CLERK MAXWELL'S ELECTROMAGNETIC THEORY [1])

When I had the honour to be invited to deliver a lecture in the Rede Foundation, I thought I might perhaps present to you a brief review of the electromagnetic theory of your great physicist JAMES CLERK MAXWELL. The choice seemed the more appropriate as it is now exactly fifty years ago that the work which raised him at once to the very first rank of investigators of all ages, the Treatise on Electricity and Magnetism, was published. In this work it was proved beyond all doubt that electric and magnetic actions can be conceived as being transmitted through a medium and the theory was crowned by the wonderful revelation that light is an electromagnetic phenomenon.

MAXWELL's theory was also a great simplification. Indeed, before his time there was much uncertainty and confusion in this part of physics and many contending theories were in the field. In electrodynamics, for instance, we had the laws of AMPÈRE and GRASSMANN for the actions between elements of current, and, when we went further, we found the speculations of WEBER, RIEMANN and CLAUSIUS about the mutual actions of particles of electricity. In connexion with these theories there was a good deal of discussion on the phenomena that were to be expected in the case of closed and in that of open circuits. It was thought in those days that the current in a wire, by means of which a metallic conductor is charged, ends on that conductor, and even the discharge current of a condenser was considered not to be closed; there was a gap in the circuit, because we had no idea that something is going on in the insulating layer between the coatings.

In optics we had no less trouble. It is true that the general principles of the undulatory theory of light had been firmly established and physicists were justly proud of the success that had been achieved in the explanation of interference and dif-

[1]) The Rede Lecture for 1923, Cambridge University Press.

fraction, double refraction and polarization. Yet, when we tried to penetrate somewhat deeper, we were confronted with serious difficulties. When we wanted to account for the different optical properties of various substances, of air and water for instance, we had the choice between two assumptions. FRESNEL had sought the cause of the difference in an inequality of the density of the ether in the two substances, the elasticity being the same in both, F. E. NEUMANN, on the other hand, had supposed the densities to be the same, but the elasticities to be different. On either of these suppositions, and in no other way, it had been found possible to deduce the right value for the ratio between the amplitude of the reflected and that of the incident light. You know that in this problem two principal cases must be distinguished, the vibrations being normal to the plane of incidence in the one case and parallel to that plane in the other. The two values for the ratio in question are

$$\frac{\sin(i-r)}{\sin(i+r)} \quad \text{and} \quad \frac{\operatorname{tg}(i-r)}{\operatorname{tg}(i+r)},$$

if i is the angle of incidence and r the angle of refraction; and it is remarkable that, of the two rival theories, one led to the expression with the sines when the other required that with the tangents, and conversely. In connexion with this FRESNEL supposed the vibrations of plane polarized light to be at right angles to the plane of polarization, whereas NEUMANN wanted them to be parallel to that plane.

Here was a problem that long baffled the efforts of physicists, and many attempts were made to determine experimentally the direction of the vibrations. One cannot say that the result has been very satisfactory and the question remained open until MAXWELL's theory settled it once for all.

I have enumerated some only of the difficulties with which we had to struggle. I could have mentioned similar problems that arose in the theory of double refraction and I may add that in some cases longitudinal vibrations intruded themselves and complicated the theory.

MAXWELL relieved us of all these doubts and uncertainties. By his bold assumption that in a non-conducting body, in a dielectric as he called it, there can exist what is truly a motion of electricity, and that, if this motion, the dielectric displacement, is taken into account, electricity can always be said to move as an incompressible fluid, the open currents and the longitudinal vibrations that were closely allied to them were made to vanish from the scene. Further, the optical behaviour of non-conducting substances was shown to depend on two properties, each characterized by a physical constant, the dielectric constant, or FARADAY's specific inductive capacity, and the magnetic permeability. It is true that the way in which these constants are determined by the constitution of matter, by the structure of molecules and atoms, was not considered and that, so far, they were no less inaccessible than the ethereal density and elasticity of the old theories, but there was this important difference that, whereas these latter constants had no connexion with any other phenomena, the dielectric constant and the magnetic permeability can be measured by means of statical experiments, so that, at least in certain simple cases, we can deduce the optical properties of a substance from wholly different data. It was found that in the new theory the treatment of the reflexion problem was much like that in the old one; one is led to the two formulae which I recalled to you, if one supposes either the dielectric constant or the magnetic permeability to be the same in the two substances. The choice between these alternative suppositions again entailed a decision concerning the direction of the vibrations with respect to the plane of polarization, but the choice was not doubtful now, as it had been ascertained experimentally that the ratios between the magnetic permeabilities of transparent substances are little different from unity, whereas the dielectric constants diverge to a much greater extent. It was therefore at once established that the electric vibrations are normal to the plane of polarization. This implies that the magnetic vibrations are in that plane, so that, in a sense, the contending parties both had their will.

