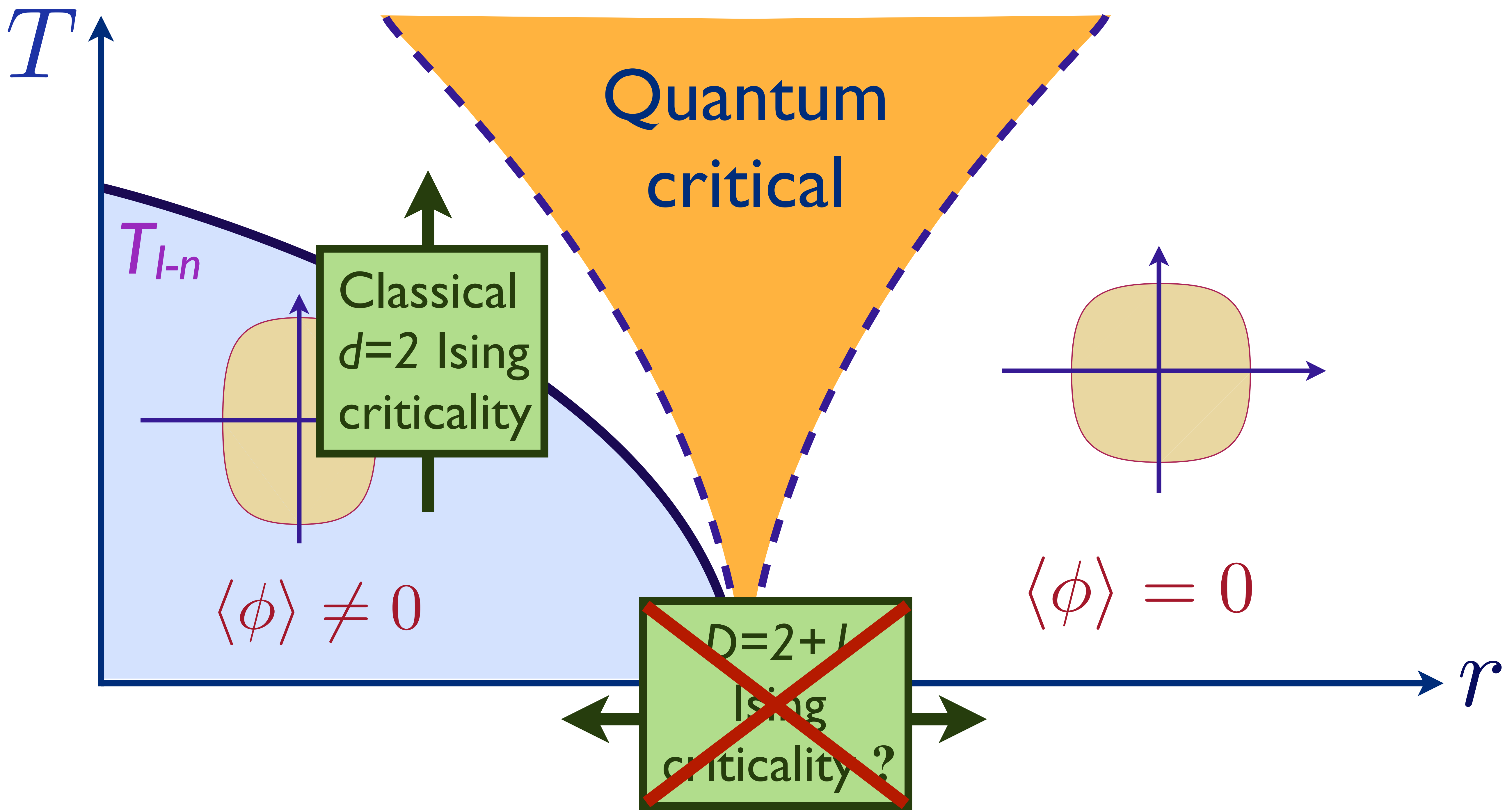

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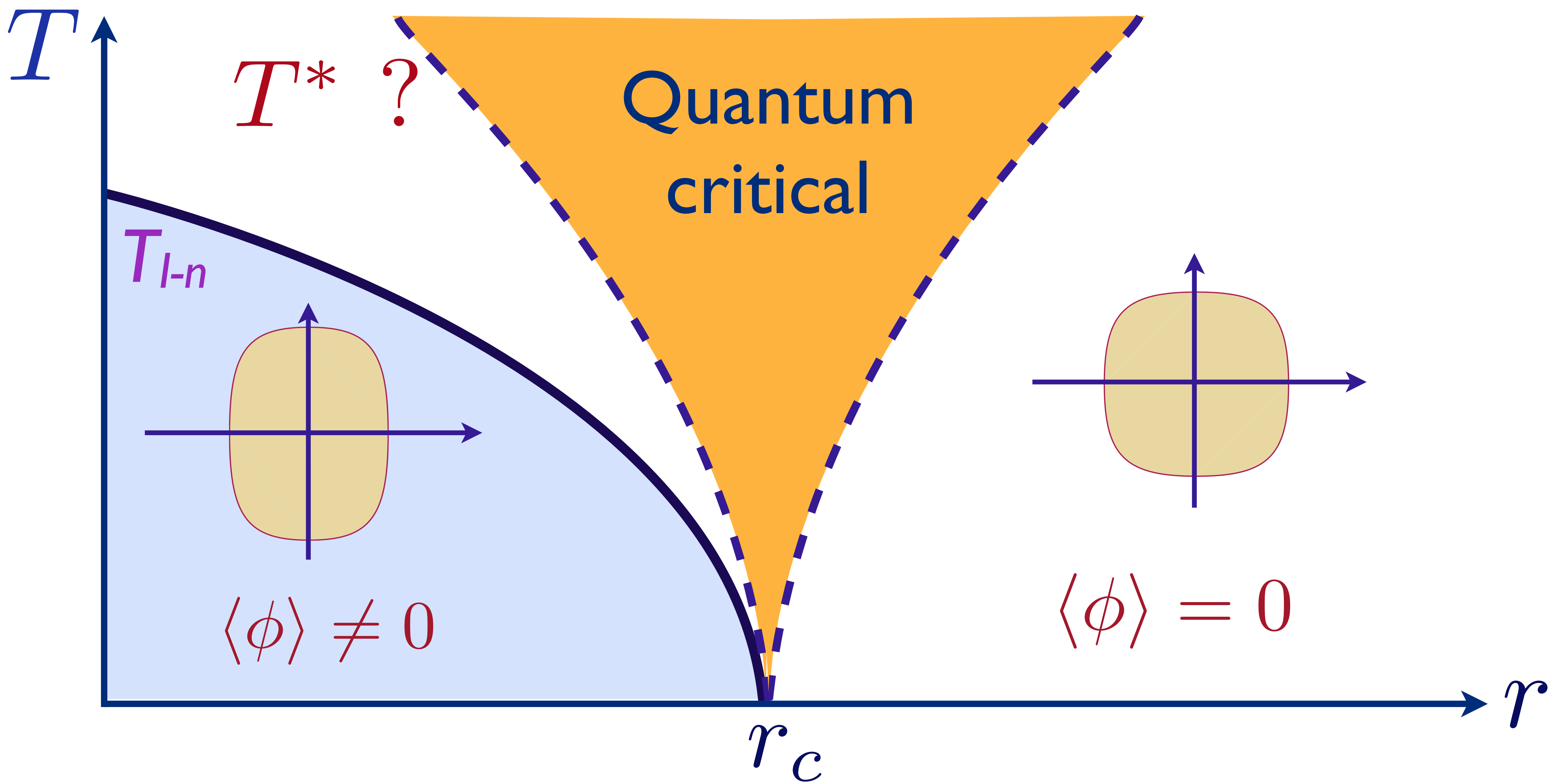

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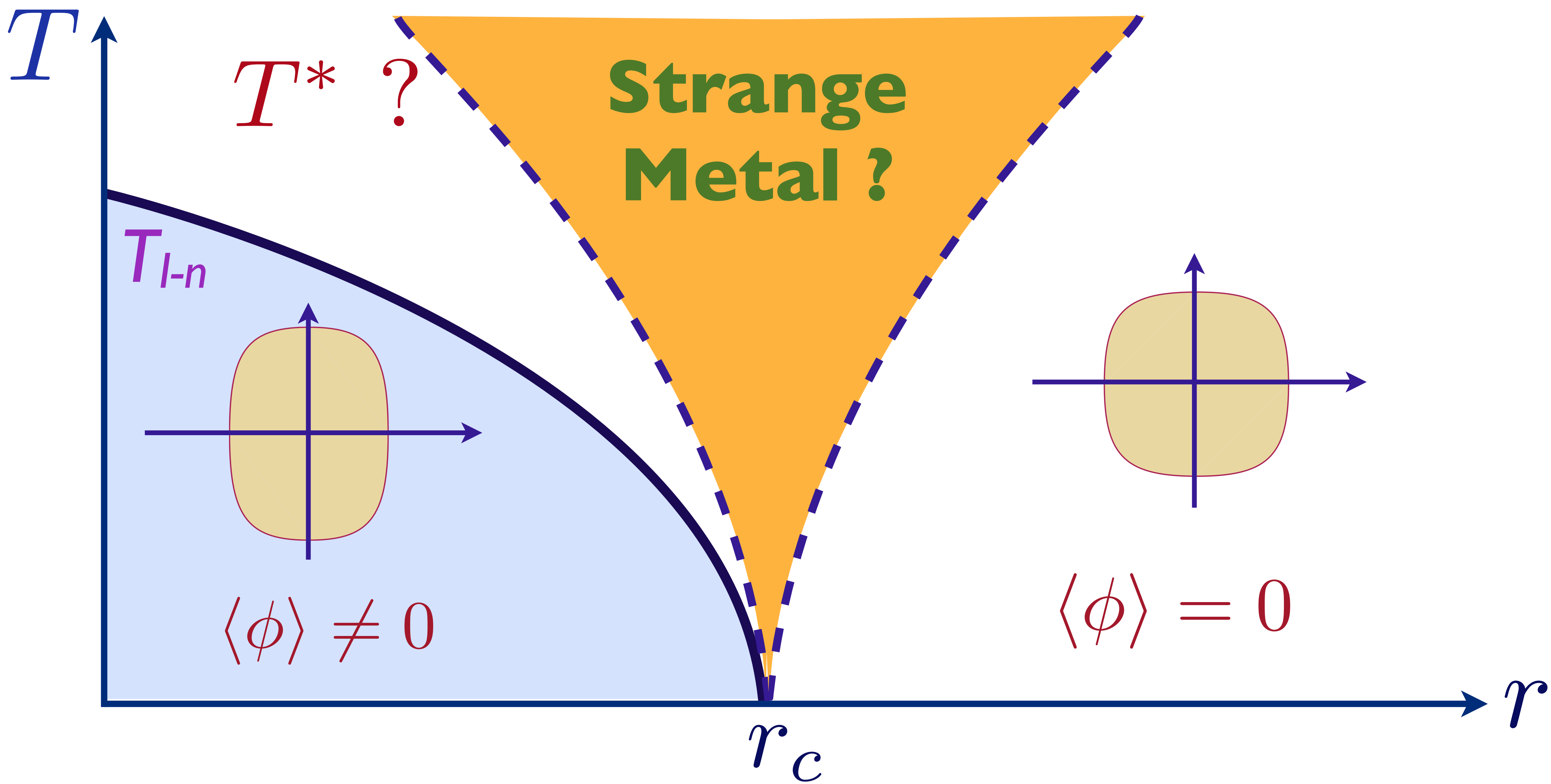

Phase diagram as a function of T and r

Quantum criticality of Ising-nematic ordering

Effective action for Ising order parameter

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

Quantum criticality of Ising-nematic ordering

Effective action for Ising order parameter

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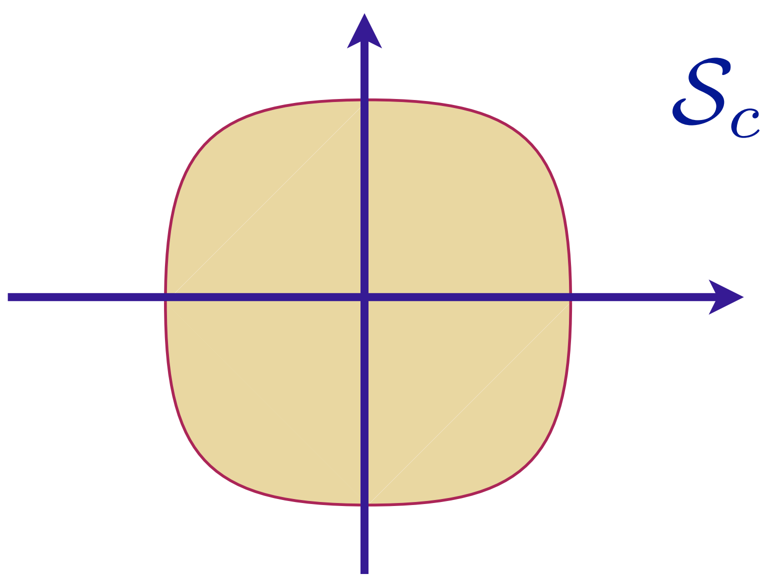
Effective action for electrons:



$$\mathcal{S}_c = \int d\tau \sum_{\alpha=1}^{N_f} \left[\sum_i c_{i\alpha}^\dagger \partial_\tau c_{i\alpha} - \sum_{i<j} t_{ij} c_{i\alpha}^\dagger c_{i\alpha} \right]$$

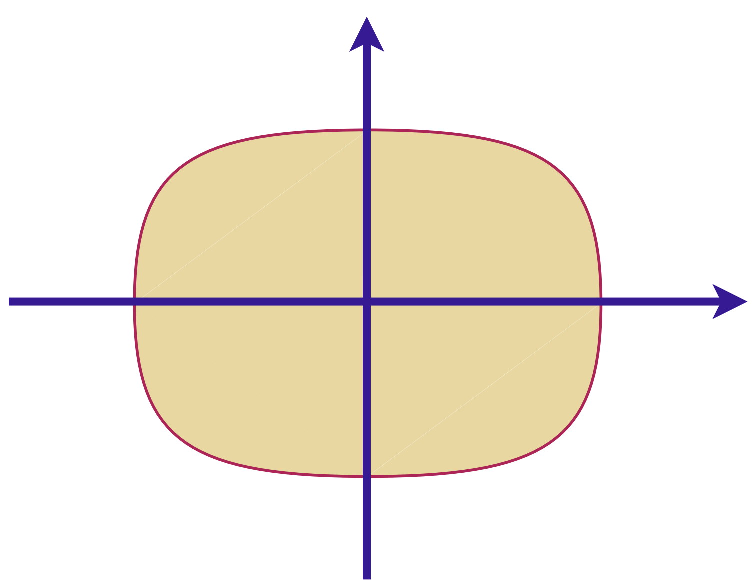
$$\equiv \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

Quantum criticality of Ising-nematic ordering

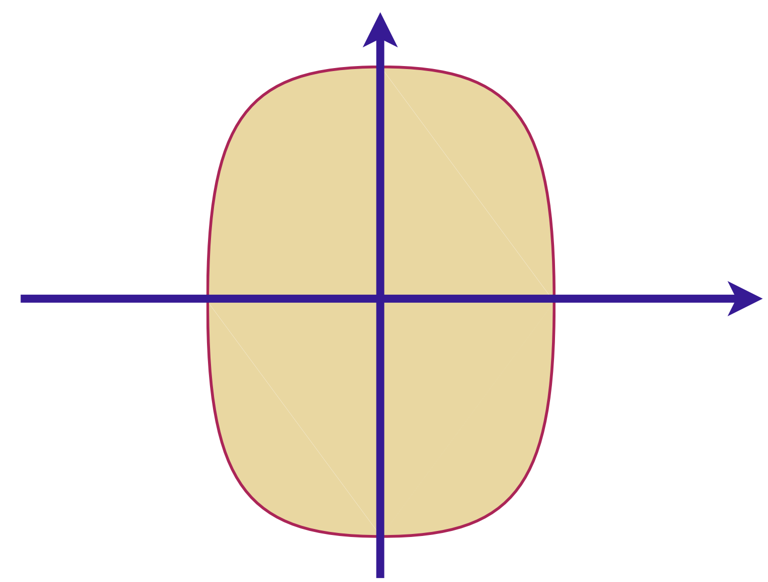
Coupling between Ising order and electrons

