E. VAN EVERDINGEN Jr., *Measurements concerning the dissymmetry of the* HALL-*effect in bismuth, and the mean* HALL-*effect in bismuth and antimony.*

1. Firstly, I tried to find out whether there is a relation between the dissymmetry and the increase of resistance in a magnetic field [1]).

The measurement of the magnetic force was performed with a ROWLAND's proof-plate, just in the manner described by LEBRET in Chapt III § 1, p. 26 of his inaug. dissertation. The values, obtained in this way for five different strengths of the magnetising current were gathered in a graphical representation, by means of which afterwards for each strength of current the magnetic force was interpolated. The distance between the poles was, except on a single occasion, left constant = 13 mM.

With a distance of 16,5 mM. between the poles and a current of 9 ampères I found instead of 3000 c. g. s., 6000 c. g. s. for the magnetic force.

(If we calculate with this value the constant R of HALL from the experiments of LEBRET, we find for bismuth I and II instead of 7 and 13 [2]) respectively 3,5 and 6,5.

In a magnetic field of 6000 c. g. s. VON ETTINGSHAUSEN and NERNST [3]) found for pure bismuth R = 7,3)

[1]) See LEBRET, Dis. p. 92; Comm. N⁰. 19, p. 25.
[2]) „ „ „ p. 102; „ N⁰. 19, p. 31.
[3]) Wied. Ann. 33. p. 474, 1888.

In the following tables we mean by:

M : the magnetic force in c. g. s. units.
S_A : „ HALL-current for the direction of magnetisation A.
S_B : „ HALL-current for the direction of magnetisation B.
S : „ mean HALL-current.
D : „ difference between S_A and S_B.
S_0 : „ secondary current in a zero magnetic field.
$S_{0(c)}$: „ same, corrected for the increase of resistance by magnetisation.

The numbers for the secondary currents are obtained by dividing 1000 by the compensative resistances.

The value of S_0 is added here to allow a judgment of the accuracy in the results for the dissymmetry.

Round plate N⁰. 1.

This is the same plate, which has served in the investigation of LEBRET, described Chapt. VIII § 5 and 6 of his inaug. diss. [1]).

M	S_A	S_B	S	D	S_0	$S_{0(c)}$	Q
1350	3,44	3,94	**3,69**	**0,50**	0,66⁵	0,68	0,17
2700	5,78	7,34	**6,56**	**1,56**	0,66	0,71	0,21
5050	6,94	10,66	**8,80**	**3,72**	0,38	0,45	0,20
6800	6,69	12,23	**9,46**	**5,54**	0,35	0,45	0,19
8600	6,22	13,75	**9,98⁵**	**7,53**	0,34⁵	0,48	0,19

S_0 was in the same direction as S_A.

[1]) Communications N⁰. 19. p. 24.

The last column contains the quotients of D and the numbers, which express according to HENDERSON the increase of resistance in percents.

According to the theory, exposed by LEBRET in Chapt. X § 2 [1]), in the position where the dissymmetry has its maximum:

$$e_A = \left\{ H + \tfrac{1}{2}(K_{11} - K_{22}) \right\} \frac{I}{d}$$

$$e_B = \left\{ -H + \tfrac{1}{2}(K_{11} - K_{22}) \right\} \frac{I}{d} \quad \text{or}$$

$$= \left\{ H - \tfrac{1}{2}(K_{11} - K_{22}) \right\} \frac{I}{d}$$

if we reckon here positive a difference of potential in opposite direction, hence

$$e_A - e_B = (K_{11} - K_{22}) \frac{I}{d} \qquad e_A + e_B = 2H\frac{I}{d}.$$

So D is proportional to $K_{11} - K_{22}$.

With quadratic plates we would be able to determine $K_{11} - K_{22}$ in absolute measure from the above data, where $\frac{e}{I}$ should be calculated as indicated on p. 102 of LEBRET's dissertation.