In the case of crystals it became certain that their double refraction is due to an inequality of the dielectric properties in different directions.

So, many difficulties and outstanding problems melted away as snow before the sun. Indeed, a reviewer in *Nature* actually compared MAXWELL's work to the sun, his only criticism being that there are spots on the sun itself, which, however, „are not visible save to those whose eyes can bear the full glare of the glowing orb." My eyes certainly were not as strong as that. I could not see the spots, but what I could see was that the sun was not entirely unclouded; what sun always was? It was not always easy to grasp MAXWELL's ideas, and one feels a want of unity in his book, due to the fact that it faithfully reproduces his gradual transition from old to new ideas. When we read what MAXWELL says of AMPÈRE and FARADAY, of the former having removed all traces of the scaffolding by which he had built up a perfect demonstration of his law, whereas FARADAY „shews us his unsuccessful as well as his successful experiments, and his crude ideas as well as his developed ones", we feel that, great though the difference may be between the *Experimental Researches* and MAXWELL's largely mathematical Treatise, yet the two works were written in the same spirit. In fact, MAXWELL repeatedly expresses his indebtedness to FARADAY, from whom he had borrowed part of his fundamental ideas, so that, when there is question of MAXWELL's theory, we must often think of FARADAY also.

MAXWELL's followers, of whom there were many, in this country and elsewhere, have perfected the theory in its form and extended it by the introduction of new ideas. Think, for instance, of POYNTING's beautiful and important theorem on the flow of energy, determined at every point by the electric and the magnetic force existing in the field, a theorem that has produced more clearness perhaps than any other and which is now so essential that we can hardly recall the state in which physics was when we did not know it. Yet, notwithstanding all innovations of this kind, we always speak, and with full justice, of „MAXWELL's Theory". We continue to do so now that we have been led to introduce electric charges supposed to exist in the interior of molecules and atoms, by which we have come to the theory of electrons. And when we refer to those wonderfully simple equations in which the fundamental laws of electromagnetism are embodied with a conciseness that could never have been dreamed of before, we call them „MAXWELL's Equations". Surely, though MAXWELL did not use them in their modern form, no name could be more appropriate, for the general relations which they express are those that were constantly in his mind.

Time does not permit me to dwell at length on the verifications of MAXWELL's theory, but I should like to make an exception for two of them.

Allow me, in the first place, to say some words on the optical properties of metals.

„If the medium", so we read in MAXWELL, „instead of being a perfect insulator, is a conductor, the disturbance"(viz. that which is produced by an incident beam of light) „will consist not only of electric displacements but of currents of conduction, in which electric energy is transformed into heat, so that the undulation is absorbed by the medium." After having stated in these words one of the most important consequences drawn from his theory, MAXWELL goes on to calculate the coefficient of absorption as a function of the conductivity, and he proceeds: „Gold, silver and platinum are good conductors, and yet, when formed into very thin plates, they allow light to pass through them. From experiments, which I have made on a piece of gold leaf, it appears that its transparency is very much greater than is consistent with our theory, unless we suppose that there is less loss of energy when the electromotive forces are reversed for every semi-vibration of light than when they act for sensible times, as in our ordinary experiments." Later researches have amply confirmed what MAXWELL says here; obviously, bodies, both conductors and dielectrics, behave in general differently towards rapidly alternating electric forces and towards stationary ones. Yet, HAGEN and RUBENS have been able to show that when, instead of working with visible light, one uses infra-red rays of sufficiently great wavelength, the properties of metals will, in the limit, exactly conform to the theory, if we reckon with the ordinary conductivity.