$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y\right) c^{\dagger}_{\mathbf{k}+\mathbf{q}/2,\alpha} c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

for spatially dependent ϕ



$\langle \phi \rangle > 0$



$\langle \phi \rangle < 0$

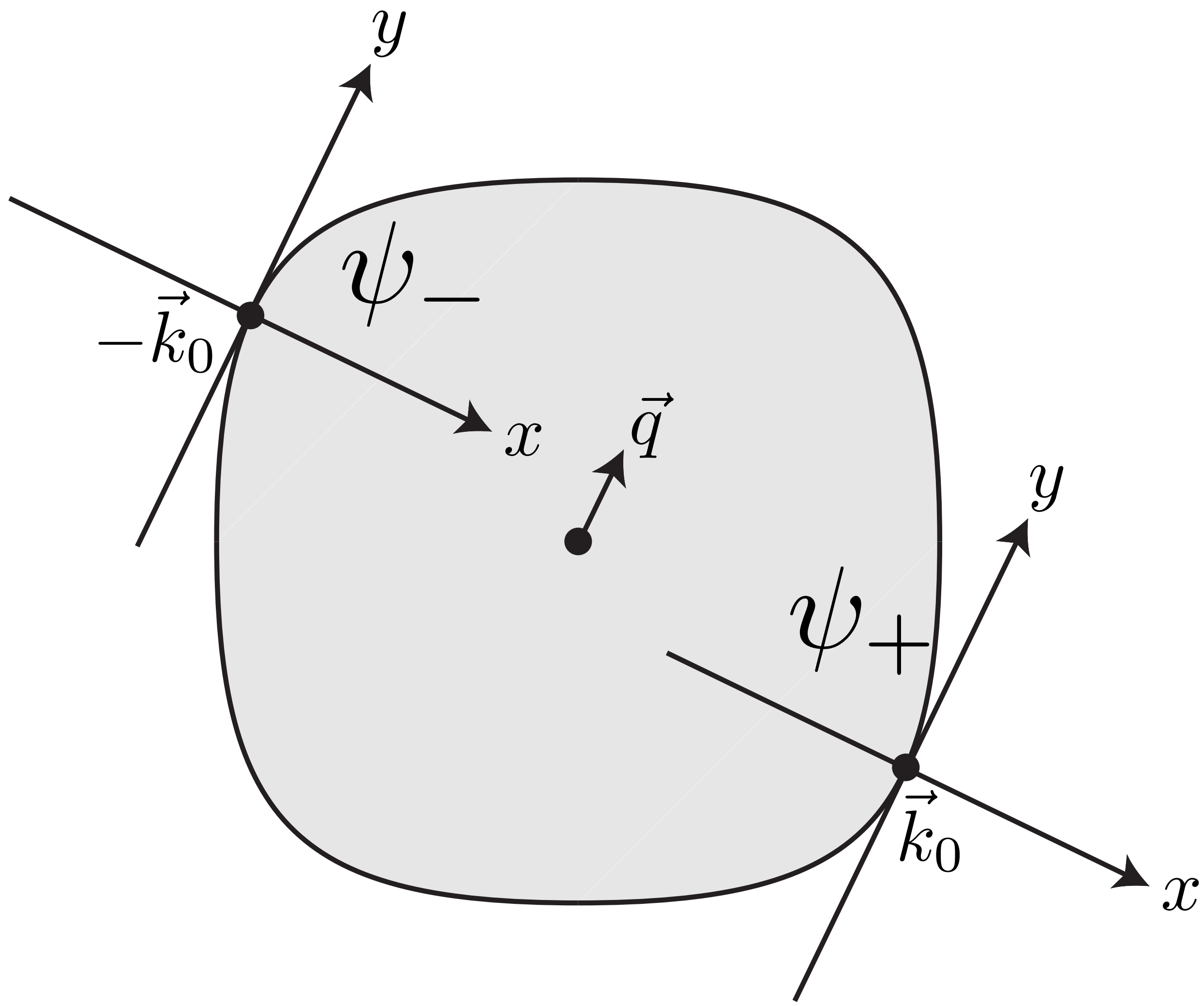
Quantum criticality of Ising-nematic ordering

$$\mathcal{S}_\phi = \int d^2r d\tau \left[(\partial_\tau \phi)^2 + c^2 (\nabla \phi)^2 + (\lambda - \lambda_c)\phi^2 + u\phi^4 \right]$$

$$\mathcal{S}_c = \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k}} \int d\tau c_{\mathbf{k}\alpha}^\dagger \left(\partial_\tau + \varepsilon_{\mathbf{k}} \right) c_{\mathbf{k}\alpha}$$

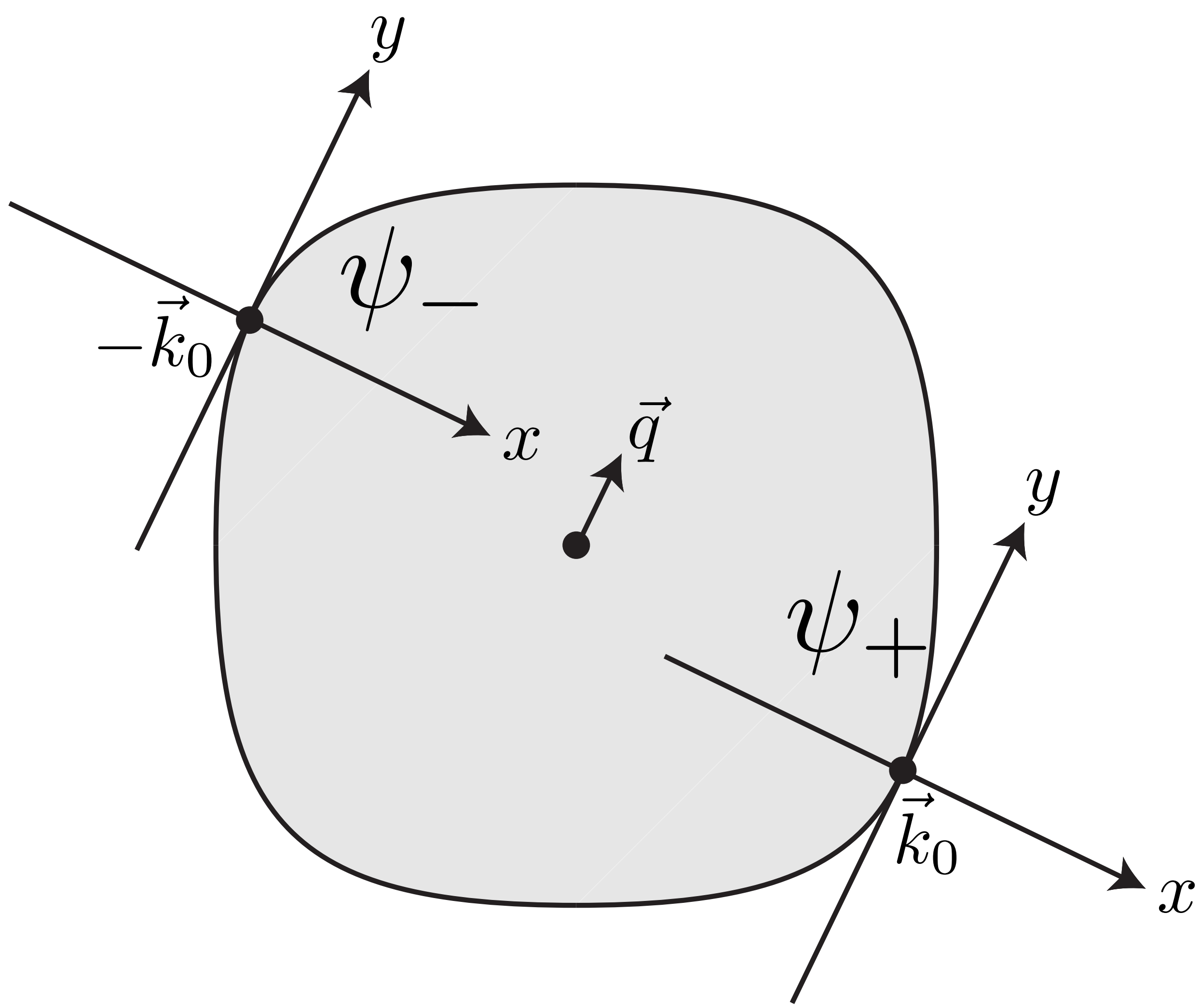
$$\mathcal{S}_{\phi c} = -\gamma \int d\tau \sum_{\alpha=1}^{N_f} \sum_{\mathbf{k},\mathbf{q}} \phi_{\mathbf{q}} \left(\cos k_x - \cos k_y \right) c_{\mathbf{k}+\mathbf{q}/2,\alpha}^\dagger c_{\mathbf{k}-\mathbf{q}/2,\alpha}$$

Quantum criticality of Ising-nematic ordering



- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

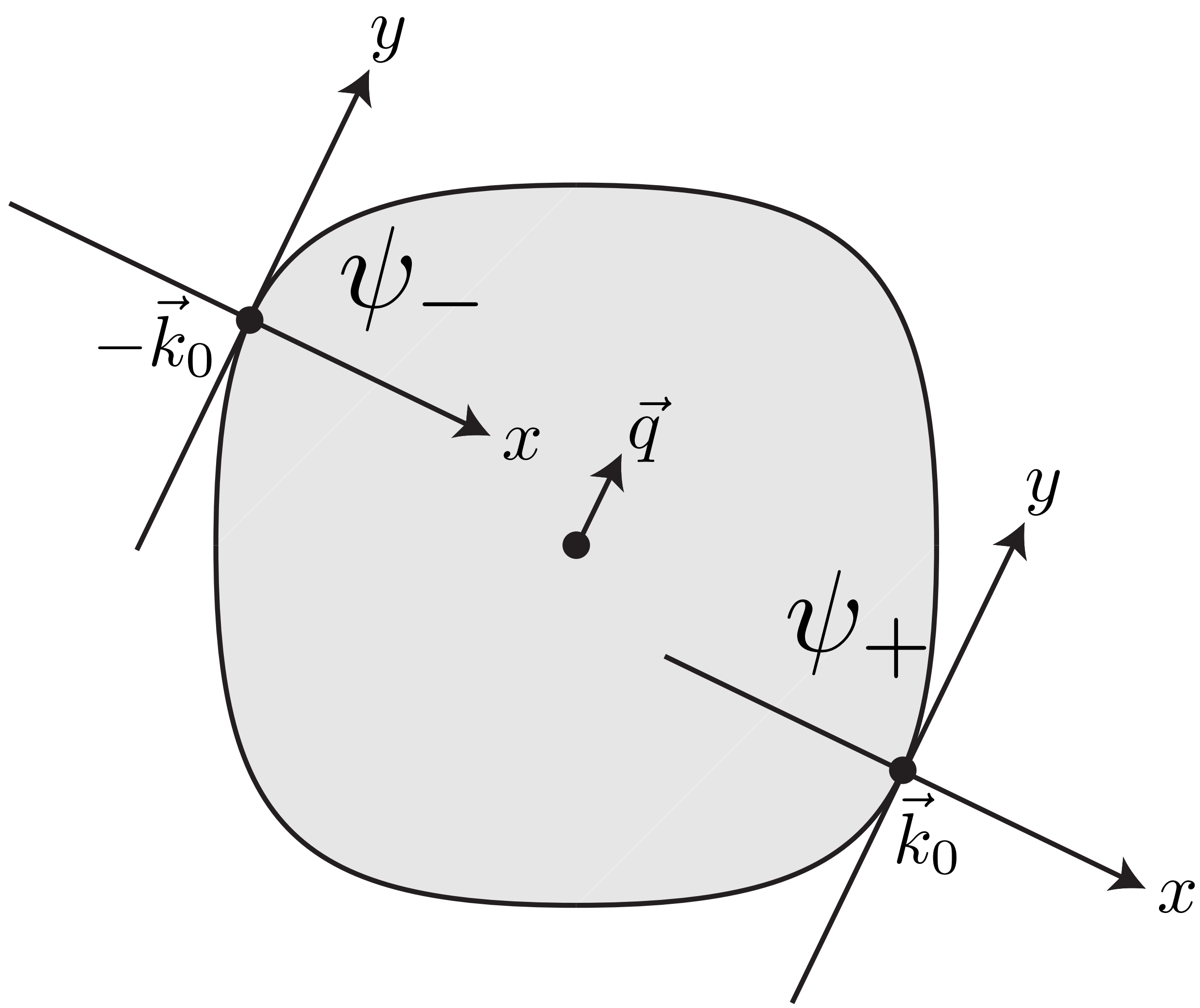
Quantum criticality of Ising-nematic ordering



- ϕ fluctuation at wavevector $\vec{q}$ couples most efficiently to fermions near $\pm\vec{k}_0$.

- Expand fermion kinetic energy at wavevectors about $\pm\vec{k}_0$ and boson (ϕ) kinetic energy about $\vec{q} = 0$.

Quantum criticality of Ising-nematic ordering



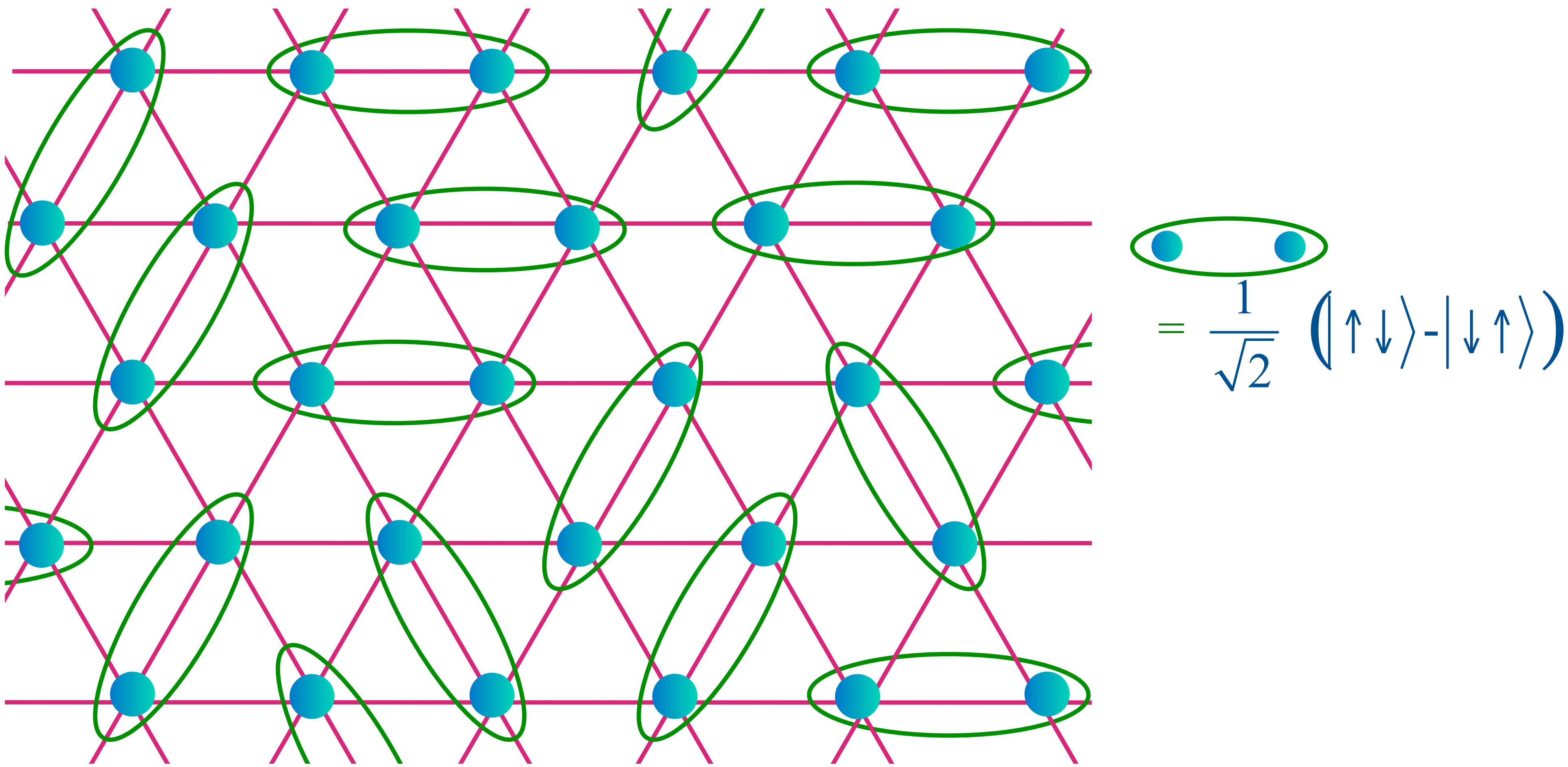
$$\mathcal{L}[\psi_\pm, \phi] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_-$$

