antisymmetrized correlator decreasing as an exponent. It is also clear that the smaller the ratio $8T_x/\hbar\gamma$ characterizing the ammeter, the better the separation. If the ratio is rather small, the contribution of the symmetrized current correlator is always dominant.

The problem considered in this section is probably similar to that of the phase breaking of electrons by an electric field. This phase breaking does occur at zero temperature, but for basic reasons, it can hardly be calculated by the substitution of the symmetrized field correlator into the expressions for the phase breaking in a classical fluctuating field. This assertion can be proved by analysing diagrams describing the damping of the 'Cooperon', where a certain class of the diagrams, which is substantial for the phase breaking is zero in the quantum limit at zero temperature due to specific cancellations.

The author is thankful to M Feĭgel'man, S V Iordanskiĭ, V Kravtsov, L Levitov, V Fal'ko, Yu Belousov, and G Blatter for stimulating discussions.

This work was supported by the Russian Foundation for Basic Research (grant No. 96-02-19568) and the Swiss National Foundation.

References

1. Lesovik G B, Loosen R *Pis'ma Zh. Eksp. Teor. Fiz.* **65** 280 (1997) [*JETP Lett.* **65** 295 (1997)
2. Lesovik G B, Levitov L S *Phys. Rev. Lett.* **72** 538 (1994)
3. Eric Yang S-R *Solid State Commun.* **81** 375 (1992)
4. de C. Chamon C, Freed D E, Wen X G *Phys. Rev. B* **51** 2363 (1995)
5. Levitov L S, Lee H, Lesovik G B *J. Math. Phys.* **37** 4845 (1996)
6. Reznikov M et. al. *Phys. Rev. Lett.* **75** 3340 (1995)
7. Schoelkopf R J et al. *Phys. Rev. Lett.* **78** 3370 (1997)
8. Mohanty P, Jariwala E M Q, Webb R A *Phys. Rev. Lett.* **78** 3366 (1997)
9. Landau L D, Lifshitz E M *Statistical Physics* (London: Pergamon Press, 1959)
10. Nyquist H *Phys. Rev.* **32** 110 (1928)
11. Perina J *Coherence of Light* (Dordrecht: D. Reidel Publ. Co., 1985)
12. Lamb W E, Retherford R C *Phys. Rev.* **72** 241 (1947)
13. Casimir H B G *Proc. Koninkl. Ned. Akad. Wetenschap* **51** 793 (1948)
14. Mandel L *Phys. Rev.* **152** 438 (1966)

Exchange effects in shot noise in multi-terminal devices

Ya M Blanter, S A van Langen, M Büttiker

1. Introduction

The interest in shot noise in mesoscopic systems [1] increased considerably over the last decade after it was found that the study of shot noise provides information on the system which is not contained in the conductance [2–4].

For metallic diffusive wires the bimodal distribution of transmission coefficients [5] yields a spectacular 1/3-suppression of shot noise in the low frequency limit with respect to the Poisson value,

$$S(\omega = 0) = \frac{1}{3} eGV . \qquad (1)$$

Here $S(\omega)$ is the Fourier transform of the current–current correlator, $S(t) = \langle \Delta I(t) \Delta I(0) \rangle$, while G and V are the conductance of the wire and the applied voltage, respectively; $\Delta I = I(t) - \langle I \rangle$. Equation (1) was derived theoretically in different ways [6–10] and shown to be insensitive to dephasing at least in the semi-classical theory [11]. After the initial experimental confirmation of suppressed shot noise [12] new experiments [13] followed in an interaction-dominated regime [14] and a regime where shot noise is suppressed by inelastic scattering [6, 15, 16]. A macroscopic metal exhibits no shot noise [6, 17].

Similar results have been found theoretically for a ballistic cavity with chaotic classical dynamics. Transport through such a system can be described under the assumption that the scattering matrix is a random matrix drawn from a circular ensemble [18, 19]. As in the diffusive system, a bimodal distribution of transmission eigenvalues reduces the shot noise of a many-channel cavity below the Poisson value, but with a suppression factor of 1/4 instead of 1/3 [19]. This result does not require phase coherence. However, inelastic scattering further suppresses the shot noise [1].