HAGEN and RUBENS did not measure the amount of radiation that is transmitted through a thin plate but the coefficient of reflexion of a thick mirror. For the case of normal incidence, this coefficient and therefore also the loss of energy, i.e. the quantity that is absorbed by the mirror, can easily be calculated as a function of the conductivity. For the residual rays of sylvin, whose wavelength is 12 μ, and for silver, copper, gold and platinum, the absorbed energy was found to be respectively 1.15, 1.6, 2.1 and 3.5 per cent of the incident energy, whereas it ought to have been 1.3, 1.4, 1.6 and 3.5 per cent according to the theoretical formula.

The agreement became still better when the residual rays of fluorite with a wavelength of 25.5 μ were used. Since, however, for waves of this length the reflexion becomes nearly complete, it was not possible to determine the loss of energy with sufficient precision. HAGEN and RUBENS overcame this difficulty by measuring the emissivity of the different metals, or rather the ratio between this emissivity and that of a perfectly black body at the same temperature, a ratio which, by KIRCHHOFF's law, is equal to that between the absorbed and the incident energy for a beam falling on the metal from the outside, so that it can be calculated by the same formula as this latter ratio. For the four metals just mentioned (at a temperature of 170° C) the ratio in question was found to be (after multiplication by 100) 1.13, 1.17, 1.56 and 2.82. The theoretical values were 1.15, 1.29, 1.39 and 2.96.

These numbers show conclusively that, however complicated things may be for shorter waves, we can calculate the optical properties of metals in the extreme infra-red by means of MAXWELL's equations, simply substituting for the conductivity the value that has been deduced from experiments with constant or slowly alternating currents. This is certainly a most splendid confirmation, the counterpart to the verification, which for gaseous bodies at least has been very satisfactory, of MAXWELL's relation of the dielectric constant to the index of refraction.

The phenomenon of the pressure of radiation may serve as a second example of verification. That a beam of light falling, say in the normal direction, on a mirror exerts on it a pressure proportional to the intensity of the beam was deduced by MAXWELL from his formulae, and he calculated the force that may be expected in the case of sunlight. It lasted a quarter of a century before LEBEDEW succeeded in observing this small force, which, for sunlight, amounts to no more than about a ten millionth part of a gramme weight per cm^2 and which it is therefore difficult to disentangle from other forces that are caused by the surrounding gas, even when this is highly rarefied. Some years later E. F. NICHOLS and HULL repeated the experiment with the utmost care and were able to measure the pressure and to prove that its intensity agrees with MAXWELL's calculation.

We are now quite sure of this phenomenon which has come to play a great part in stellar physics. When we are concerned with very small particles near or in a star, the radiation pressure may very well become greater than the force of gravitation, and it is taken into account by many astronomers in their speculations about the state of heavenly bodies.

The forces exerted by rays of light or heat are a special case of what we call ponderomotive forces, i.e. of the forces with which the electromagnetic field acts on material bodies. MAXWELL showed how, in general, these forces can be deduced from the values of the electromagnetic energy corresponding to different positions of a system of bodies, or from a consideration of certain stresses which exist in the electromagnetic field and of which he taught us to determine the direction and the intensity. Every student, even of rather elementary physics, now knows that the mutual attraction of two conducting plates between which there is a difference of potential, e.g. of the plates of an absolute electrometer, may be considered as due to stresses along the lines of force, that the same may be said of the attraction between a magnetic pole and a piece of iron, and that in the case of an electromagnetic motor we are concerned with the tangential stresses acting at the surface of the revolving system. Here again there has been a great deal of later development, but we continue to speak of „MAXWELL's stresses".

The notion of the electromagnetic momentum, which MAXWELL seems not to have had, though he was quite near it, has also proved very fruitful. A beam of light has a definite momentum, much like a moving ball, and when the beam is normally reflected by a mirror, so that the momentum is inverted, we can deduce the force acting on the mirror from the change of the momentum, exactly as we can do in the case of the ball or of a stream of material particles.