With round plates it is not allowed to assume the streamlines to run everywhere in the direction of the line uniting the primary electrodes. In order to get nevertheless an idea about the values of K_{11} and K_{22}, we will derive from the experiments the ratio of $(K_{11} - K_{22})$ to H on the supposition, that the HALL-currents remain at least proportional to the terms

[1]) Communications N°. 19. p. 21.

$$\left\{ H + \tfrac{1}{2}(K_{11} - K_{22}) \right\} \frac{I}{d}$$

$$\text{and } \left\{ H - \tfrac{1}{2}(K_{11} - K_{22}) \right\} \frac{I}{d}.$$

Then, we take $H = RM$, taking into account the change of R with the magnetisation. From the above formulae follows immediately:

$$\frac{K_{11} - K_{22}}{H} = 2\frac{S_A - S_B}{S_A + S_B} = \frac{D}{S}.$$

In a magnetic field of 8600 we find $\frac{D}{S} = 0{,}754$.

In a field of 6000, for bismuth I, $R = 3{,}5$, so $RM = 21000$.

For S we find here by interpolation 9,20.

RM is proportional to S, so we find in the field of 8600:

$$H = RM = 21000 \times \frac{9{,}98^5}{9{,}20} = 22792$$

and

$$K_{11} - K_{22} = 0{,}754 \times 22792 = 17185.$$

The resistance of bismuth, which perhaps may be represented best by $\frac{K_{11} + K_{22}}{2}$, is at 20° in a zero magnetic field $1{,}36.10^5$ in c. g. s. units, in a field of 8600 c. g. s. $1{,}88.10^5$.

So we would find here

$$K_{11} = \pm\, 1{,}97.10^5 \qquad K_{22} = \pm\, 1{,}79.10^5.$$

These experiments confirm therefore the results of LEBRET and indicate, that in this plate the dissymmetry varies indeed in the same manner with the magnetic force as the increase of resistance of bismuth does.

Relation between the dissymmetry of the HALL-*effect in bismuth and the state of crystallization.*

2. Already LEBRET has observed, that the dissymmetry may be connected with the planes of slit found in the plates, and tried to obtain a big crystal of bismuth, or crystals regularly grown together, from dealers in minerals. These efforts were continued by me, but without success [1]). In the meantime I have tried to construct plates myself, wherein the crystals should be placed more uniformly side by side, in order to come thus a little nearer to the wished-for end. I succeeded best with the round plate N^0. 2, which showed a very considerable dissymmetry.

This plate was made out of a larger round plate of bismuth, cast of bismuth I in a mould of glass, heated beforehand to ± 300°, which was placed in a sand-bath and after the casting was covered with sand of the same temperature, in order to cause the cooling to take place very slowly.

For the sake of comparing the degree of regularity in the grouping of the crystals, this plate and a quadratic one were polished and heated a moment in dilute nitric acid; the round one looked then wholly equal of tint, except in three little spots, whilst on the quadratic one, that had shown almost no dissymmetry, after the treatment some hundred irregularly shaped and very differently tinted spots were to be seen. Under the microscope the round plate showed in several places parallel lines in two directions almost at right angles, the same over all the plate and on front and back, which directions did not wholly coincide with the axes of symmetry, found afterwards upon it (see the engravings).

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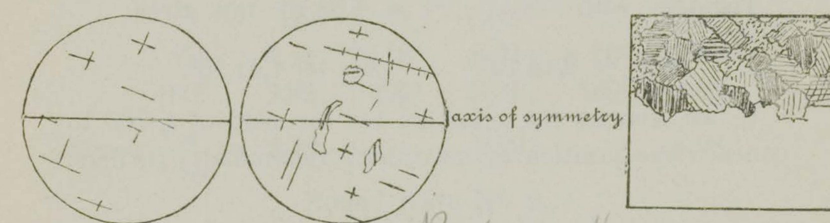
[1]) On a request, directed then to the „Königliches Blaufarbenwerk Oberschlema," this has been so very kind as to send me very fine specimens of crystals grown together, which were received while printing this communication, and give rise to the hope, that a suitable plate may be constructed from them.



N°674 = N°890.