Other recent developments of the field include experimental investigation of the shot noise in the fractional quantum Hall regime [20, 21], the theory of shot noise in the half-filled Landau level [22], and investigation of the frequency dependence of shot noise, experimentally [23] as well as theoretically [24–27].

Below we are interested in exchange effects in shot noise. The phenomenon considered is known in optics as the Hanbury Brown and Twiss effect [28] and was investigated in mesoscopic conductors in Ref. [29]. We consider a conductor, connected to four reservoirs α, β, γ, and δ at equilibrium (examples are shown in Fig. 1), and discuss three types of experiments. In experiment A the current is incident from the probe β, i.e. $\mu_\alpha = \mu_\gamma = \mu_\delta$; $\mu_\beta - \mu_\alpha = eV$, μ_λ being the chemical potential of electrons in the reservoir λ. In experiment B the current is incident from probe δ: $\mu_\alpha = \mu_\beta = \mu_\gamma$; $\mu_\delta - \mu_\alpha = eV$. Finally, in experiment C the current is incident from both probes β and δ: $\mu_\alpha = \mu_\gamma$; $\mu_\beta = \mu_\delta$; $\mu_\beta - \mu_\alpha = eV$. The current correlation in probes α and γ is measured in all the experiments, $S_j(t) \equiv -\langle \Delta I_\alpha(t) \Delta I_\gamma(0) \rangle$, $j = A, B, C$.

A general analysis of Ref. [29] allows these quantities to be expressed in terms of scattering matrices $s^{\lambda\nu}$, with the indices λ and ν labeling the probes. Thus, for zero frequency and temperature† one obtains

$$\begin{Bmatrix} S_A \\ S_B \\ S_C \end{Bmatrix} = \frac{e^2}{\pi} e|V| \begin{Bmatrix} \Xi_1 \\ \Xi_2 \\ \Xi_1 + \Xi_2 + \Xi_3 + \Xi_4 \end{Bmatrix} , \qquad (2)$$

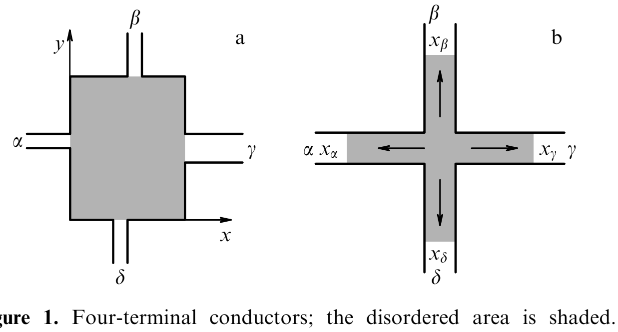

Figure 1. Four-terminal conductors; the disordered area is shaded. Reproduced from Ref. 10.

† We set $\omega = T = 0$ and discuss only the regime linear in voltage V throughout the paper. We also set $\hbar = 1$.

with quantities Ξ_i defined as follows,

$$\Xi_1 = \mathrm{Tr}\,(s^{\dagger\alpha\beta} s^{\alpha\beta} s^{\dagger\gamma\beta} s^{\gamma\beta})\,,$$
$$\Xi_2 = \mathrm{Tr}\,(s^{\dagger\alpha\delta} s^{\alpha\delta} s^{\dagger\gamma\delta} s^{\gamma\delta})\,,$$
$$\Xi_3 = \mathrm{Tr}\,(s^{\dagger\alpha\beta} s^{\alpha\delta} s^{\dagger\gamma\delta} s^{\gamma\beta})\,,$$
$$\Xi_4 = \mathrm{Tr}\,(s^{\dagger\alpha\delta} s^{\alpha\beta} s^{\dagger\gamma\beta} s^{\gamma\delta})\,. \qquad (3)$$

The scattering matrices are evaluated at the Fermi surface, and the trace is taken with respect to channel indices.

Thus, $S_C \neq S_A + S_B$: experiments A and B are not additive due to the interference terms Ξ_3 and Ξ_4. It was shown in Ref. [29] that these terms come with different signs for fermions and bosons; hence we will call them exchange terms, and define the exchange contribution to the current correlation as

$$\Delta S = S_C - S_A - S_B\,.$$

It follows from the unitarity of matrices $s^{\lambda\nu}$ that the quantities Ξ_1 and Ξ_2 (to be referred to below as direct terms) are positively defined. At the same time, $\Xi_3 + \Xi_4$ can have either sign. This means that exchange may either suppress or enhance the direct value.