In modern theory — I allude to the theory of relativity — one has found good reasons for combining into one unity, which we call the stress-energy-tensor, all these quantities of which I have spoken, viz. the energy, the flow of energy, MAXWELL's stresses and the electromagnetic momentum. In EINSTEIN's theory of gravitation this tensor determines the gravitation field that is produced by an electromagnetic system and in virtue of which such a system has an influence on the motion of material particles, unfortunately much too small to be observed.

I should be led too far astray if I dwelt on these questions, but what I want to point out is this, that we could never have gone so far if we had contented ourselves with the actions at a distance, if we had not fixed our attention on the intervening medium, localizing the energy in it and considering it as the seat of momenta and stresses which manifest themselves in the observed motions of bodies. All these modern ideas have their origin in MAXWELL's work.

We are also concerned with a stress-energy-tensor, similar to the electromagnetic one, when we consider a system of material particles, whether unconnected like the molecules of a gas, or held together by internal forces as in an elastic body or a fluid. The question naturally arises: are these stress-energy-tensors, the electromagnetic and, let me say, the material one, wholly independent or can one be reduced to the other? One has often tried to do so and, more particularly, to imagine electromagnetic phenomena as produced by some invisible mechanism moving according to the laws of dynamics.

This was a favourite idea of MAXWELL's and one of his most brilliant chapters is devoted to the dynamical theory of electromagnetism. It is the more important because it shows that such a theory can be developed on very general lines, it not being necessary to make definite assumptions regarding the underlying mechanism. MAXWELL showed that in the case of linear circuits carrying electric currents we can account for the ponderomotive forces and for the phenomena of self and mutual induction by LAGRANGE's or HAMILTON's equations of motion, provided that we introduce, besides the coordinates which determine the positions of the material circuits, a certain number of new coordinates, one for each circuit, the velocities belonging to these coordinates for the several circuits being proportional to the current intensities. In fact, the new or „internal" coordinate for each circuit represents the total quantity of electricity that has traversed some section since a fixed instant that is chosen as the origin of time. When the internal coordinates are given this meaning, the magnetic energy becomes the kinetic energy of the system, whereas the electric energy has to be identified with the potential energy and is comparable to the energy of deformation of an elastic body.

While he was applying the laws of dynamics in this very general way, MAXWELL was led to discuss certain phenomena that might perhaps be expected to exist and some of which have been actually observed in our days, though MAXWELL was not able to detect them with the means at his disposal.

In the theory of dynamical systems there are as many velocities as there are coordinates and the kinetic energy is a homogeneous quadratic function of these velocities, in which in general not only the squares but also the products of the velocities appear. When we have one or more circuits carrying electric currents, we can distinguish in the kinetic energy one part that depends on the material velocities only, and this is the kinetic energy of ordinary mechanics, and a second part containing only the velocities corresponding to the internal coordinates; this is the magnetic energy that manifests itself in so many ways. Now, is this all? There would certainly be a third part of the kinetic energy if an electric current consisted in a real motion of some substance along the conducting wire, for, if the wire were moving, say in the direction of its length, with the velocity v and if v' were the internal velocity proportional to the current, the total velocity of the moving substance would be $v + v'$ and in its square we should have the term $2vv'$. One is led to a similar conclusion on other less simple assumptions and so, independently of any special conception, the question arises whether any part of the kinetic energy consists of products of ordinary velocities and strengths of electric currents. MAXWELL thinks this question to be of great importance and deems it „desirable that experiments should be made on the subject with great care."

He then proceeds to examine different ways in which the terms in question might be made to reveal themselves, the first of which he explains as follows:

„If any part of the kinetic energy depends on the product of an ordinary velocity and the strength of a current, it will probably be most easily observed when the velocity and the current are in the same or in opposite directions. We therefore take a circular coil of a great many windings, and suspend it by a fine vertical wire, so that its windings are horizontal and the coil is capable of rotating about a vertical axis, either in the same direction as the current in the coil, or in the opposite direction.

We shall suppose the current to be conveyed into the coil by means of the suspending wire, and, after passing round the windings, to complete its circuit by passing downwards through a wire in the same line with the suspending wire and dipping into a cup of mercury. A vertical mirror is attached to the coil to detect any motion in azimuth.