$$-\phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

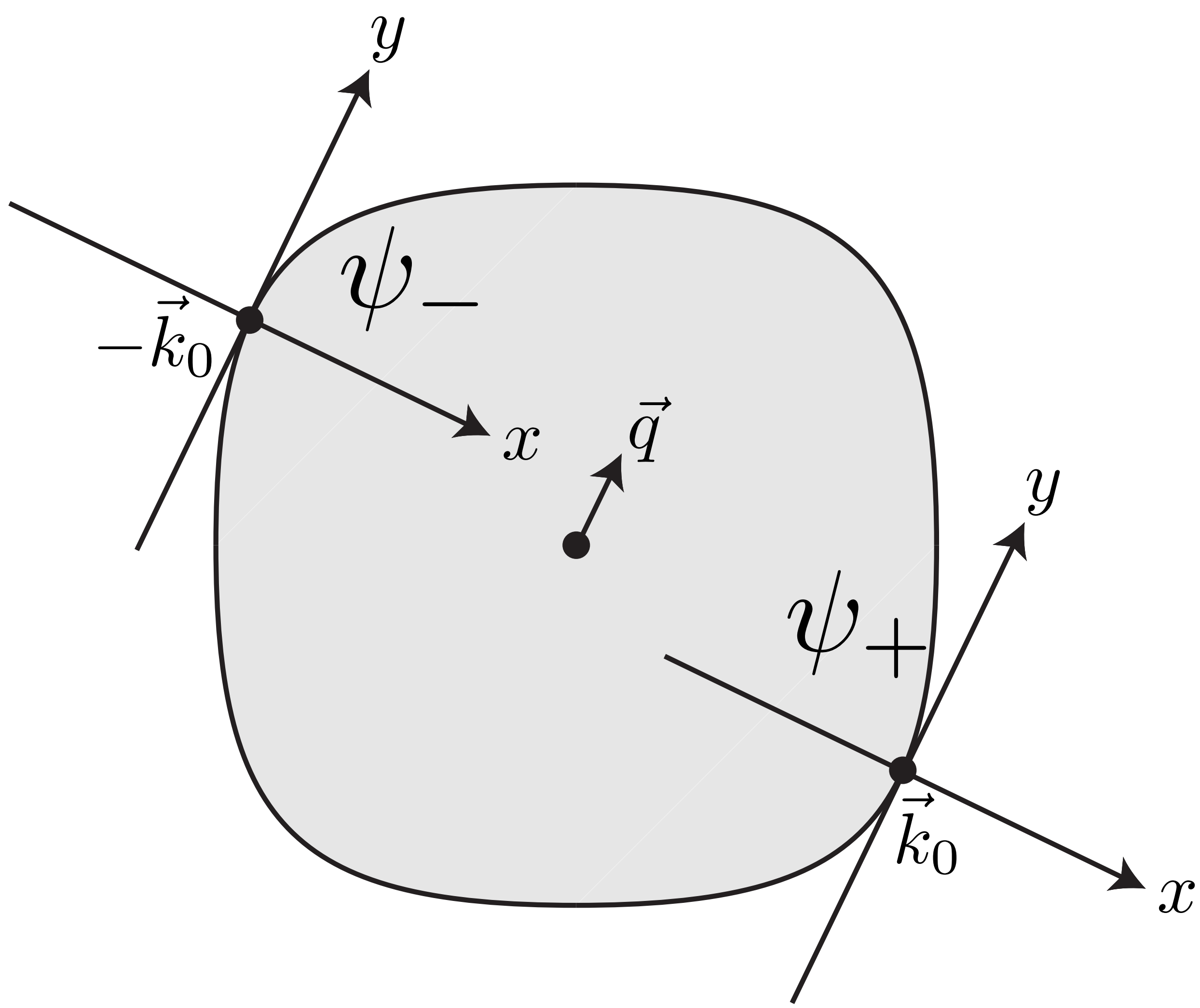
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

- Model of a spin liquid ("Bose metal"): couple fermions to a dynamical gauge field A_μ.



$$\mathcal{L} = f_\sigma^\dagger \left(\partial_\tau - i A_\tau - \frac{(\nabla - i\mathbf{A})^2}{2m} - \mu \right) f_\sigma$$

Quantum criticality of Ising-nematic ordering



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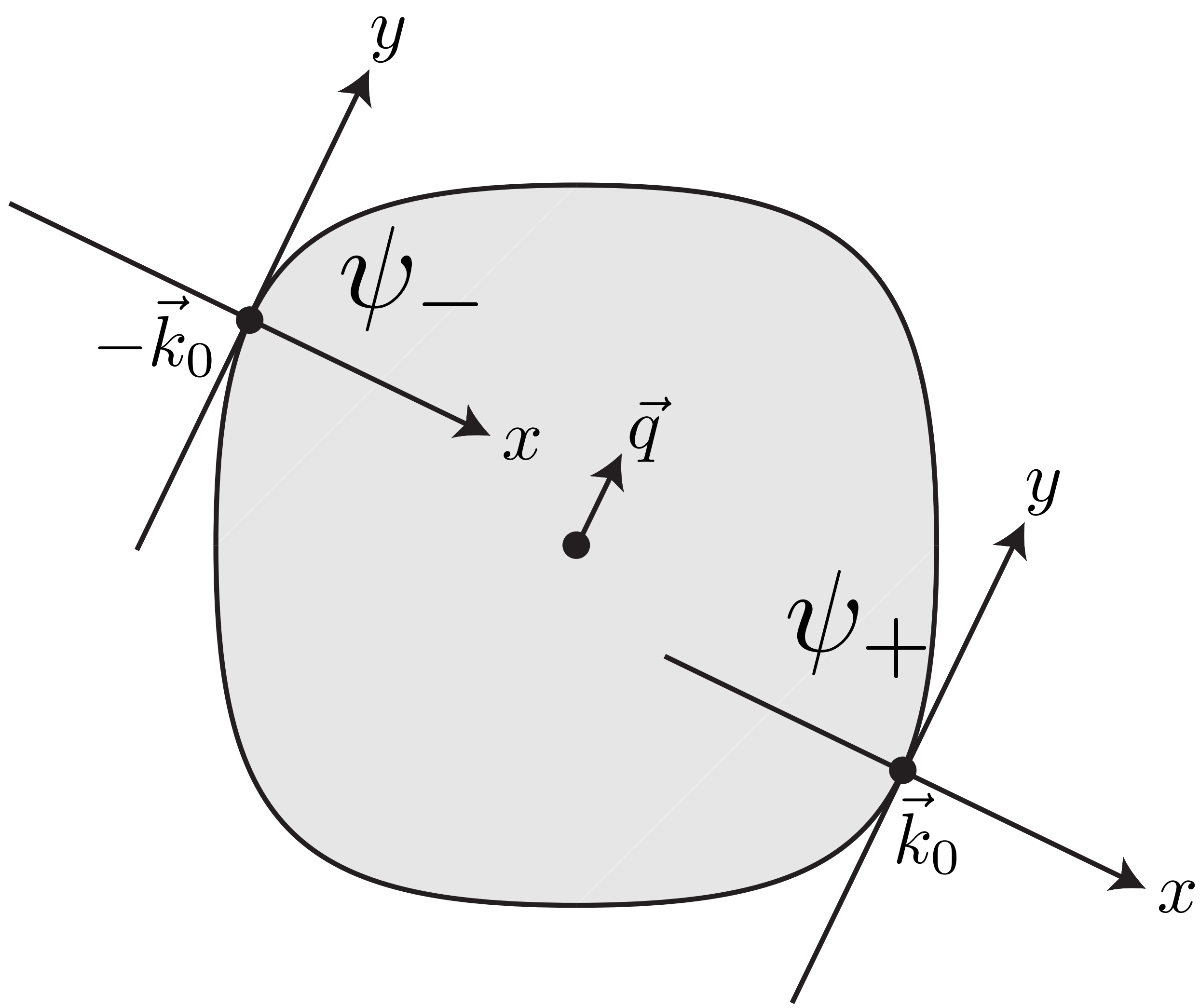
Field theory of U(1) spin liquid



$$\mathcal{L}[\psi_\pm, a] =$$

$$\psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_-$$

$$-a\left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y a\right)^2$$

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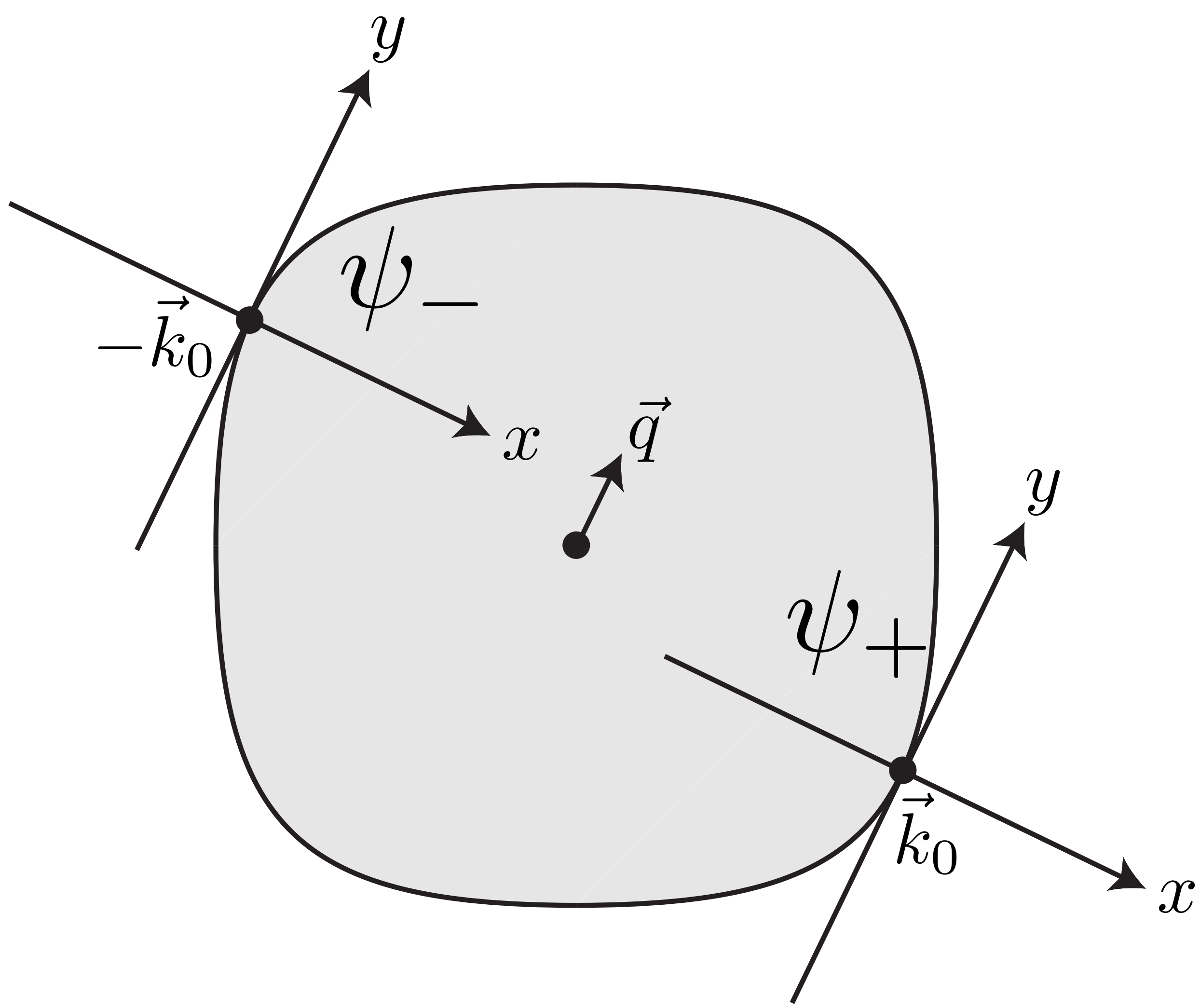
Quantum criticality of Ising-nematic ordering



$$\mathcal{L}[\psi_\pm, \phi] =$$

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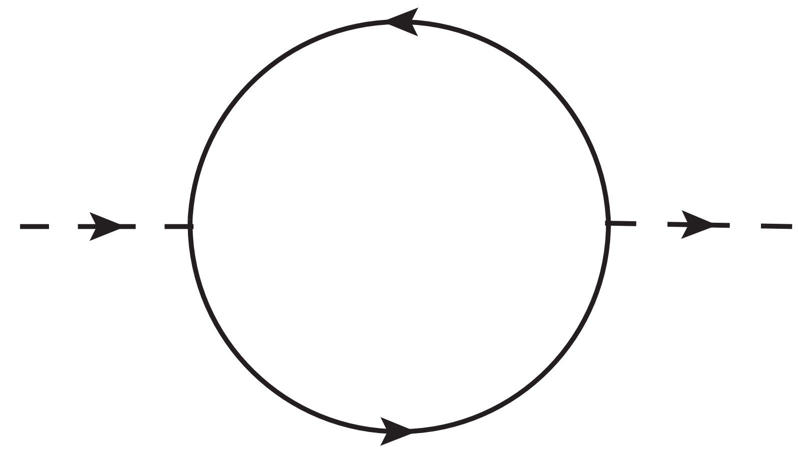
Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$



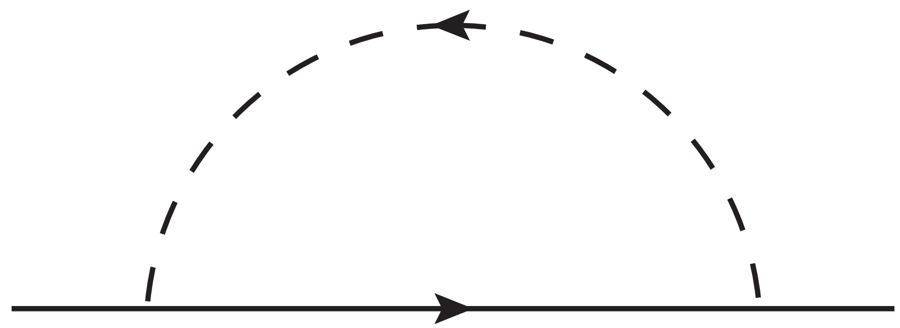
One loop ϕ self-energy with N_f fermion flavors:

$$D(\vec{q},\omega) = N_f \int \frac{d^2k}{4\pi^2}\frac{d\Omega}{2\pi}\frac{1}{\left[-i(\Omega+\omega)+k_x+q_x+(k_y+q_y)^2\right]\left[-i\Omega-k_x+k_y^2\right]}$$
$$= \frac{N_f}{4\pi}\frac{|\omega|}{|q_y|}$$

Landau-damping

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$



Electron self-energy at order $1/N_f$:

$$\Sigma(\vec{k},\Omega) = -\frac{1}{N_f}\int\frac{d^2q}{4\pi^2}\frac{d\omega}{2\pi}\frac{1}{\left[-i(\omega+\Omega)+k_x+q_x+(k_y+q_y)^2\right]\left[\frac{q_y^2}{g^2}+\frac{|\omega|}{|q_y|}\right]}$$

$$= -i\frac{2}{\sqrt{3}N_f}\left(\frac{g^2}{4\pi}\right)^{2/3}\mathrm{sgn}(\Omega)|\Omega|^{2/3}$$

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- - \phi \left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2} \left(\partial_y \phi\right)^2$$

Schematic form of ϕ and fermion Green's functions

$$D(\vec{q}, \omega) = \frac{1/N_f}{q_y^2 + \frac{|\omega|}{|q_y|}} \quad , \quad G_f(\vec{q}, \omega) = \frac{1}{q_x + q_y^2 - i\,\mathrm{sgn}(\omega)|\omega|^{2/3}/N_f}$$

In *both* cases $q_x \sim q_y^2 \sim \omega^{1/z}$, with $z = 3/2$. Note that the bare term $\sim \omega$ in G_f^{-1} is irrelevant.

Strongly-coupled theory without quasiparticles.

Quantum criticality of Ising-nematic ordering

$$\begin{aligned}\mathcal{L} &= \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right) \psi_- \\ &\quad - \phi\left(\psi_+^\dagger \psi_+ + \psi_-^\dagger \psi_-\right) + \frac{1}{2g^2}\left(\partial_y \phi\right)^2\end{aligned}$$

Simple scaling argument for $z = 3/2$.

Quantum criticality of Ising-nematic ordering

$$\mathcal{L}_{\text{scaling}} = \psi_+^\dagger \left(-i\partial_x - \partial_y^2\right) \psi_+ + \psi_-^\dagger \left(+i\partial_x - \partial_y^2\right) \psi_- - g\,\phi \left(\psi_+^\dagger \psi_+ - \psi_-^\dagger \psi_-\right) + \left(\partial_y \phi\right)^2$$

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Quantum criticality of Ising-nematic ordering

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Simple scaling argument for $z = 3/2$.

Under the rescaling $x \to x/s$, $y \to y/s^{1/2}$, and $\tau \to \tau/s^z$, we find invariance provided

$$\begin{aligned} a &\to a\,s^{(2z+1)/4} \\ \psi &\to \psi\,s^{(2z+1)/4} \\ g &\to g\,s^{(3-2z)/4} \end{aligned}$$

So the action is invariant provided $z = 3/2$.