This is a general result, valid for an arbitrary multiterminal conductor. If one considers a metallic disordered or a chaotic system, it is necessary to average all these quantities over an ensemble of impurity or cavity configurations, respectively. Naively, one might think that due to the phases contained in the quantities Ξ_3 and Ξ_4 these will average to zero, and thus the average of the exchange term $\langle \Delta S \rangle$ vanishes (here angular brackets are used to indicate the corresponding average). Below we present the results of explicit calculations of averaged correlation functions S_j, and demonstrate that this is not the case. The average exchange correlator $\langle \Delta S \rangle$ generally has a nonzero value. For a diffusive system we also provide a simple explanation of this phenomenon. Details of the calculation can be found in Refs [10, 30].

Below we disregard the electron–electron interaction. For the diffusive system, since in the ensemble averaged quantities the effect is local and electron trajectories enclosing a large area are suppressed, the effect is not expected to be sensitive to dephasing. For chaotic cavity, dephasing can be modeled by an additional fictitious voltage lead. It is found that the results presented below do not require phase coherence [30].

2. Diffusive system

We consider a disordered two-dimensional system, connected to reservoirs by ideal leads. The transverse motion of electrons in each lead is quantized, and we assume that all leads are wide, i.e. the number of channels at the Fermi surface in the lead λ is large, $N_\lambda = p_{\mathrm{F}} W_\lambda \gg 1$. Here p_{F} is the Fermi momentum, while W_λ is the width of the lead.

General relations [31] allow one to express scattering matrices for an arbitrary geometry through retarded and advanced Green's functions of the system. The resulting expressions should be averaged with the use of the standard impurity diagram technique [32]. At the intermediate stage one should solve the diffusion equation with boundary conditions appropriate for a corresponding geometry. This program was carried out in Ref. [10], where two particular geometries, shown in Fig. 1, were considered. Before outlining our results, however, we would like to note that the diagrams which give the main contribution to the exchange terms in the current correlation function, can be translated back to the language of electron motion in real space. The typical electron trajectory contributing to these quantities is shown in Fig. 2. The electron motion is essentially a diffusion between different leads with ballistic propagation (described by disorder-averaged single-particle Green's function) close to the leads and somewhere in the middle of the sample. The motion in the center is described by the Hikami-box [33], in which all four points are separated by a distance which does not exceed the wavelength. This means that within the standard treatment of metallic diffusive conductors the trajectories do not enclose any area. This is the reason for the existence of an ensemble averaged effect. If, as one might assume naively, the electrons moving along typical trajectories for the exchange contribution assemble phases, the ensemble average would vanish. The absence of phase accumulation also makes it evident that the ensemble averaged correlations are not sensitive to dephasing.

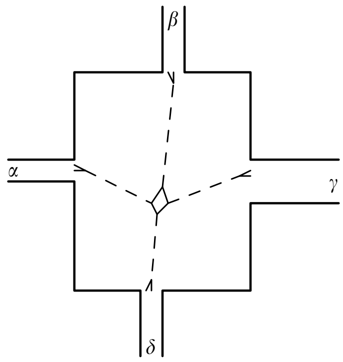

Figure 2. Typical electron trajectories, contributing to the exchange terms in the shot noise. Solid lines denote ballistic propagation (described by averaged single-particle Green's function), and dashed lines denote diffusive propagation (described by the diffusion). Reproduced from Ref. 10.

Now we describe the results for two particular geometries.

2.1 Box geometry

For the geometry of Fig. 1a we assume all the leads to be wide, $W_\lambda \gg l$, l being the mean free path, and then the diffusion equation can be approximately solved. First of all, the results both for direct correlation functions S_A and S_B, the exchange contribution ΔS, and their ratio are not universal, and depend on the geometry of the sample: the widths and positions of the contacts etc.