Now let a current be made to pass through the coil in the direction N.E.S.W. If electricity were a fluid like water, flowing along the wire, then, at the moment of starting the current and as long as its velocity is increasing, a force would require to be supplied to produce the angular momentum of the fluid in passing round the coil, and as this must be supplied by the elasticity of the suspending wire, the coil would at first rotate in the opposite direction or W.S.E.N., and this would be detected by means of the mirror. On stopping the current there would be another movement of the mirror, this time in the same direction as that of the current."

It does not appear that MAXWELL actually tried the experiment; he only says: „no phenomenon of this kind has yet been observed."

Now, if for MAXWELL's coil we substitute a rod of iron, the magnetization and demagnetization of which are comparable to the starting and stopping of a current in the coil, we have exactly the RICHARDSON-EINSTEIN-DE HAAS effect that was really observed by EINSTEIN and DE HAAS and by some other physicists. You know that it amounts to this, that a cylindrical rod of iron suspended in a vertical direction is set rotating, with a sudden jerk, when it is rapidly magnetized or demagnetized. When the magnetization is periodically reversed, the rod is made to oscillate and the amplitude of this motion may be increased by adjusting the frequency of the reversals to that of the free oscillations of the rod.

Whereas MAXWELL seems not to have tried the above experiment with the coil, he tried to observe another effect. A coil, through which a current could be passed and which could be provided with an iron core, was placed in a system rapidly revolving about a vertical axis, the arrangement being such that the coil was free to rotate in the revolving system, so that the axis of the coil could be inclined to different degrees with respect to the vertical axis of rotation. If in the formula for the kinetic energy there were terms of the kind which MAXWELL wanted to detect, there would be a tendency for the coil to place itself with its axis parallel to the axis of rotation; it would behave as a gyroscope and might be called an electromagnetic gyroscope. No trace of a phenomenon of this kind could, however, be observed. I may insert the remark that, if the tendency of which I spoke existed to an appreciable extent, a magnetic needle would, even in the absence of the terrestrial magnetic field, still take a definite position that would be determined by the rotation of the earth; in fact, in virtue of its internal motions the magnetic needle would be comparable to a gyroscopic compass, such as has lately come into use in navigation.

Now that we know the intensity of the EINSTEIN effect we can say with certainty that, if only his instrumental means had been more refined, MAXWELL's experiment would have had a positive result. We can evaluate the magnitude of the effect and we can also calculate that the force which a magnetic needle experiences on account of its internal motions and of the rotation of the earth is thousands of millions of times smaller than the force due to the earth's magnetic field, so that you need not fear from this cause any error in measurements in terrestrial magnetism.

In the experiments just discussed we were concerned with forces acting on the material bodies. MAXWELL next considers cases in which, always on account of the terms in question, not the material system but the electricity contained in it is set in motion. Here he reverts again to the suspended circular coil, and he points out that when a rotation is suddenly imparted to it there could be produced a transient electric current. Similarly, there would be a current, but now in the opposite direction, when the motion of the coil is stopped.

A very simple experiment may serve to give you an idea of these phenomena. We take a cylindrical tumbler, partly filled with water, and set it suddenly in rotation about its vertical axis. Then the friction between the glass and the water will make the fluid rotate likewise, but this will take some time; during a certain period the water will lag behind. Let us next suppose that the motion of the vessel has been kept constant for a sufficient length of time, so that the water has acquired the full angular velocity, and then let the vessel be suddenly brought to rest. It is clear that the circulation of the water will continue for a certain time, until it is exhausted by the friction.

Similar phenomena could be observed with a closed circular tube capable of rotating about its axis, and there must be a corresponding electromagnetic phenomenon if a metallic wire contains something like movable electricity, as we are now in a position to assert, because we have good reasons for believing that an electric current consists in a motion of negative electrons. Though the experiment is much more delicate with electricity than with water, TOLMAN and STEWART have performed it with very satisfactory results. Using a coil with a great number of windings, rapidly rotating about its axis, they were able to observe with a sensitive galvanometer the transient electric current that was produced on stopping the motion by means of a brake. The electrons continued to move over some distance just as the water did in the experiment with the circular tube. The direction of the current showed that the movable particles really have negative charges and the observed deflections agreed with what can be inferred from the ratio between the charge and the mass of the electrons, a ratio that was found for the first time by ZEEMAN and Sir J. J. THOMSON somewhat more than twenty-five years ago and has been repeatedly determined in later years.