Quantum criticality of Ising-nematic ordering

$$\mathcal{L} = \psi_+^\dagger \left(\partial_\tau - i\partial_x - \partial_y^2\right)\psi_+ + \psi_-^\dagger \left(\partial_\tau + i\partial_x - \partial_y^2\right)\psi_- - \phi\left(\psi_+^\dagger\psi_+ + \psi_-^\dagger\psi_-\right) + \frac{1}{2g^2}\left(\partial_y\phi\right)^2$$

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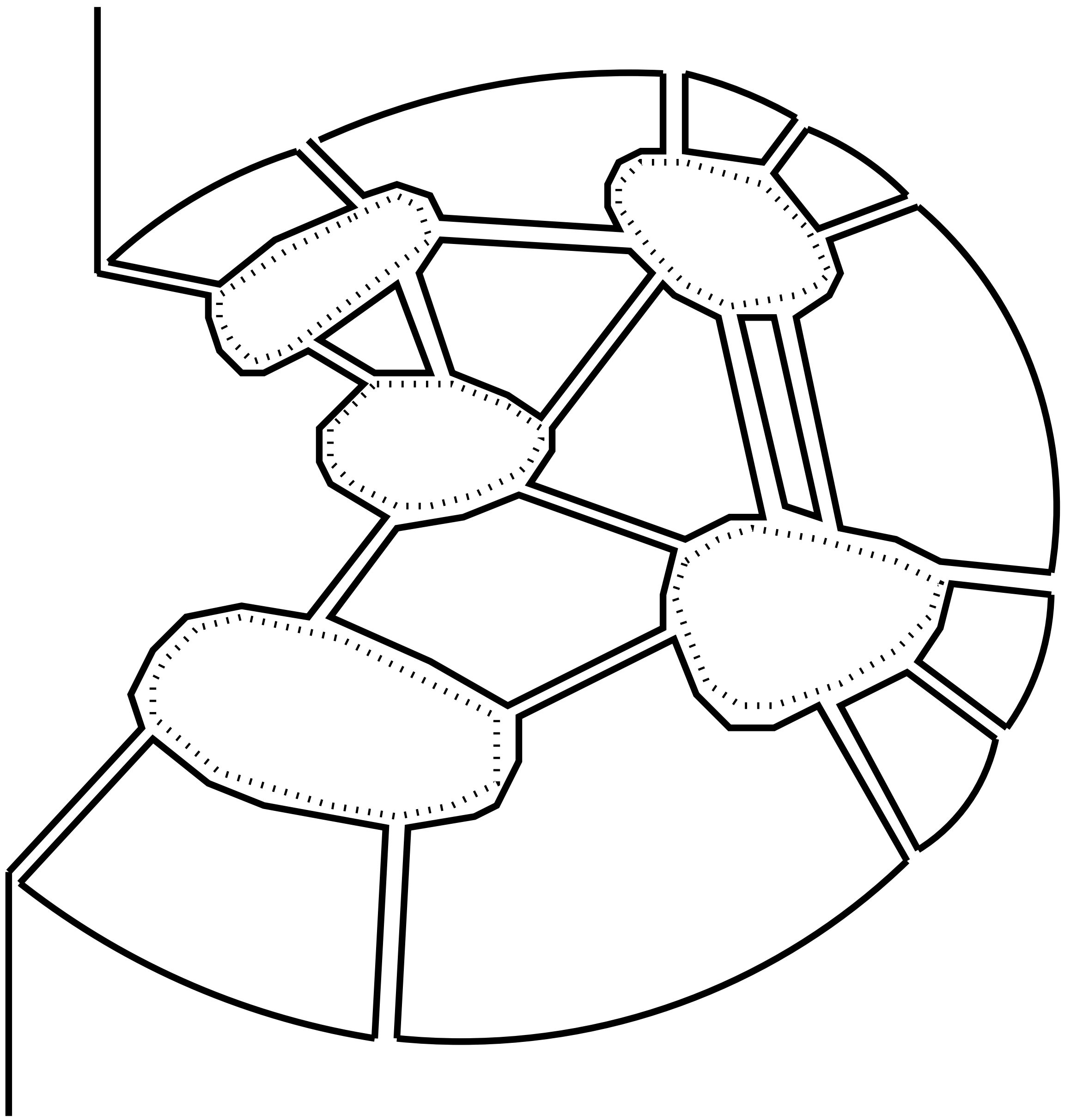
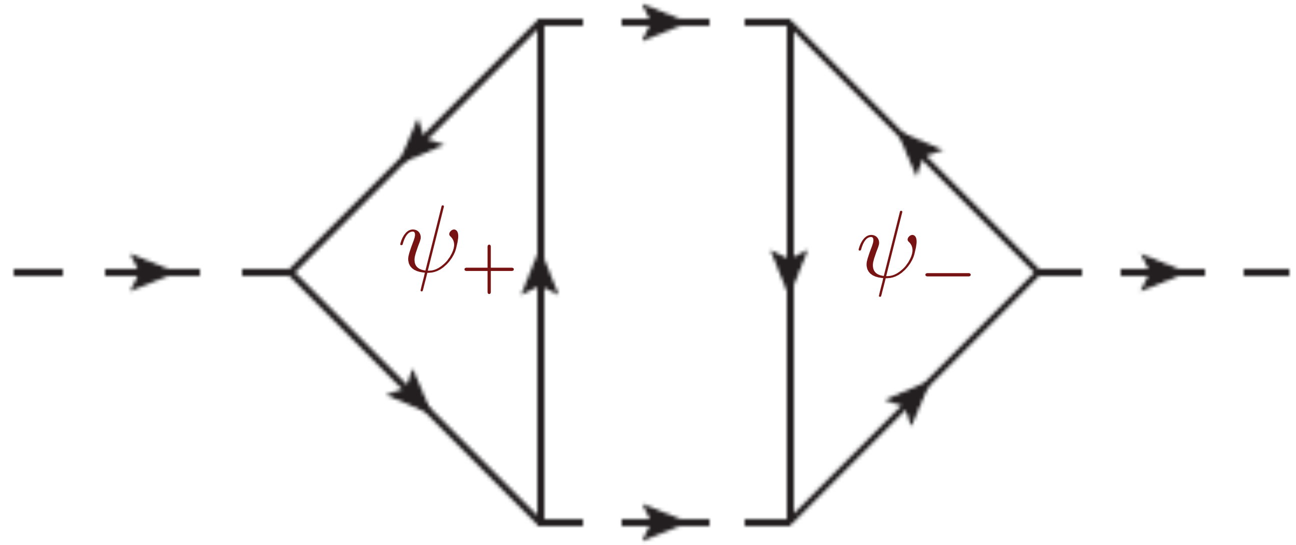
The $1/N_f$ expansion is *not* determined by counting fermion loops, because of infrared singularities created by the Fermi surface. The $|\omega|^{2/3}/N_f$ fermion self-energy leads to additional powers of N_f, and a breakdown in the loop expansion.

Computations in the $1/N$ expansion





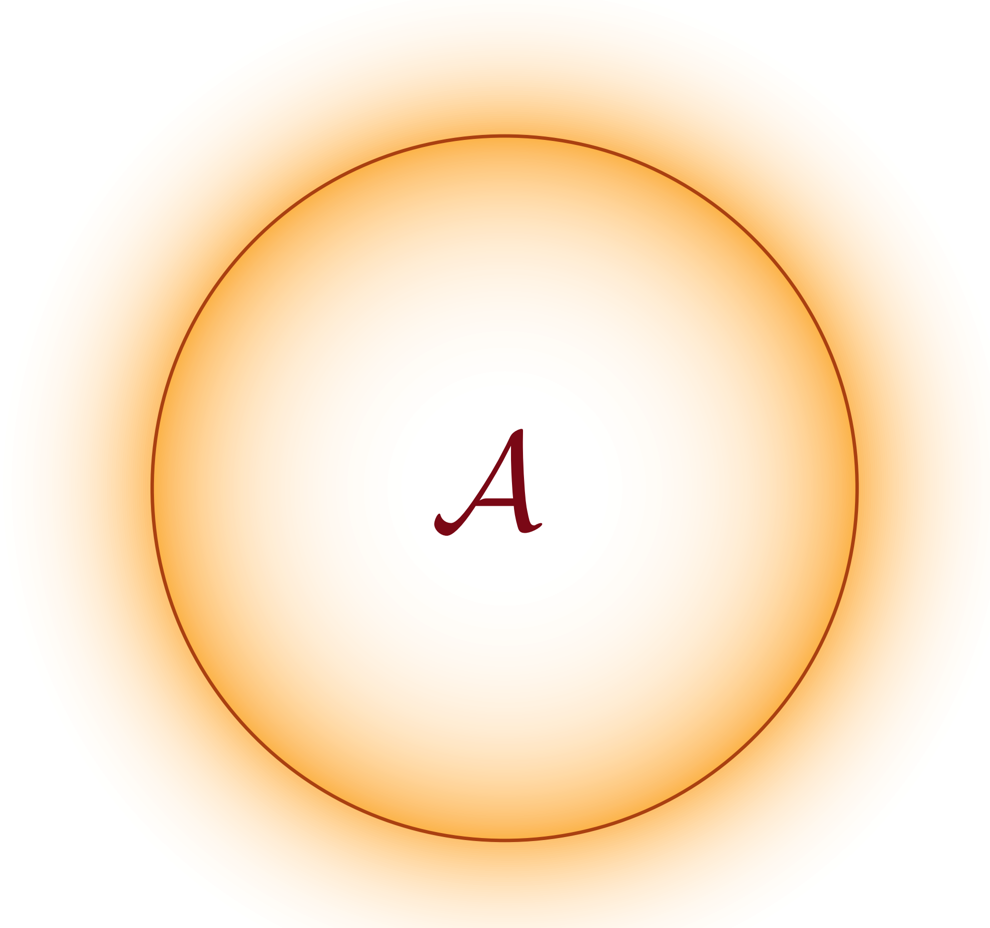
Graph mixing ψ_+ and ψ_- is $\mathcal{O}\left(N^{3/2}\right)$ (instead of $\mathcal{O}\left(N\right)$), violating genus expansion

All planar graphs of ψ_+ alone are as important as the leading term

M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)

Sung-Sik Lee, *Physical Review* B **80**, 165102 (2009)

Properties of the strange metal at the Ising-nematic critical point



- There is a sharp Fermi surface defined by the fermion Green's function: $G_f^{-1}(|\mathbf{k}| = k_F, \omega = 0) = 0$.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
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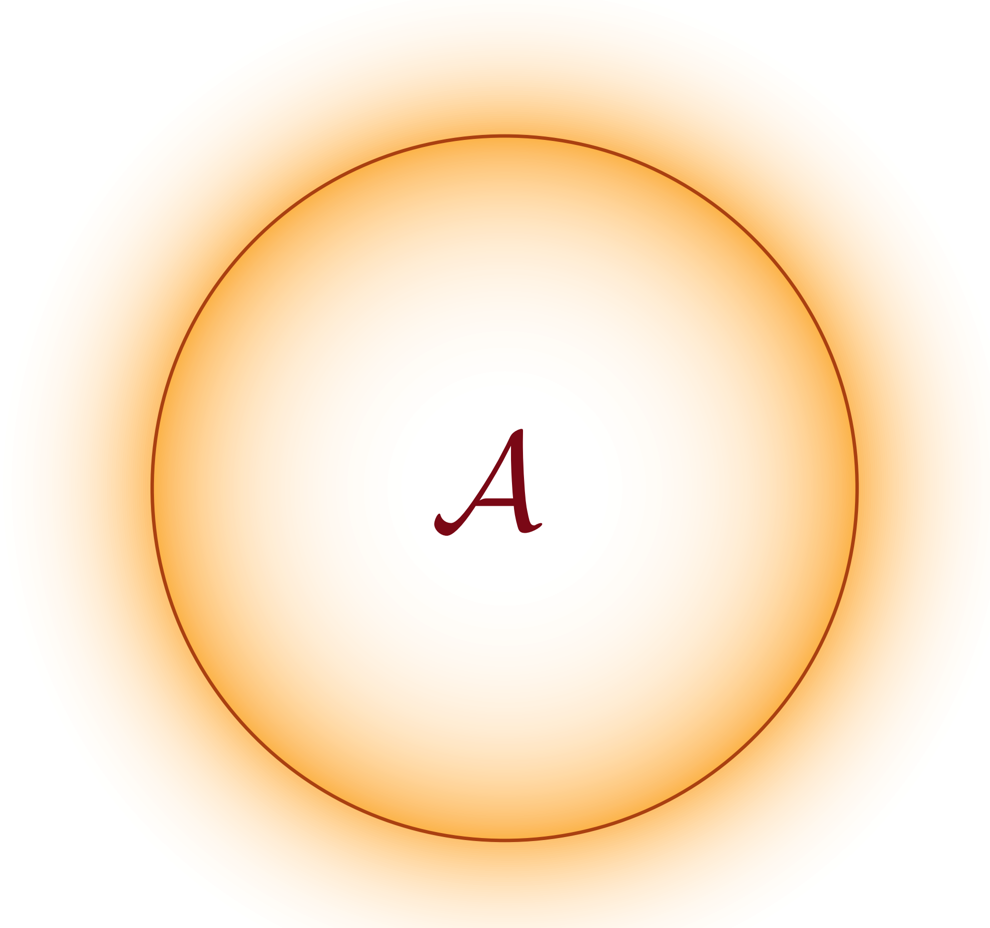

- There is a sharp Fermi surface defined by the fermion Green's function: $G_f^{-1}(|\mathbf{k}| = k_F, \omega = 0) = 0$.

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

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D. F. Mross, J. McGreevy, H. Liu, and T. Senthil, Phys. Rev. B **82**, 045121 (2010)

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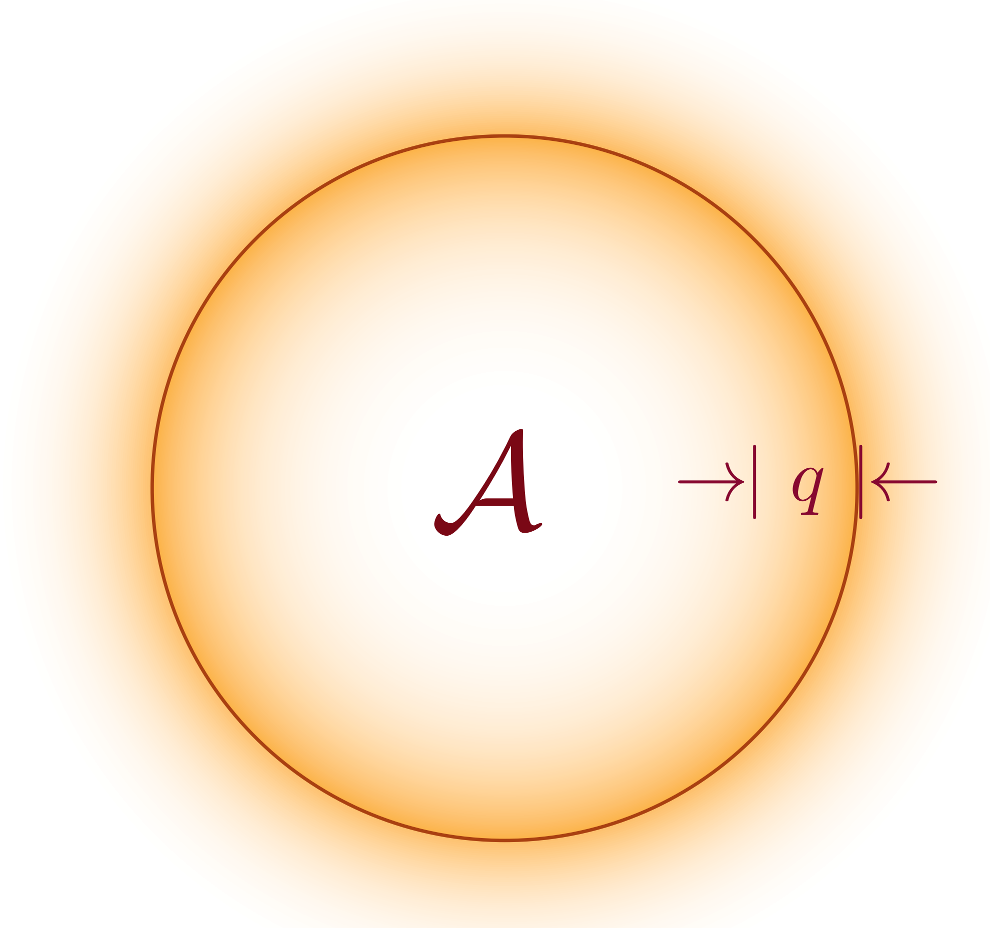

- There is a sharp Fermi surface defined by the fermion Green's function: $G_f^{-1}(|\mathbf{k}| = k_F, \omega = 0) = 0$.