Furthermore, for the symmetric sample (square $L \times L$, four identical contacts of width W centered at each side of the sample) one obtains [10]

$$\left\{ \begin{matrix} \Xi_1 = \Xi_2 \\ \Xi_3 = \Xi_4 \end{matrix} \right\} = \left\{ \begin{matrix} \eta_1 \\ -\eta_3 \end{matrix} \right\} p_{\mathrm{F}} l \left(\frac{W}{L} \right)^4 , \qquad (4)$$

with positive constants

$$\eta_1 = \frac{1}{2\sinh^3 \pi} (\cosh \pi - 1)(2\pi \cosh \pi - \sinh \pi) \approx 0.21\,,$$

$$\eta_3 = \frac{1}{\sinh^3 \pi} (2\pi \cosh \pi - \sinh \pi) \approx 0.03\,.$$

It is seen that the exchange effect exists, and has a *negative sign* (i.e. the exchange suppresses the result of experiment C in comparison with the sum of the results of experiments A and B). Although the relative value of the effect is $\Xi_3/\Xi_1 \sim 0.1$, it should be clearly observable.

2.2 Cross geometry

We now consider the cross geometry of Fig. 1b, and assume that all arms of the cross have equal lengths L and widths W. For $L \gg W$ one can consider the diffusion as one-dimensional. We also assume that the center of the cross is described by a reflection coefficient R and a transmission coefficient $T = (1 - R)/3$ between any two different arms. Since the area of the cross is negligible in comparison with the areas of the arms, we can rule out the possibility of finding the Hikami box inside the cross, and allow it to be situated only in one of the arms. Then it follows [10] that

$$\Xi_1 = \Xi_2 = \frac{l}{3L} W p_{\mathrm{F}} \frac{3(1+\epsilon^2)+4}{(3+\epsilon)^4},$$

$$\Xi_3 = \Xi_4 = \frac{4l}{L} W p_{\mathrm{F}} \frac{\epsilon - 1}{(3+\epsilon)^4},$$

with the quantity ϵ defined as

$$\epsilon = \begin{cases} 1 + l(LT)^{-1}(1-2T), & T \gg l/L, \\ l(LT)^{-1}, & T \ll l/L. \end{cases} \tag{6}$$

Thus, in the case $T \gg l/L$, when the overall transmission through the sample is governed by the diffusive arms rather than by the center of the cross, one has $\epsilon \sim 1$. The quantities Ξ_1 and Ξ_2 are regular for $\epsilon = 1$, and therefore assume the finite value, $\Xi_1 = \Xi_2 = (5/192)(p_{\mathrm{F}} Wl/L)$. At the same time, the exchange terms Ξ_3 and Ξ_4 are strongly suppressed in the parameter l/L,

$$\Xi_3 = \Xi_4 = \frac{1}{64} \frac{p_{\mathrm{F}} Wl^2}{L^2 T}(1-2T).$$

In the less realistic case $T \ll l/L$ (the transmission is determined by the center of the cross) one obtains $\epsilon \gg 1$. All quantities Ξ_i are small, since now all channels are nearly closed (cf. the situation for two-terminal shot noise [4, 2]), however the exchange terms are additionally suppressed in the parameter ϵ^{-1}.

Thus, in the cross geometry of Fig. 1b the exchange noise $\langle \Delta S \rangle$ is suppressed in comparison with the regular terms $\langle S_A + S_B \rangle$ irrespectively of the transmission properties of the center of the cross. It is also quite remarkable that for the cross geometry the exchange contribution is *positive*, although small: the total effect is enhanced by the exchange.

The results obtained for the cross geometry allow us to make predictions for experiments in real systems. Indeed, we found that the exchange contribution is suppressed strongly with respect to the average noise intensities $\langle S_A \rangle$ and $\langle S_B \rangle$. This result was obtained by assuming that the intermediate scattering, described by the Hikami box, does not occur in the center of the cross, i.e., strictly speaking, for ballistic propagation through the center. One can imagine that the center of the cross is itself a disordered or a chaotic system, and then the entire exchange effect will be determined by properties of the center of the cross. If the motion within the center is diffusive, one can apply the results obtained above for the box geometry. The total exchange effect is expected to be negative. However, since the arms of the cross (which correspond to disordered leads in real experiments) contribute to the intensities $\langle S_A \rangle$ and $\langle S_B \rangle$, but not to the exchange contribution, the latter will still be suppressed, if disorder extends far into the leads. Finally, if the center of the cross is a chaotic cavity (see below), the disordered arms play the role of high barriers, separating the cavity from the ideal leads. In this case the exchange contribution is positive and enhances the effect.