No less remarkable than TOLMAN and STEWART's experiments are those made by Mr. and Mrs. BARNETT. They found that a cylindrical rod of iron, rotating about its geometrical axis, becomes thereby magnetized in the direction of its length. Like the EINSTEIN effect, to which it forms a counterpart, this new phenomenon can be predicated on the assumption that magnetization consists in a motion of electrons in the molecules of the metal. This allows us to assign the direction of the effects, but for the sake of truth I must add that both the rotation produced by the magnetization of rods and the magnetization caused by a rotation are, for some reason which we do not yet understand, only about half what the theory of electrons had led us to expect.

In MAXWELL's time the electron was unknown and the mechanism of conduction was even more mysterious than it is now. It was precisely for this reason that he attached so much importance to the phenomena to which I have drawn your attention. He says in this connexion:

„It appears to me that, while we derive great advantage from the recognition of the many analogies between the electric current and a current of a material fluid, we must carefully avoid making any assumption not warranted by experimental evidence, and that there is, as yet, no experimental evidence to shew whether the electric current is really a current of a material substance, or a double current, or whether its velocity is great or small as measured in feet per second.

A knowledge of these things would amount to at least the beginning of a complete dynamical theory of electricity, in which we should regard electrical action, not, as in this treatise, as a phenomenon due to an unknown cause, subject only to the general laws of dynamics, but as the result of known motions of known portions of matter, in which not only the total effects and final results, but the whole intermediate mechanism and details of the motion are taken as the object of study."

MAXWELL did not always express himself so cautiously: at other times he did not shrink from imagining an elaborate mechanical model. All physicists know its principal features. The magnetic energy is considered as a true *vis viva*, the magnetic field being the seat of invisible motions, rotations of small particles about the lines of force. The system of these particles may be compared to a wheelwork, and MAXWELL has to explain how it can be that all the wheels in an element of volume are rotating in the same direction. This shows that the motion is not transmitted directly from one wheel to the next. So MAXWELL is led to assume that, between these wheels of the magnetic field and in contact with them, there are smaller ones which transmit the motion in the manner of friction wheels.

What I called wheels might in reality be spheres capable of turning about axes in any direction and, as MAXWELL showed, a system of this kind is amenable to mathematical analysis. In his image the friction wheels represent what we call electricity; in a conductor we must conceive them to be freely movable, whereas the centres of the larger wheels or balls have fixed positions. It is easily seen how a motion of translation imparted to the friction wheels can give rise to rotations of the larger balls; this is how a current produces a magnetic field. Other phenomena can be explained on the same lines; it is sufficient for this that MAXWELL's equations can be deduced by means of the model.

Similar models have been invented, with more or less success, by other physicists, in such quantities even, that nearly all that could be done has been tried. The outcome of these various attempts is, in my opinion, this: that we must admit the possibility of mechanical representations, a possibility that is already shown by the fact that the formulae of the electromagnetic field can be given the form of the general equations of dynamics. On the other hand it cannot be denied that, when we desire them to be applicable to a comparatively wide range of phenomena, the theories that have been proposed become so complicated that they can give us but little satisfaction. So, I think, the majority of physicists will agree to attach less importance to mechanical models and even to the general dynamical analogies, however helpful these may be to us, than to MAXWELL's equations that summarize so admirably all that is essential.

Will it be possible to maintain these equations? I am not thinking here of the comparatively slight modifications that have been found necessary in the theory of relativity, and by which we explain, for instance, the curvature of a ray of light in a gravitation field. A greater and really serious danger is threatening from the side of the quantum theory, for the existence of amounts of energy that remain concentrated in small spaces during their propagation, to which several phenomena seem to point, is in absolute contradiction to MAXWELL's equations. However this may be, even if further development should require profound alterations, MAXWELL's theory will always remain a step of the highest importance in the progress of physics.

But I must not trespass any longer on your patience. Let me rather thank you for the attention with which you have listened to what I had to say, which was mainly intended as a tribute to the memory of one of the greatest Masters of Science.