- Area enclosed by the Fermi surface $\mathcal{A} = \mathcal{Q}$, the fermion density

- Critical continuum of excitations near the Fermi surface with energy $\omega \sim |q|^z$, where $q = |\mathbf{k}| - k_F$ is the distance from the Fermi surface and z is the dynamic critical exponent.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
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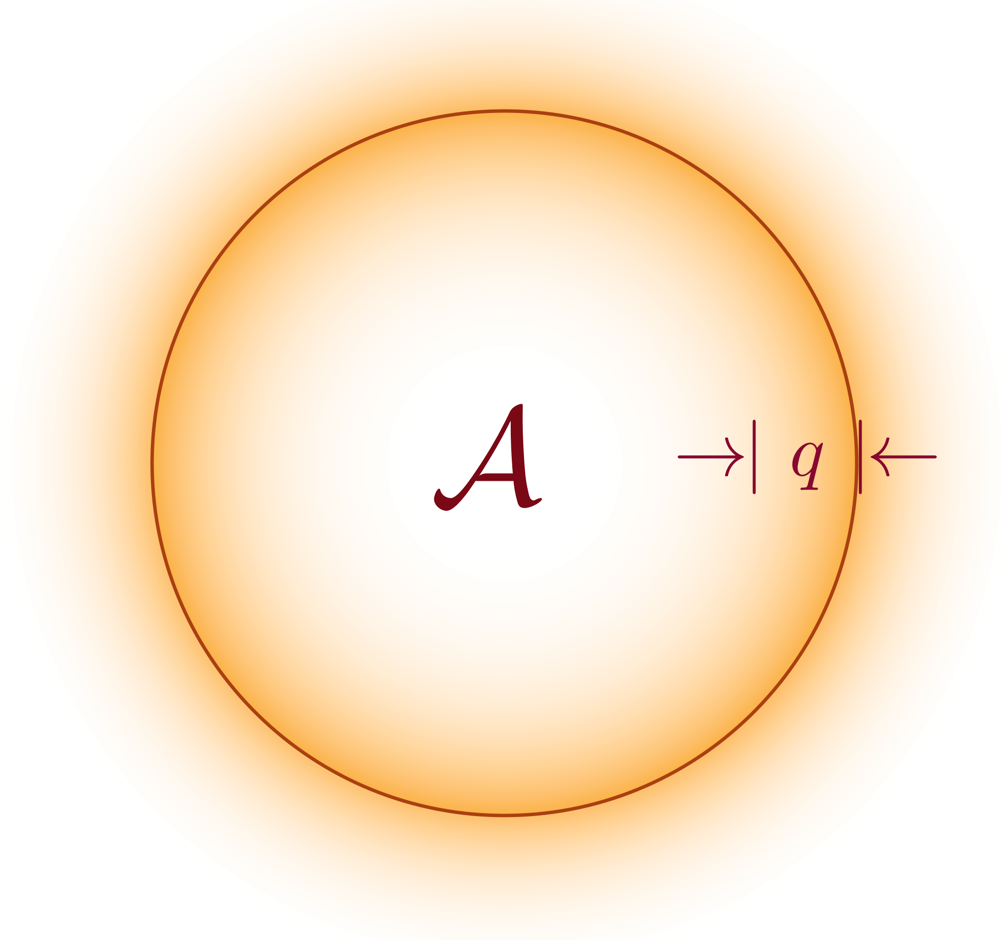

- Fermion Green's function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$. Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.

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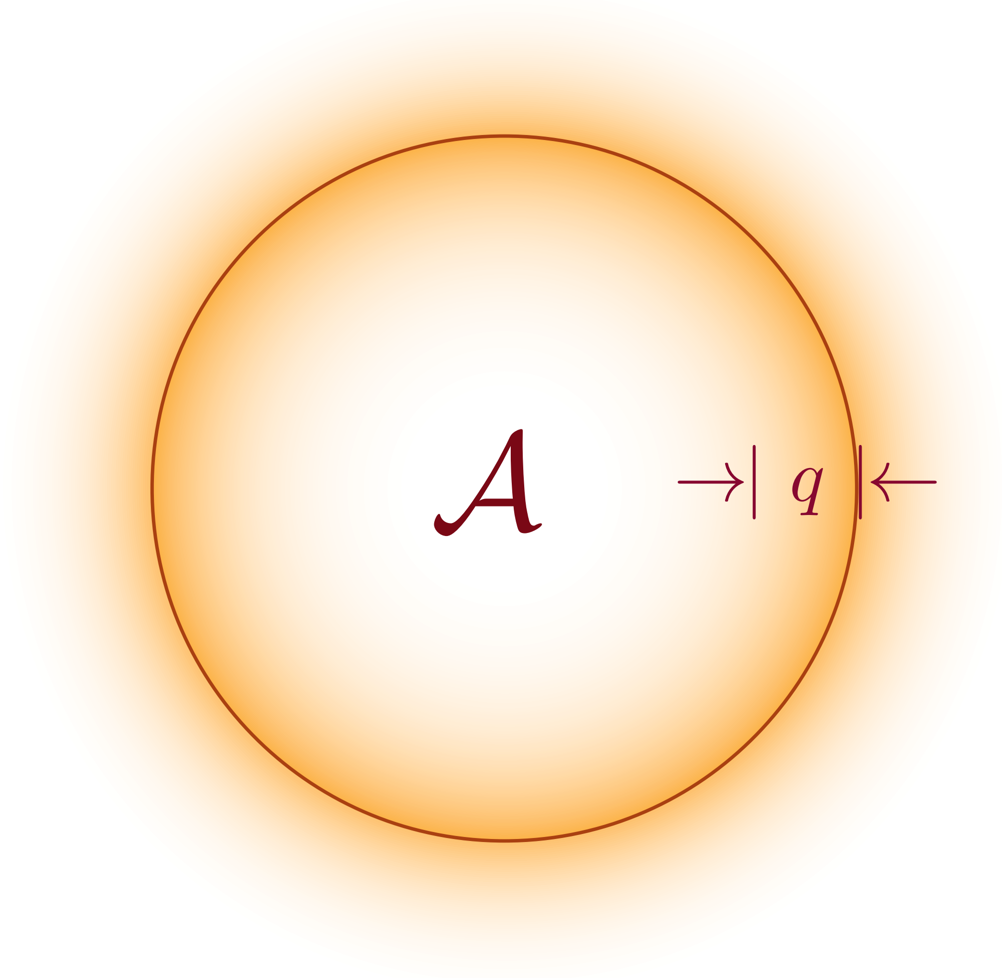

- Fermion Green's function $G_f^{-1} = q^{1-\eta} F(\omega/q^z)$. Three-loop computation shows $\eta \neq 0$ and $z = 3/2$.
- The phase space density of fermions is effectively one-dimensional, so the entropy density $S \sim T^{d_{\text{eff}}/z}$ with $d_{\text{eff}} = 1$.

S.-S. Lee, Phys. Rev. B **80**, 165102 (2009)
M. A. Metlitski and S. Sachdev, Phys. Rev. B **82**, 075127 (2010)
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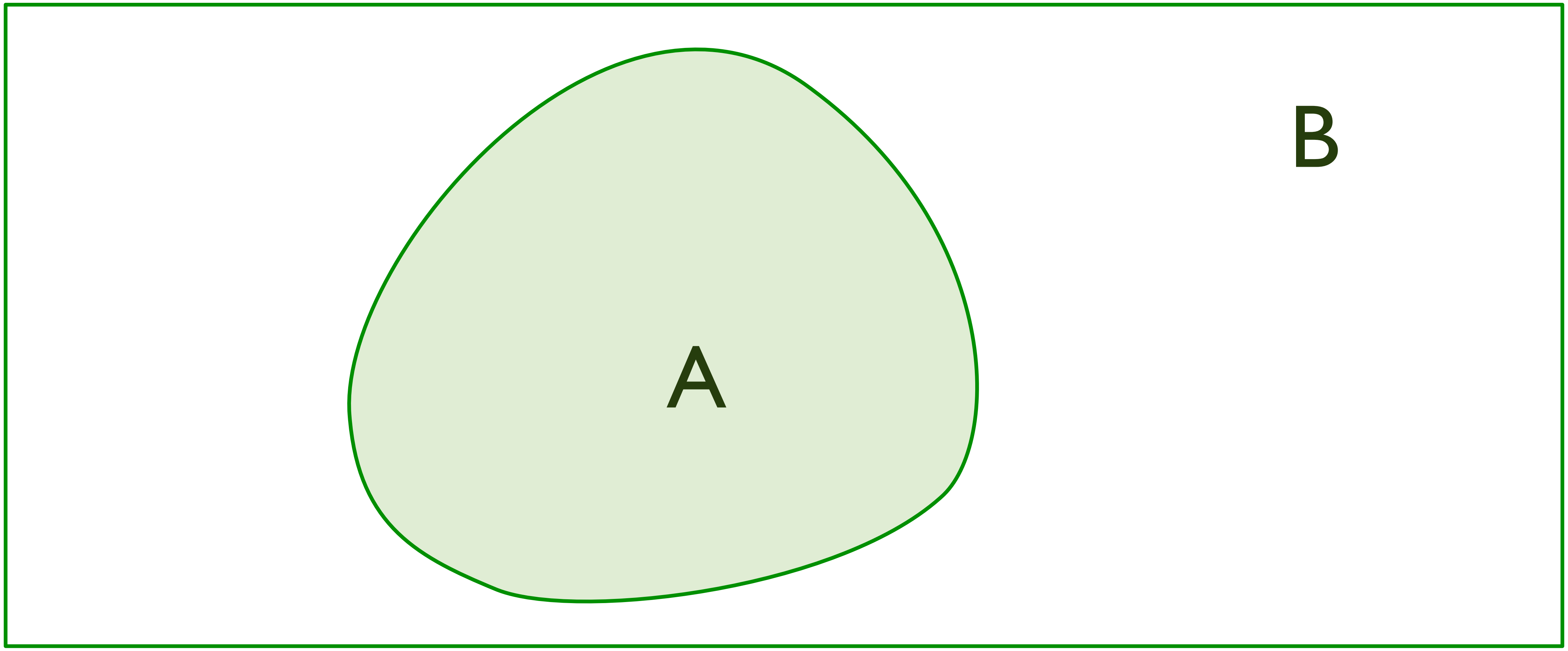
Entanglement entropy

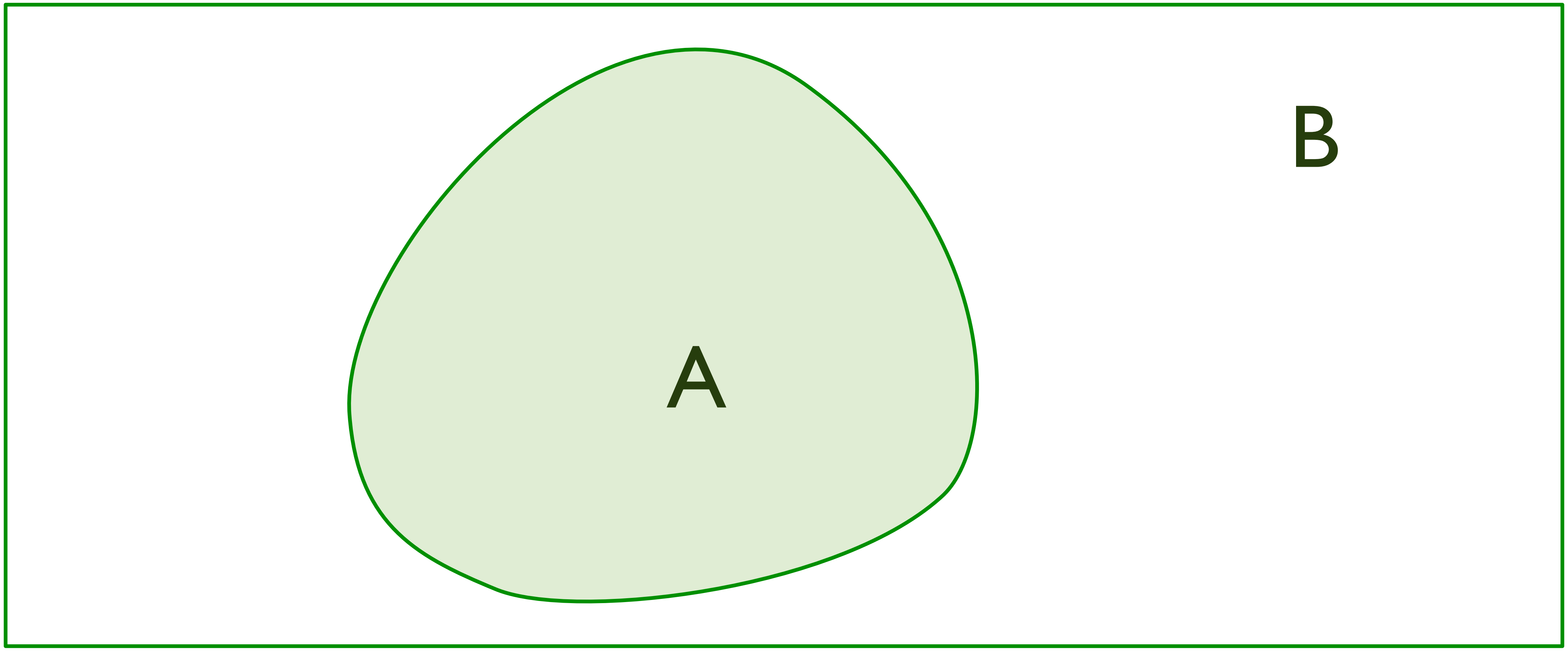


Measure strength of quantum entanglement of region A with region B.

$$\rho_A = \mathrm{Tr}_B \rho = \text{density matrix of region } A$$

$$\textbf{Entanglement entropy } S_{EE} = -\mathrm{Tr}\left(\rho_A \ln \rho_A\right)$$

Entanglement entropy of Fermi surfaces



Logarithmic violation of "area law": $S_{EE} = \frac{1}{12}(k_F P)\ln(k_F P)$

for a circular Fermi surface with Fermi momentum k_F,
where P is the perimeter of region A with an arbitrary smooth shape.

D. Gioev and I. Klich, *Physical Review Letters* **96**, 100503 (2006)
B. Swingle, *Physical Review Letters* **105**, 050502 (2010)

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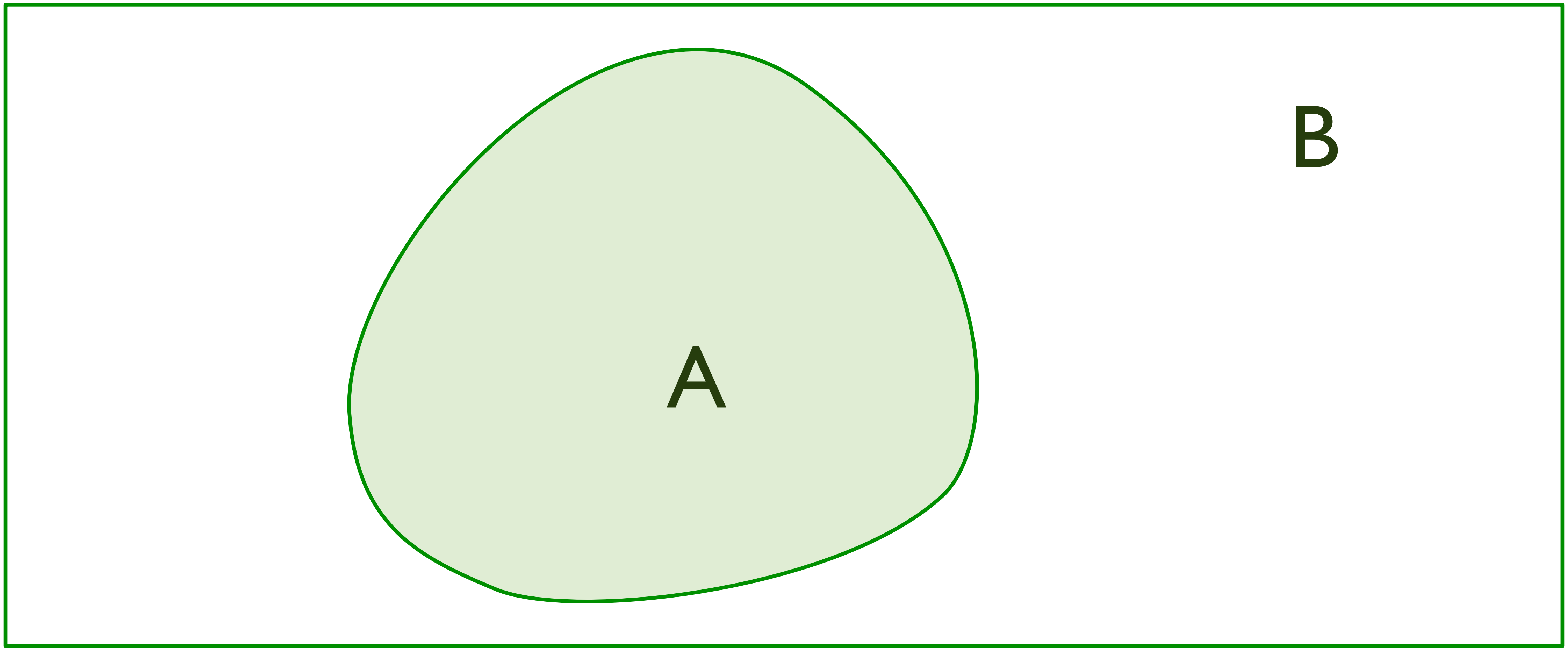

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Non-Fermi liquids have, at most, the "1/12" prefactor modified.