3. Chaotic cavity

We now consider a chaotic cavity, coupled to four reservoirs, each lead carrying N channels. (For simplicity we assume that time-reversal symmetry is broken by a small magnetic field.) Using the uniform distribution on the unitary group of the $4N \times 4N$ s-matrix [18, 19], one can easily average products of four scattering matrix elements [34], as required for the current correlations. One finds [30]

$$\Xi_1 = \Xi_2 = -3\Xi_3 = -3\Xi_4 = \frac{3}{4} \frac{N^3}{16N^2 - 1}. \tag{7}$$

Note that the quantities Ξ_i depend only on the number of channels. The sign of the exchange contribution is negative, i.e. the effect is reduced, as in the case of the disordered box geometry.

The above results hold for ideal coupling of the leads to the cavity. A set of interesting results is obtained if the cavity is coupled to the reservoirs via tunnel barriers with a transmission probability Γ (Fig. 3). In this case the effective scattering matrix of the system is a rational function of the scattering matrix s of the cavity. One can still average the products of this effective scattering matrix for $N \gg 1$, by expanding the fractions, and evaluating each term in the series using the diagrammatic theory of Ref. [35]. For $\Gamma \gg N^{-1}$ one obtains [30]

$$\left\{ \begin{array}{l} \Xi_1 = \Xi_2 \\ \Xi_3 = \Xi_4 \end{array} \right\} = \frac{N\Gamma}{64} \left\{ \begin{array}{c} \Gamma + 2 \\ -3\Gamma + 2 \end{array} \right\}. \tag{8}$$

Thus, for $\Gamma = 2/3$ the exchange effect changes sign: if the barriers separating the cavity from the reservoirs are high enough, the exchange enhances the correlations. In the other limit $\Gamma \ll N^{-1}$ of very weak coupling, the transport is dominated by a single state at the Fermi level, yielding very different results. In this case, the two-terminal shot noise

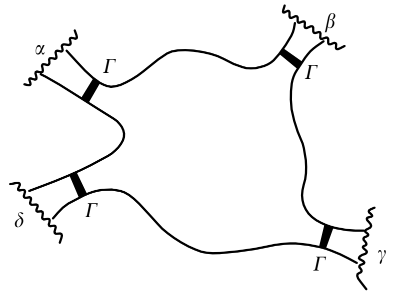

Figure 3. Chaotic cavity connected to four reservoirs via N-channel leads. Tunnel barriers of transparency Γ model the non-ideal coupling of the leads to the cavity. Reproduced from Ref. [30].

vanishes, as it does for resonant tunneling through a symmetric two-barrier system. In the four-terminal configuration of experiment C, the fluctuations and correlations of the current do not vanish [30]. This is purely due to the partition and unification of the noiseless total current from lead β and δ to lead α and γ.

4. Conclusions

In conclusion, we investigated exchange effects in the shot noise of multi-lead devices. We have demonstrated that this effect exists not only in each individual experiment, but also in the average over an ensemble of disordered or chaotic systems. For the disordered case, the explanation is that typical electron trajectories do not enclose any area, and therefore do not average to zero. Furthermore, in the disordered case both the sign and magnitude of the effect are not universal and depend on the geometry of the sample. In particular, for the box geometry shown in Fig. 1a, the exchange suppresses the effect, and the magnitude of the corrections is of the same order as the current correlations themselves. In contrast, in the cross geometry, the exchange enhances the effect, but the correction is small in comparison with the direct terms. For chaotic cavities a universal behavior is found: the exchange terms are equal to one third of the direct ones and have opposite sign, i.e. the total effect is suppressed. The potential barriers installed between the cavity and the ideal leads increase the total effect, and for high enough barriers the exchange enhances current correlations.

This work was supported by the Swiss National Science Foundation (Y. M. B. and M. B.) and the Dutch Science Foundation NWO/FOM (S. A. v. L.).