Y. Zhang, T. Grover, and A. Vishwanath, *Physical Review Letters* **107**, 067202 (2011)

Conformal quantum matter

A. Fermi liquids:graphene

B. Holography: Reissner - Nördstrom solution

C. Non-Fermi liquids: nematic critical point (and U(1) spin liquids)

D. Holography: scaling arguments for entropy and entanglement entropy

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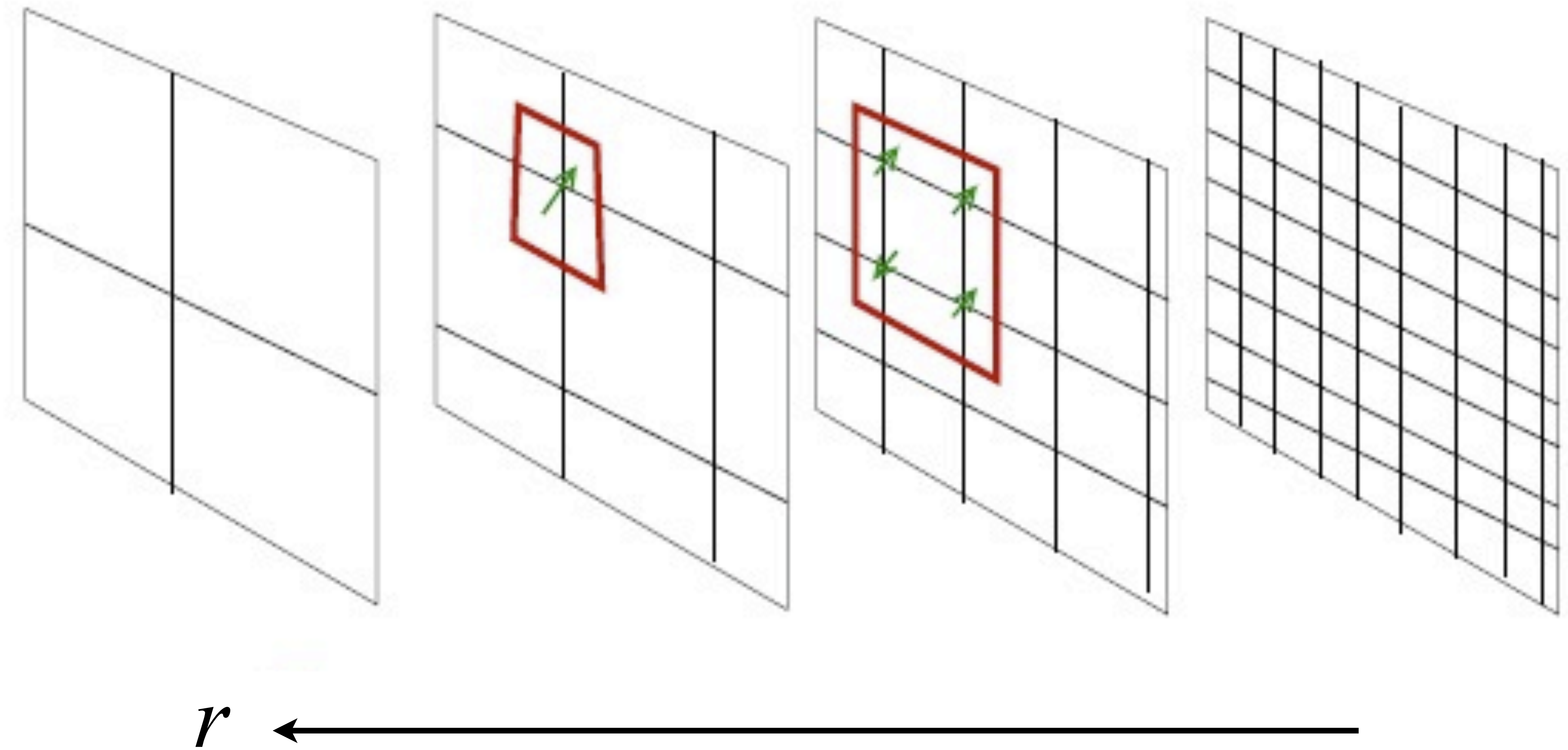
r

Consider the metric which transforms under rescaling as

$$\begin{aligned} x_i &\rightarrow \zeta\, x_i \\ t &\rightarrow \zeta^z\, t \\ ds &\rightarrow \zeta^{\theta/d}\, ds. \end{aligned}$$

This identifies z as the dynamic critical exponent ($z = 1$ for "relativistic" quantum critical points).

θ is the violation of hyperscaling exponent.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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θ is the violation of hyperscaling exponent.
The most general choice of such a metric is

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)} dr^2 + dx_i^2 \right)$$

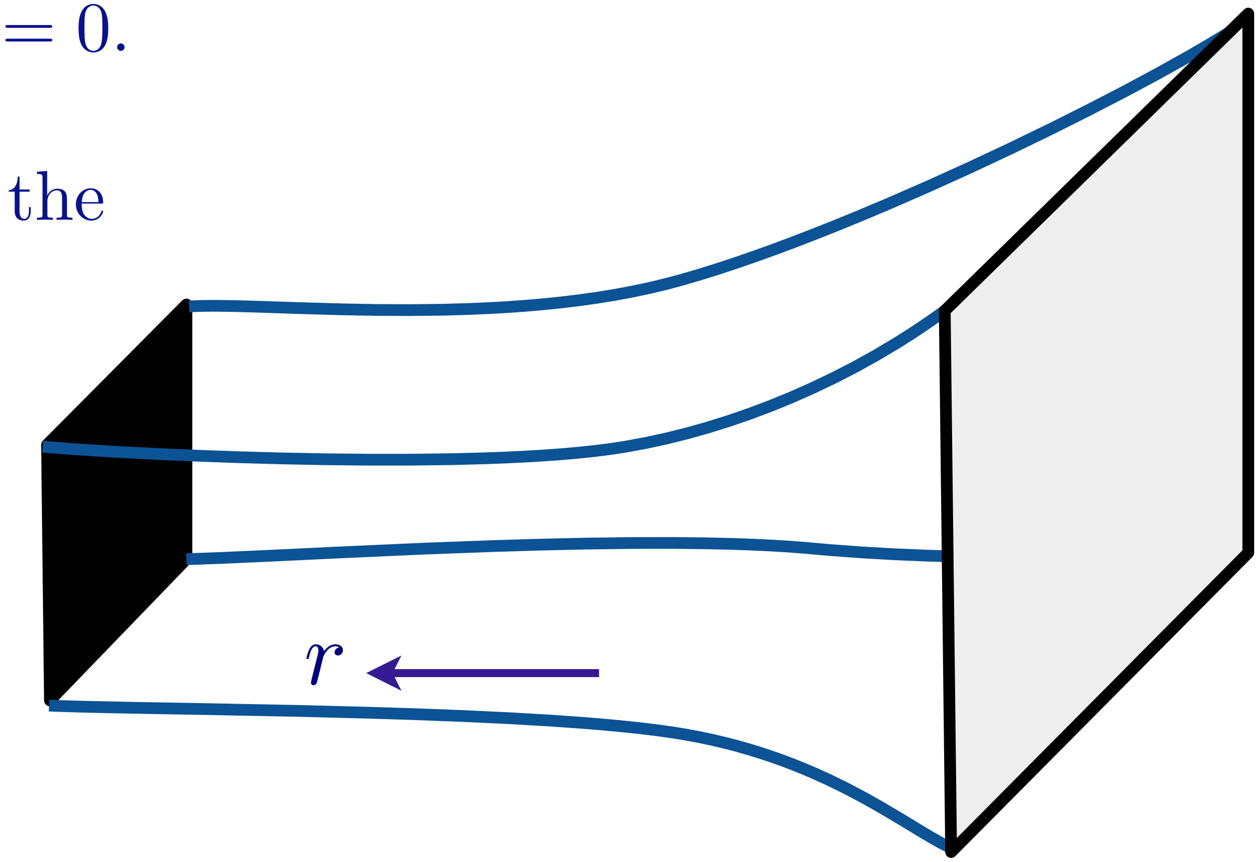
We have used reparametrization invariance in r to choose so that it scales as $r \rightarrow \zeta^{(d-\theta)/d} r$.

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

At $T > 0$, there is a "black-brane" at $r = r_h$.

The Beckenstein-Hawking entropy of the black-brane is the thermal entropy of the quantum system $r = 0$.

The entropy density, S, is proportional to the "area" of the horizon, and so $S \sim r_h^{-d}$



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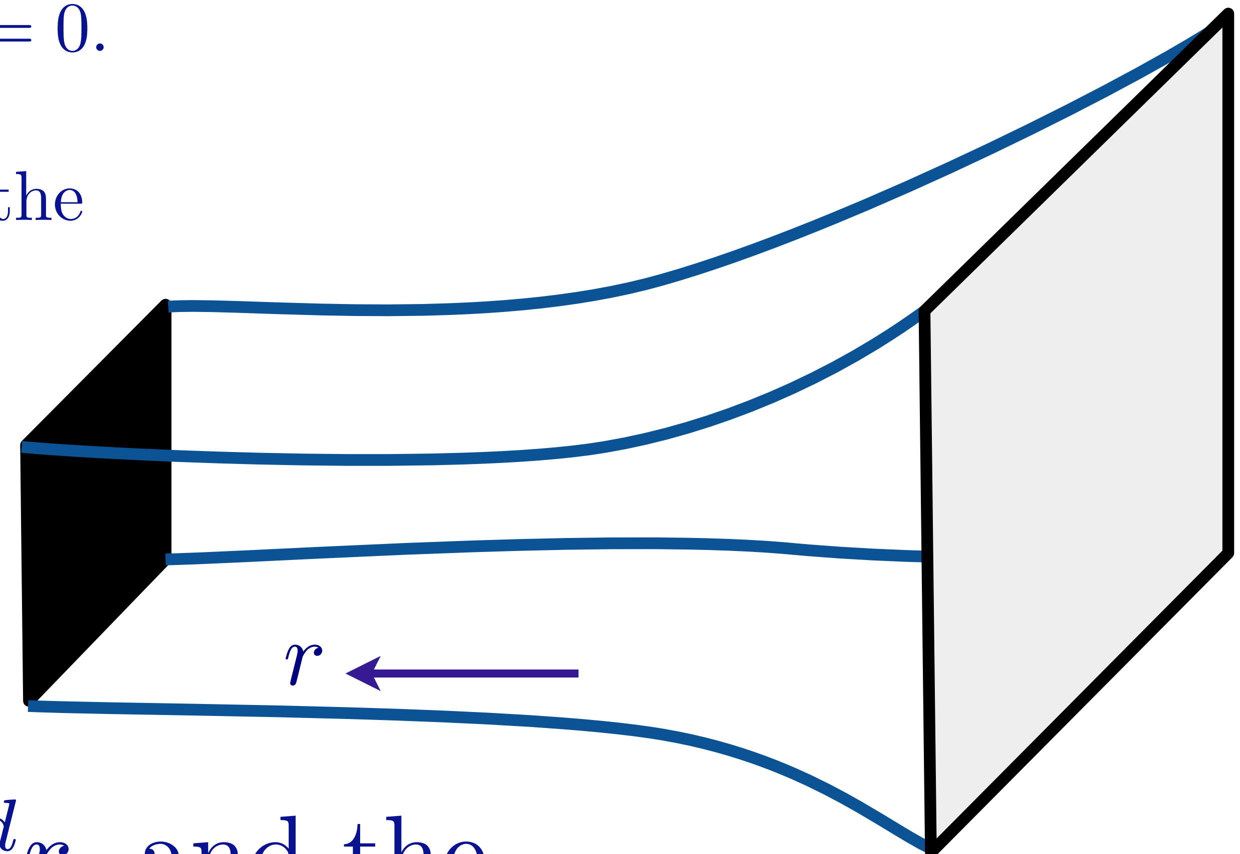


Under rescaling $r \to \zeta^{(d-\theta)/d} r$, and the temperature $T \sim t^{-1}$, and so

$$S \sim T^{(d-\theta)/z} = T^{d_{\rm eff}/z}$$

where $\theta = d - d_{\rm eff}$ measures "dimension deficit" in the phase space of low energy degrees of a freedom.

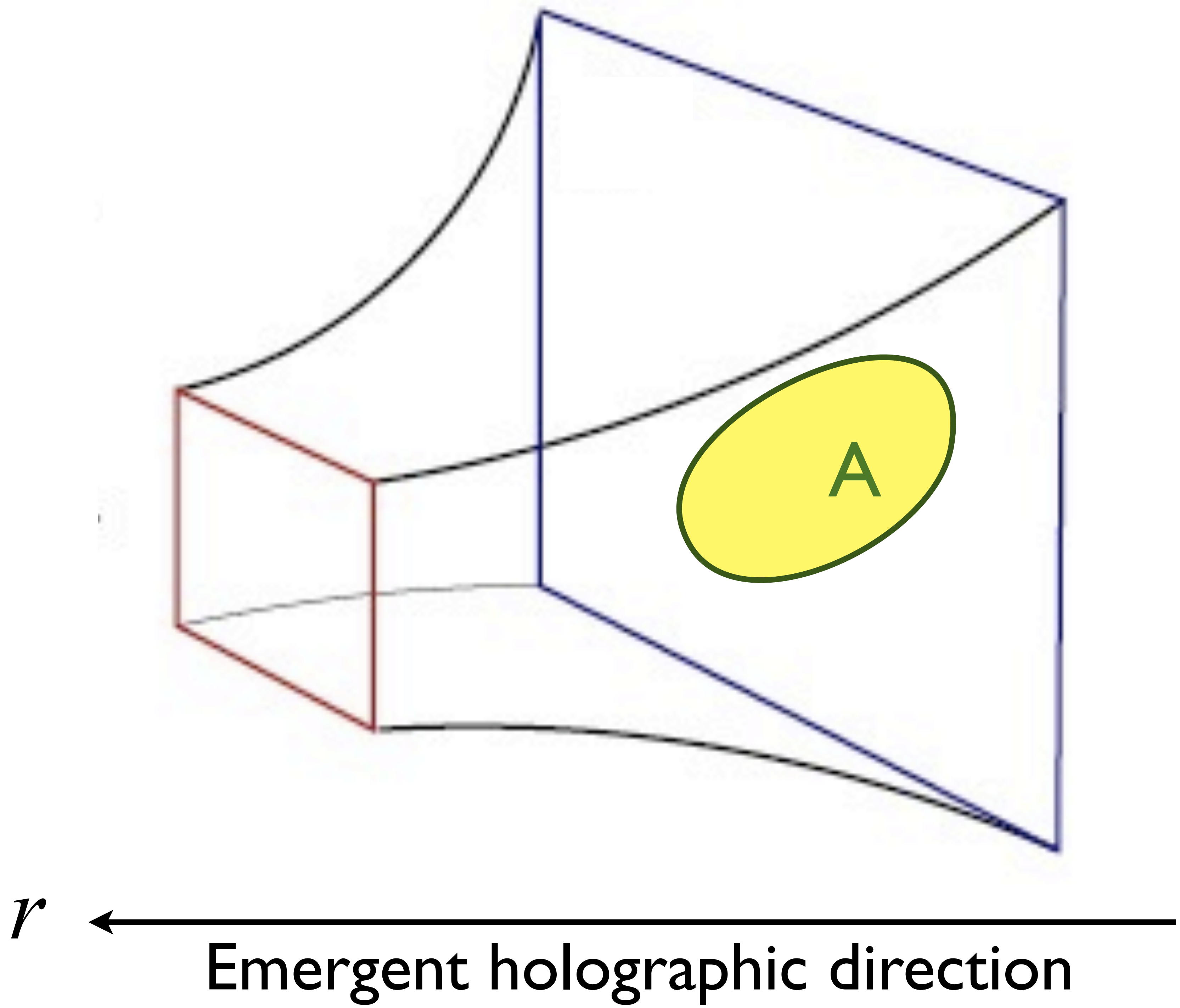
$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

- The thermal entropy density scales as

$$S \sim T^{(d-\theta)/z}.$$

The third law of thermodynamics requires $\theta < d$.

Holographic entanglement entropy



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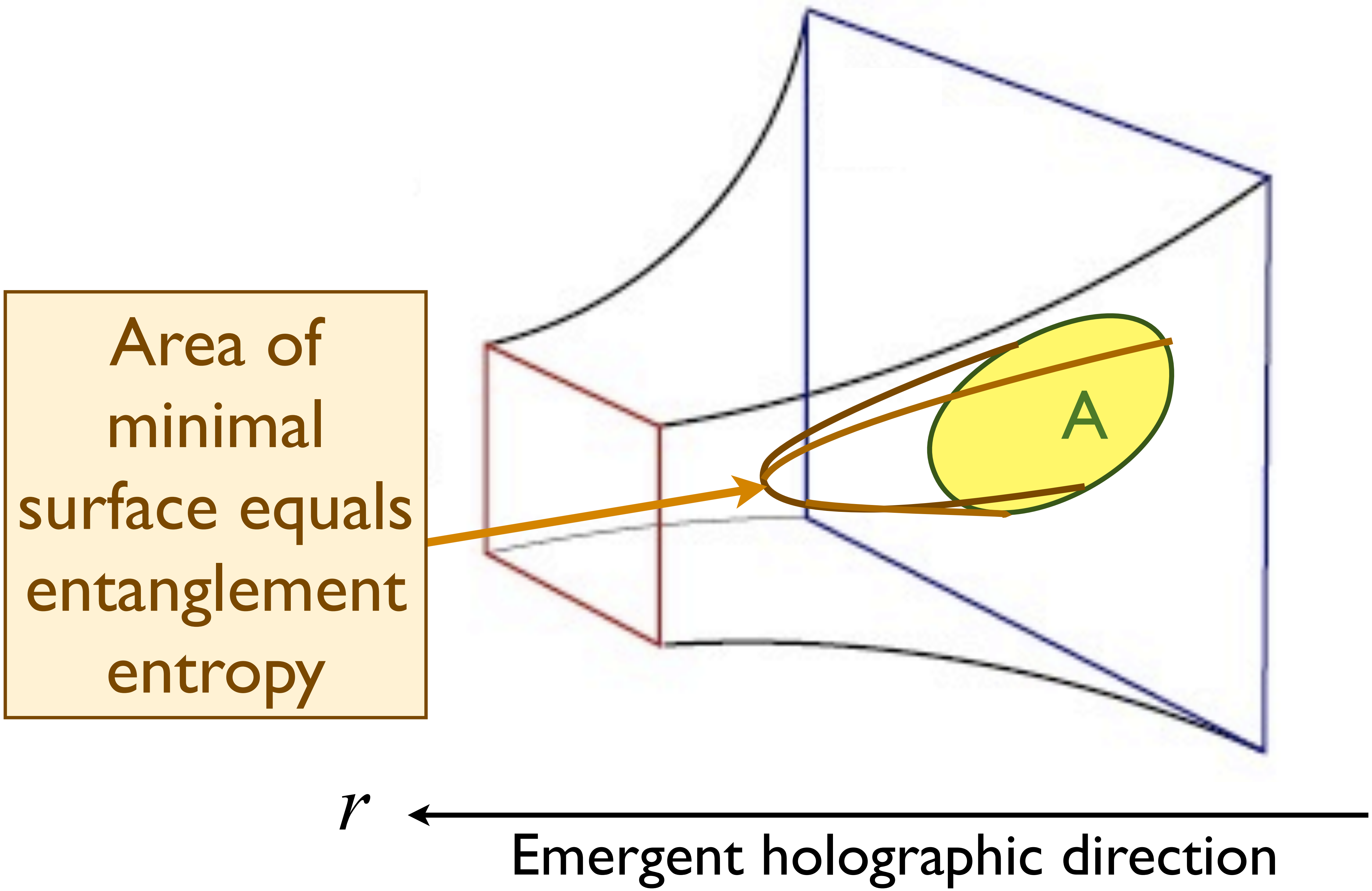

S. Ryu and T. Takayanagi, Phys. Rev. Lett. 96, 18160 (2006).

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  $$S_E \sim \begin{cases} \Sigma & , \quad \text{for } \theta < d-1 \\ \Sigma \ln \Sigma & , \quad \text{for } \theta = d-1 \\ \Sigma^{\theta/(d-1)} & , \quad \text{for } \theta > d-1 \end{cases}$$

  All local quantum field theories obey the "area law" (upto log violations) and so $\theta \le d-1$.

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$\mathrm{AdS}_2\times\mathrm{R}^d$ corresponds to $\theta = d(1-1/z)$ and $z\to\infty$.

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Holography of non-Fermi liquids

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

$$\theta = d - 1$$

- The value of θ is fixed by requiring that the thermal entropy density $S \sim T^{1/z}$ for general d.
  Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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  Conjecture: this metric then describes a compressible state with a *hidden* Fermi surface of *quarks* coupled to gauge fields

- The null energy condition yields the inequality $z \geq 1 + \theta/d$. For $d = 2$ and $\theta = 1$ this yields $z \geq 3/2$. The field theory analysis gave $z = 3/2$ to three loops !

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!

N. Ogawa, T. Takayanagi, and T. Ugajin, arXiv:1111.1023
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

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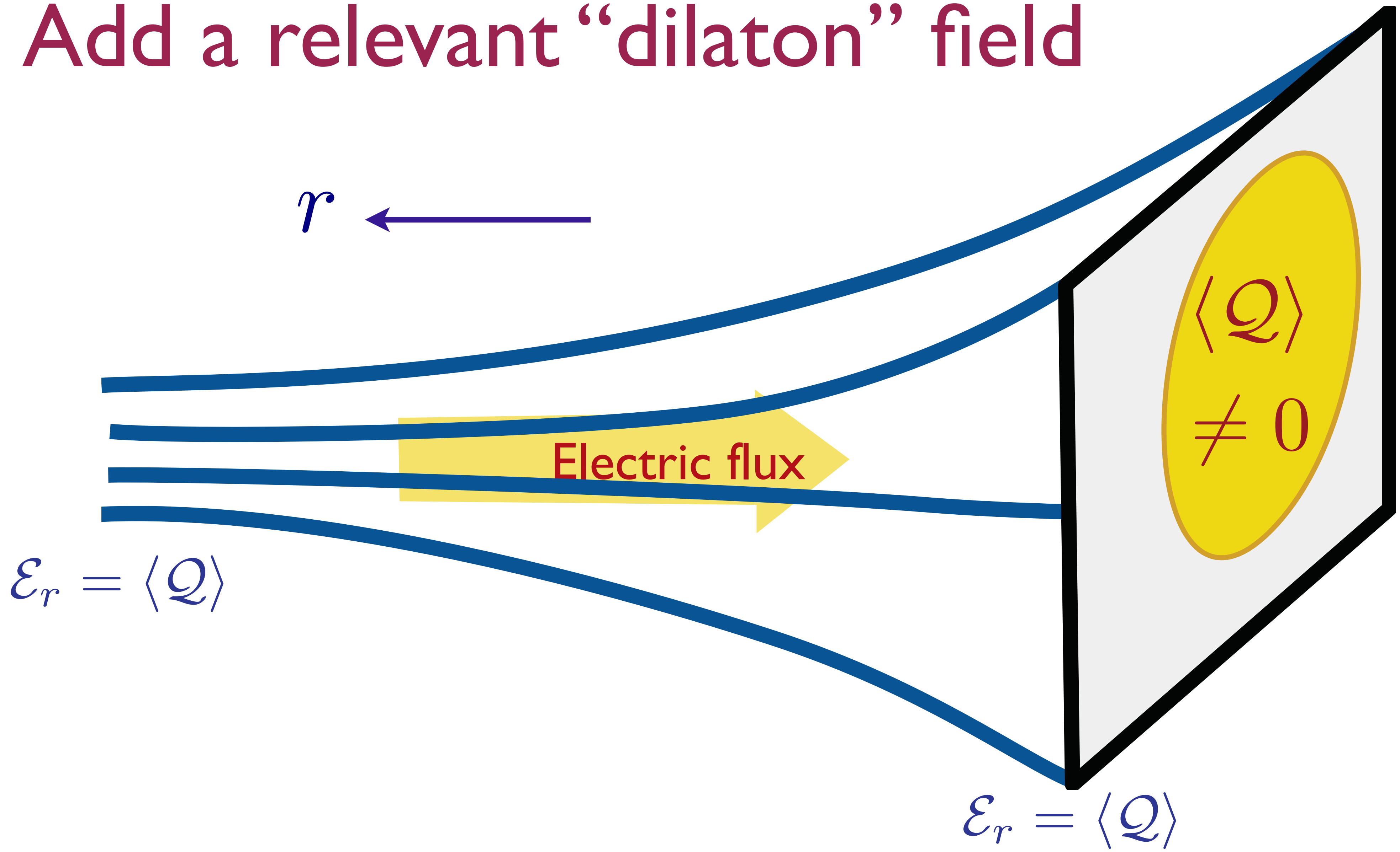
$$\theta = d - 1$$

- The entanglement entropy exhibits logarithmic violation of the area law only for this value of θ !!
- The logarithmic violation is of the form $P \ln P$, where P is the perimeter of the entangling region. This form is *independent* of the shape of the entangling region, just as is expected for a (hidden) Fermi surface !!!

L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holographic theory of a non-Fermi liquid (NFL)

Add a relevant "dilaton" field



$$\mathcal{S} = \int d^{d+2}x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left(R - 2(\nabla\Phi)^2 - \frac{V(\Phi)}{L^2}\right) - \frac{Z(\Phi)}{4e^2}F_{ab}F^{ab}\right]$$

with $Z(\Phi) = Z_0 e^{\alpha\Phi}$, $V(\Phi) = -V_0 e^{-\beta\Phi}$, as $\Phi \to \infty$.

C. Charmousis, B. Gouteraux, B. S. Kim, E. Kiritsis and R. Meyer, JHEP **1011**, 151 (2010).
S. S. Gubser and F. D. Rocha, Phys. Rev. D **81**, 046001 (2010).
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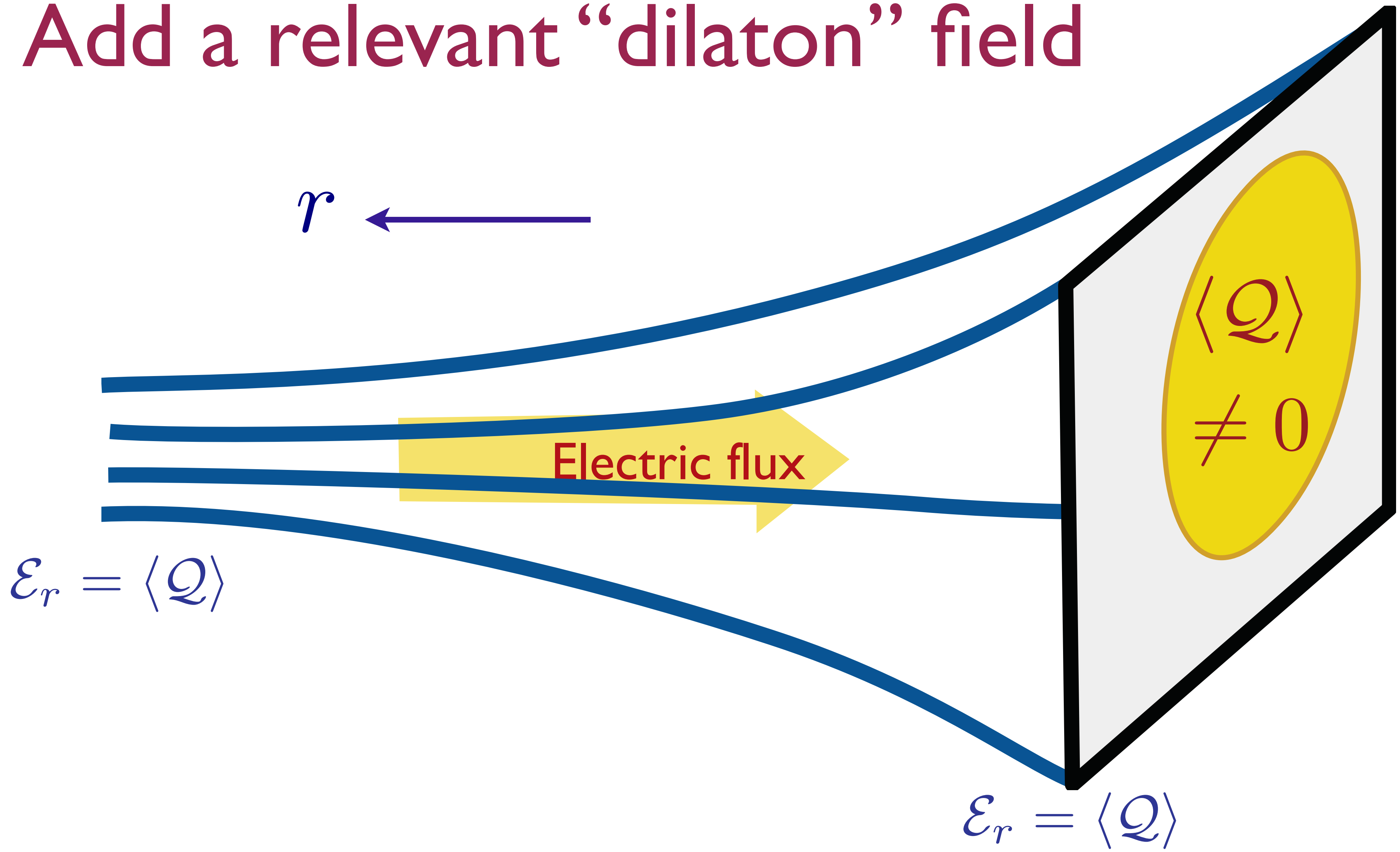

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This is a "bosonization" of the Fermi surface

Holographic theory of a non-Fermi liquid (NFL)

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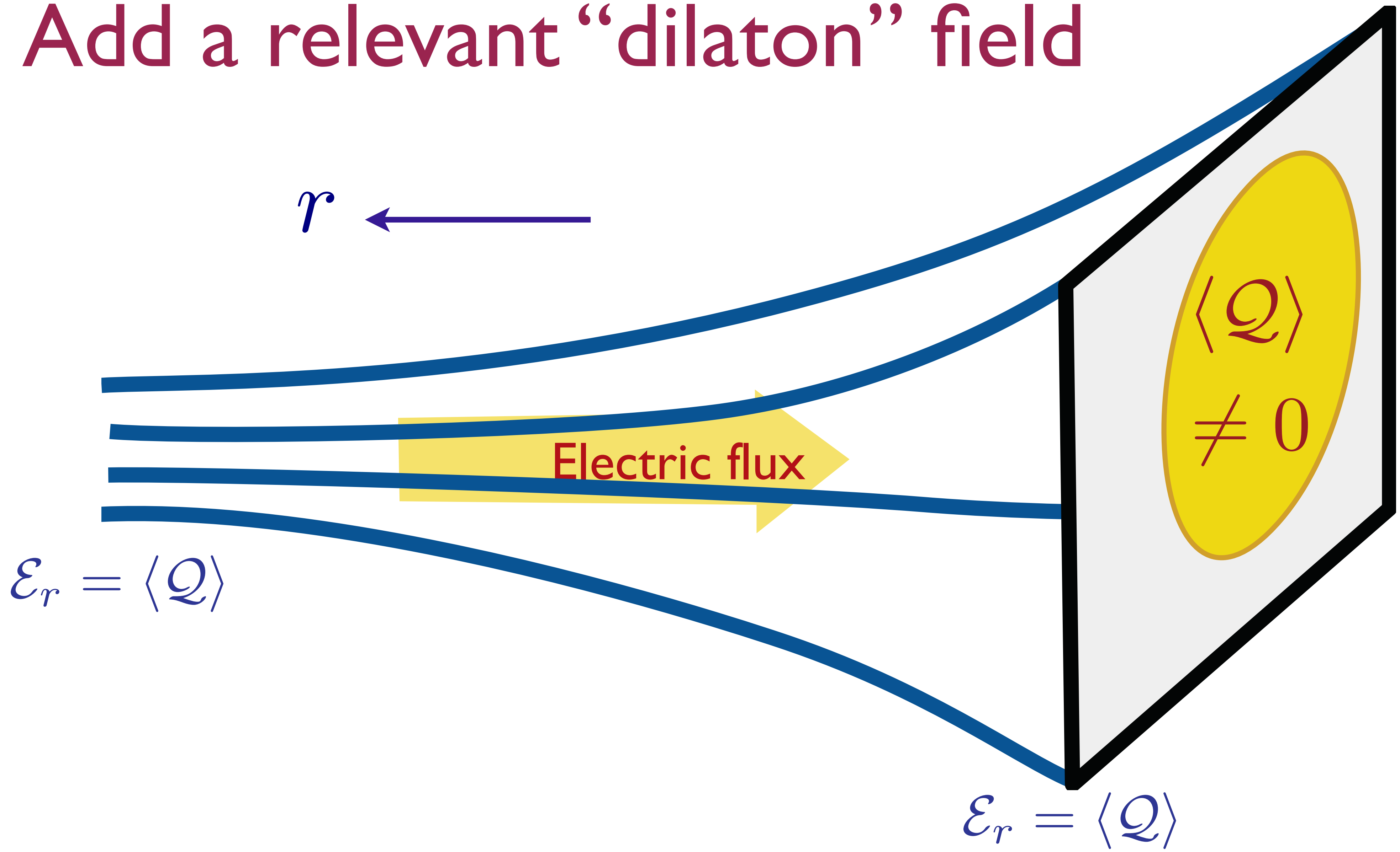

Leads to metric $ds^2 = L^2 \left(-f(r)dt^2 + g(r)dr^2 + \frac{dx^2 + dy^2}{r^2} \right)$

with $f(r) \sim r^{-\gamma}$, $g(r) \sim r^{\delta}$, $\Phi(r) \sim \ln(r)$ as $r \to \infty$.

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$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The $r \to \infty$ metric has the above form with

$$\begin{aligned}\theta &= \frac{d^2\beta}{\alpha + (d-1)\beta} \\ z &= 1 + \frac{\theta}{d} + \frac{8(d(d-\theta)+\theta)^2}{d^2(d-\theta)\alpha^2}.\end{aligned}$$

Note $z \geq 1 + \theta/d$.

Holographic theory of a non-Fermi liquid (NFL)

$$ds^2 = \frac{1}{r^2}\left(-\frac{dt^2}{r^{2d(z-1)/(d-\theta)}} + r^{2\theta/(d-\theta)}dr^2 + dx_i^2\right)$$

The solution also specifies the missing numerical prefactors in the metric. In general, these depend upon the details on the UV boundary condition as $r \to 0$. However, the coefficient of dx_i^2/r^2 turns out to be *independent* of the UV boundary conditions, and proportional to $\mathcal{Q}^{2\theta/(d(d-\theta))}$.

The square-root of this coefficient is the prefactor of the log divergence in the entanglement entropy for $\theta = d - 1$.