References

1. De Jong M J M, Beenakker C W J, cond-mat/9611140, in *Mesoscopic Electron Transport* (NATO ASI Ser. E, No. 345, Eds L L Sohn, L P Kouwenhoven, G Schön) (Dordrecht, Boston: Kluwer Academic Publishers, 1997)
2. Khlus V A *Zh. Eksp. Teor. Fiz.* **93** 2179 (1987) [*Sov. Phys. JETP* **66** 1243 (1987)]
3. Lesovik G B *Pis'ma Zh. Eksp. Teor. Fiz* **49** 513 (1989) [*JETP Lett.* **49** 592 (1989)]
4. Büttiker M *Phys. Rev. Lett.* **65** 2901 (1990)
5. Dorokhov O N *Solid State Commun.* **51** 381 (1984)
6. Beenakker C W J, Büttiker M *Phys. Rev. B* **46** 1889 (1992)
7. Nagaev K E *Phys. Lett. A* **169** 103 (1992)
8. Al'tshuler B L, Levitov L S, Yakovets A Yu *Pis'ma Zh. Eksp. Teor. Fiz.* **59** 821 (1994) [*JETP Lett.* **59** 857 (1994)]
9. Nazarov Yu V *Phys. Rev. Lett.* **73** 134 (1994)
10. Blanter Ya M, Büttiker M *Phys. Rev. B* **56** 2127 (1997)
11. De Jong M J M, Beenakker C W J *Phys. Rev. B* **51** 16867 (1995); *Physica A* **230** 219 (1996)
12. Liefrink F et al. *Phys. Rev. B* **49** 14066 (1994)
13. Steinbach A H, Martinis J M, Devoret M H *Phys. Rev. Lett.* **76** 3806 (1996)
14. Nagaev K E *Phys. Rev. B* **52** 4740 (1995); Kozub V I, Rudin A M *Phys. Rev. B* **52** 7853 (1995)
15. Shimizu A, Ueda M *Phys. Rev. Lett.* **69** 1403 (1992)
16. Landauer R *Ann. N.Y. Acad. Sci.* **755** 417 (1995); *Physica B* **227** 156 (1996)
17. Liu R C, Yamamoto Y *Phys. Rev. B* **50** 17411 (1994); *Phys. Rev. B* **53** 7555 (1996)
18. Baranger H U, Mello P A *Phys. Rev. Lett.* **73** 142 (1994)
19. Jalabert R A, Pichard J-L, Beenakker C W J *Europhys. Lett.* **27** 255 (1994)
20. Saminadayar L et al., cond-mat/9706307
21. De-Picciotto R et al., cond-mat/9707289
22. Von Oppen F, cond-mat/9707219
23. Schoelkopf R J et al. *Phys. Rev. Lett.* **78** 3370 (1997)
24. Büttiker M *J. Math. Phys.* **37** 4793 (1996)
25. Naveh Y, Averin D V, Likharev K K, cond-mat/9701095
26. Nagaev K E, cond-mat/9706024
27. Pedersen M H, Van Langen S A, Büttiker M, cond-mat/9707086
28. Hanbury Brown R, Twiss R Q *Nature* **177** 27 (1956); Goldberger M L, Lewis H W, Watson K M *Phys. Rev.* **132** 2764 (1963); Loudon R, in *Disorder in Condensed Matter Physics* (Eds J A Blackman, J Taguena) (Oxford: Clarendon Press, 1991) p. 441
29. Büttiker M *Phys. Rev. B* **46** 12485 (1992)
30. Van Langen S A, Büttiker M *Phys. Rev. B* **56** R1680 (1997)
31. Fisher D S, Lee P A *Phys. Rev. B* **23** 6851 (1981); Stone A D, Szafer A *IBM J. Res. Dev.* **32** 384 (1988); Baranger H U, Stone A D *Phys. Rev. B* **40** 8169 (1989)
32. Altshuler B L, Aronov A G, in *Electron – Electron Interactions in Disordered Systems* (Modern Problems in Condensed Matter Sciences Vol. 10, Eds by A L Efros, M Pollak) (Amsterdam: North-Holland, 1985) p. 1
33. Hikami S *Phys. Rev. B* **24** 2671 (1981)
34. Creutz M *J. Math. Phys.* **19** 2043 (1978)
35. Brouwer P W, Beenakker C W J *J. Math. Phys.* **37** 4904 (1996)