Holography of non-Fermi liquids

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$$\theta = d - 1$$

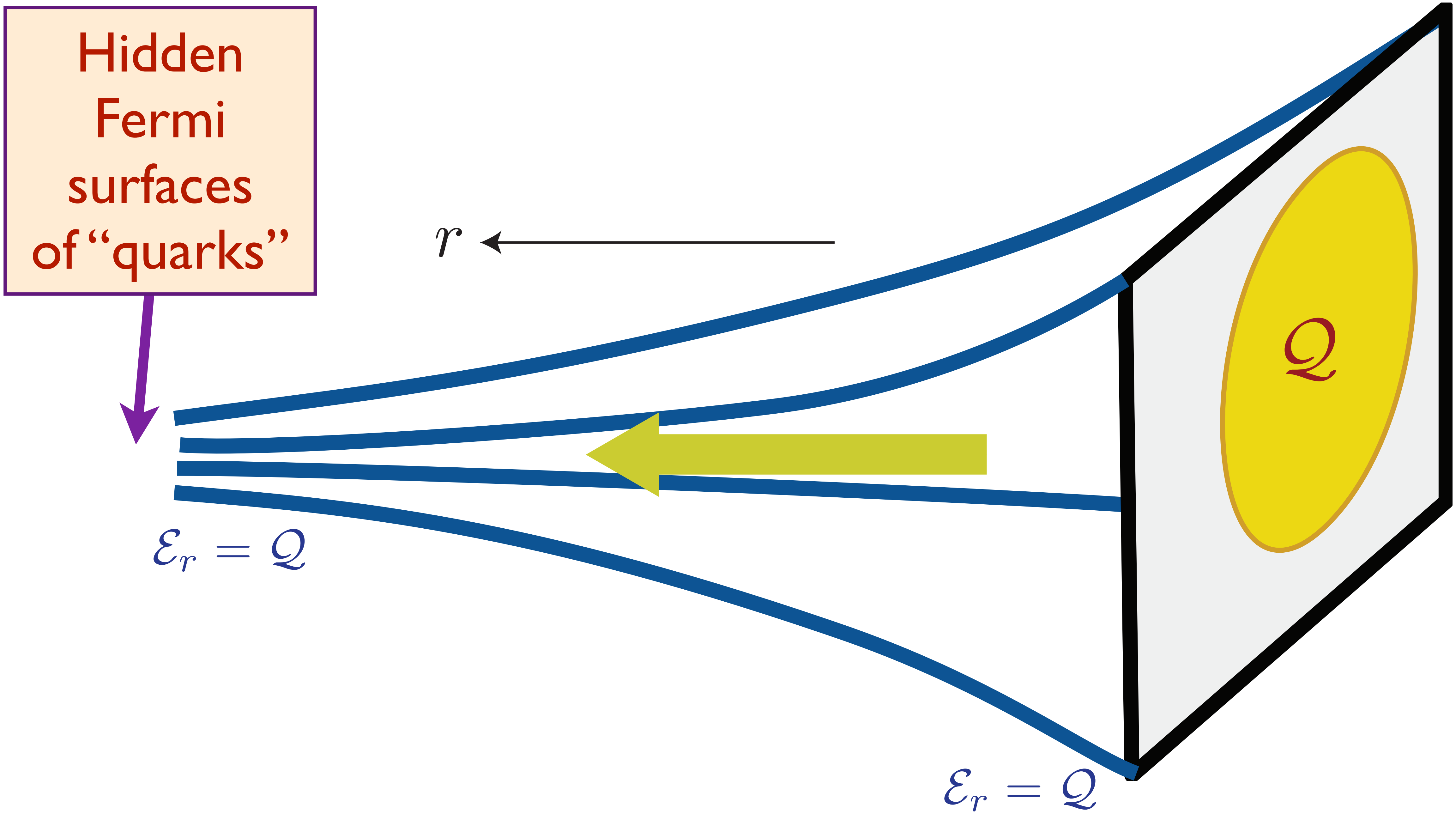
- The entanglement entropy has log-violation of the area law

$$S_E = \Xi\, \mathcal{Q}^{(d-1)/d}\Sigma\, \ln\left(\mathcal{Q}^{(d-1)/d}\Sigma\right).$$

where Σ is surface area of the entangling region, and Ξ is a dimensionless constant which is independent of all UV details, of $\mathcal{Q}$, and of any property of the entangling region. Note $\mathcal{Q}^{(d-1)/d} \sim k_F^{d-1}$ via the Luttinger relation, and then S_E is just as expected for a Fermi surface !!!!

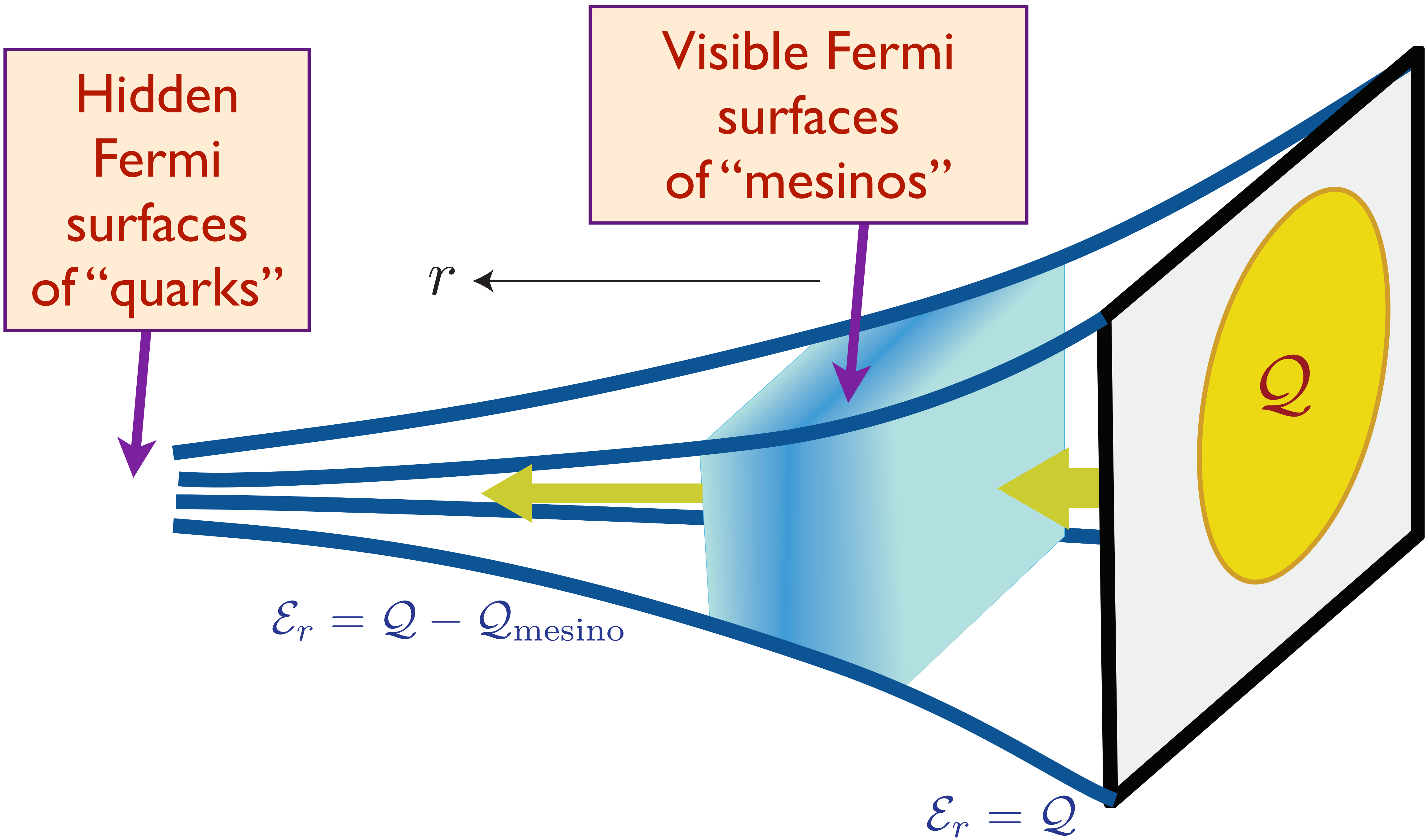
L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

Holographic theory of a non-Fermi liquid (NFL)



Gauss Law and the "attractor" mechanism
$\Leftrightarrow$ Luttinger theorem on the boundary

Holographic theory of a fractionalized-Fermi liquid (FL*)



A state with *partial* confinement

S. Sachdev, *Physical Review Letters* **105**, 151602 (2010)
S. Sachdev, *Physical Review D* **84**, 066009 (2011)

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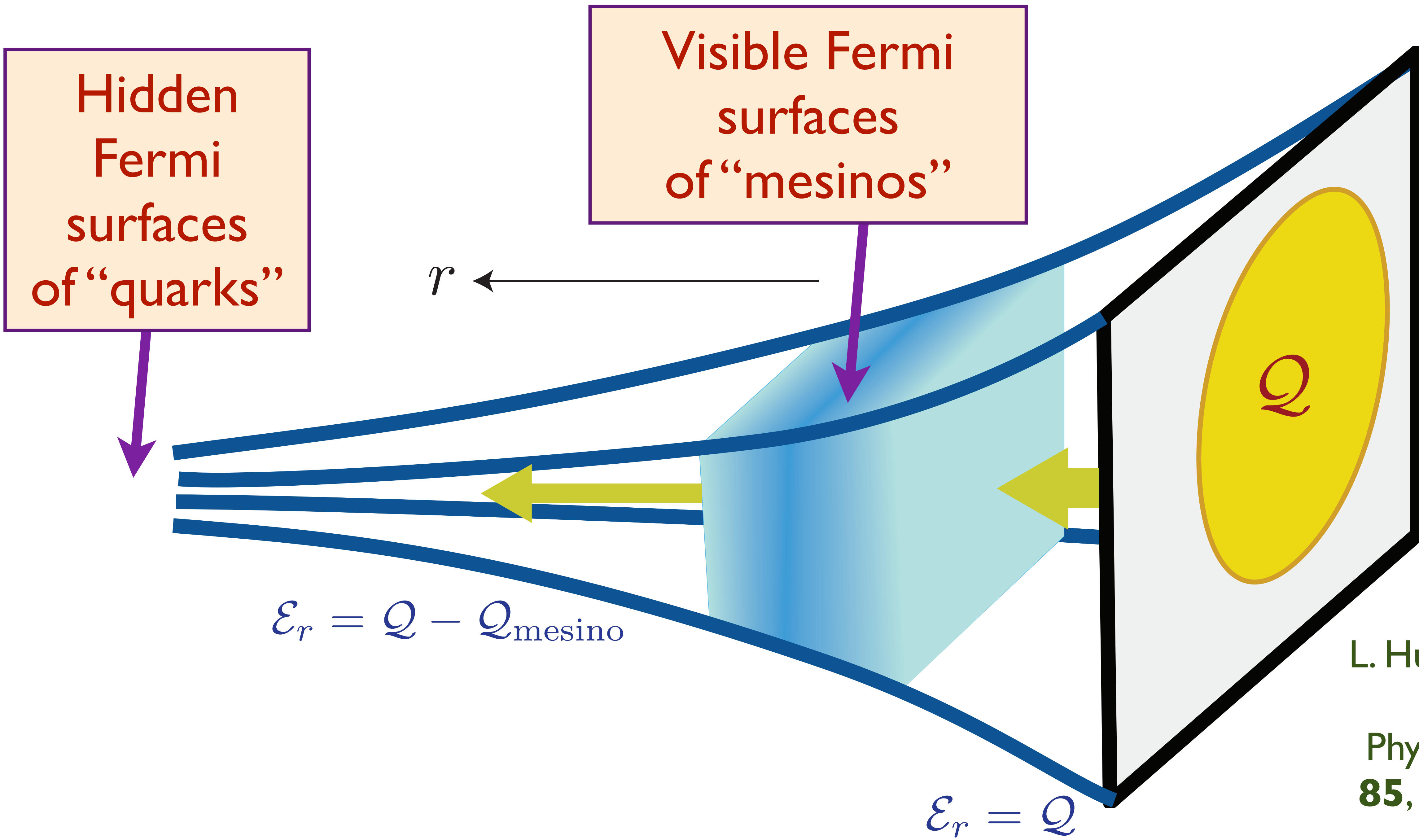
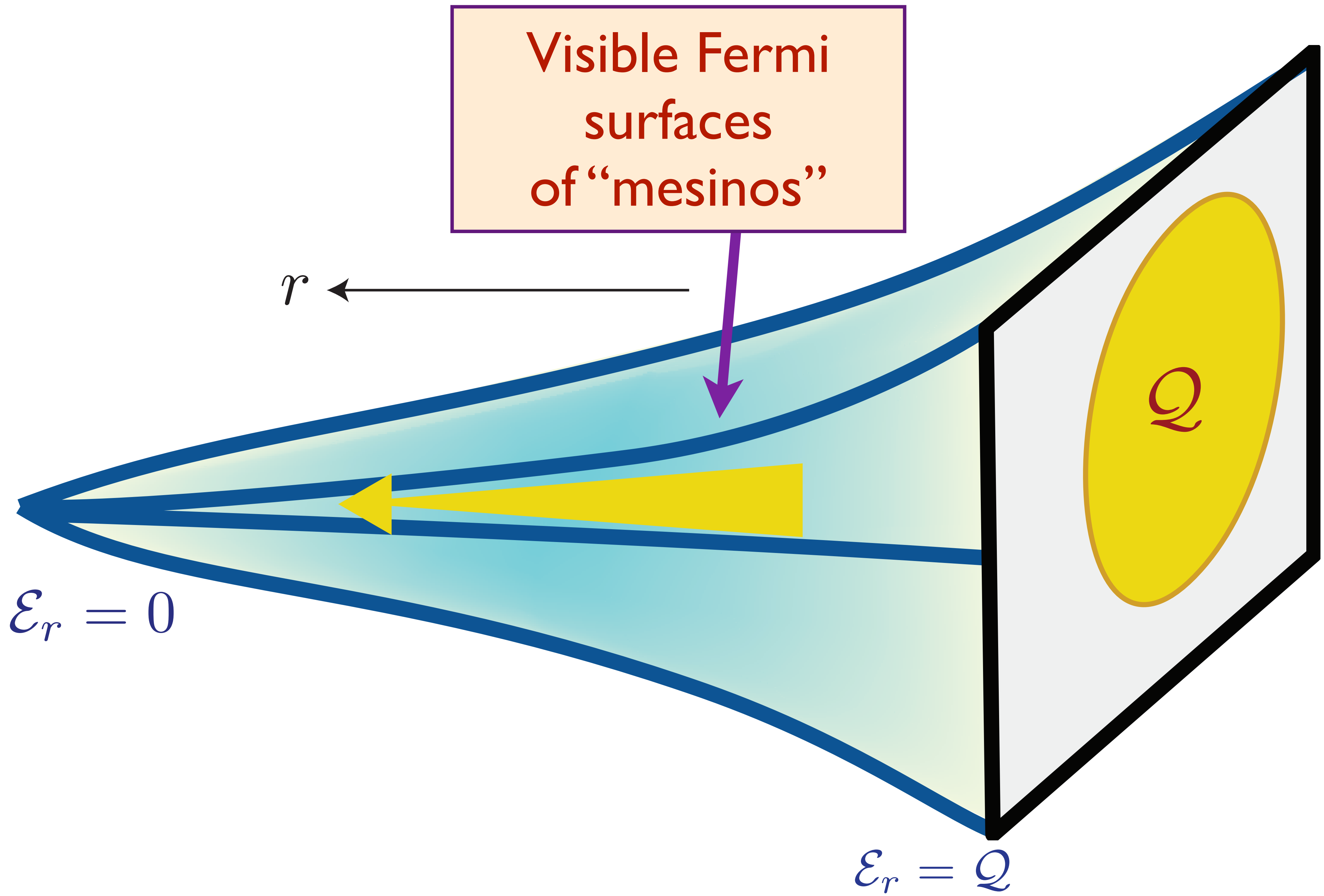


L. Huijse, S. Sachdev, B. Swingle, Physical Review B **85**, 035121 (2012)

- Now the entanglement entropy implies that the Fermi momentum of the hidden Fermi surface is given by $k_F^d \sim \mathcal{Q} - \mathcal{Q}_{\rm mesino}$, just as expected by the extended Luttinger relation. Also the probe fermion quasiparticles are sharp for $\theta = d - 1$, as expected for a FL* state.

Holographic theory of a Fermi liquid (FL)



- Confining geometry leads to a state which has all the properties of a Landau Fermi liquid.

S. Sachdev, Physical Review D **84**, 066009 (2011)

Conclusions

Compressible quantum matter

- Evidence for *hidden Fermi surfaces* in compressible states obtained for a class of holographic Einstein-Maxwell-dilaton theories. These theories describe a *non-Fermi liquid* (NFL) state of gauge theories at non-zero density.

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- Evidence for Luttinger theorem in prefactor of S_E.

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- Hidden Fermi surfaces can co-exist with Fermi surfaces of mesinos, leading to a state with *partial confinement*: the fractionalized Fermi liquid (FL*)