With this round plate I found then in a position of the secondary electrodes near one of the directions of maximum dissymmetry:

M	S_A	S_B	S	D	S_0	$S_{0\,(c)}$	Q
1200	1,79	1,66	**1,72⁵**	**0,13**	1,47	1,51	0,05
5400	9,29	3,67	**6,48**	**5,62**	1,44	1,74	0,27
7400	15,39	2,87	**9,13**	**12,52**	1,28	1,71	0,38
± 13700	29,53	— 8,39	**10,57**	**37,92**	1,10	1,92	0,52

S_0 was again in the same direction as S_A.

The sign — before S_B in the last line indicates, that here the direction of the HALL-current did *not* change sign with the reversal of the magnetic field.

In the position of maximum dissymmetry was found:

8600 21,14 — 1,36 **9,89** **22,50** 2,15 3,01.

As shown by the last column, the dissymmetry increases here much faster with magnetisation than the increase of resistance does.

The last experiment gives $\frac{D}{S} = 2{,}27$.

If we take for H also here 22792, $K_{11} - K_{22}$ becomes 51737.

Together with $\frac{K_{11} + K_{22}}{2} = 1{,}88.10^5$ this gives:

$$K_{11} = 2{,}14.10^5. \qquad K_{22} = 1{,}61.10^5.$$

If we assume the numbers for the field of 13700 to concern the position of maximum dissymmetry we find:

$$\frac{D}{S} = 3{,}59.$$

$$H = 22792 \times \frac{10.57}{9{,}89} = 24359. \qquad K_{11} - K_{22} = 0{,}874.10^5.$$

$$\frac{K_{11} + K_{22}}{2} = 2{,}35.10^5, \quad \text{so}$$

$$K_{11} = 2{,}79.10^5 \qquad K_{22} = 1{,}91.10^5.$$

Several more plates were by means of the same method constructed of bismuth, ordered from MERCK as „purissimum". None of these was formed as regularly as the plate just spoken of, and accordingly some showed no, others only moderate dissymmetry. So from these experiments may be concluded without doubt, that the dissymmetry is related to the state of crystallization.

Variation of the dissymmetry of the HALL-*effect in bismuth with temperature.*

3. Round plate N°. 1.

All observations show that the dissymmetry decreases rapidly with rise of temperature, more rapidly than the HALL-effect itself.

The most trustworthy ones (large dissymmetry and small S_0) give for a magnetic field of 8600:

		D		D		D
	17°	6,44;	12°	8,31;	16°	3,90
	100°	2,92;	100°	3,31;	100°	1,08
Hence $\frac{D_{20}}{D_{100}}$ =		2,13		2,29		1,98

Round plate N°. 2.

M = 5500 (Q has the same signification as formerly.)

T	D	Q
16°	10,59	0,49
± 63°	4,86	0,46
100°	2,87	0,48

$$\frac{D_{20}}{D_{100}} = 3{,}52$$

Round plate N°. 3.

M = 7400

T	D	Q
17°	8,88	0,27
100°	2,84	0,31

$$\frac{D_{20}}{D_{100}} = 3{,}01.$$

With plate N°. 2 also a series of observations was made in a magnetic field of 6000 c. g. s. between — 70° and + 20°. Since the increase of resistance with magnetisation at temperatures below 0° is not known, S_0 was made as little as possible and it was ascertained, that this current remained always weak. In order to reduce the importance of little variations in the resistance of the secondary circuit, 3 Ohms were added to this circuit, the usual resistance being $\pm$ 1 Ohm.

The addition of a correction to S_0 for the increase of resistance would tend to increase the values for the dissymmetry. Here follow some numbers obtained by interpolation.

M	T	S_A	S_B	S	D
6000	— 70°	— 3,60	8,50	**2,45**	**12,10**
6000	— 50°	— 1,94	8,06	**3,06**	**10,00**
6000	— 30°	— 0,47	6,97	**3,25**	**7,44**
6000	— 10°	+ 0,73⁵	5,67	**3,20**	**4,93⁶**
6000	+ 17°	+ 1.59	4,69	**3,14**	**3,10**

At the lower temperatures both HALL-currents are again in the same direction. S_0 was about 0,2.

$$\frac{D}{S} \text{ at } -70^0 \text{ almost } = 5.$$

H is there $21000 \times \frac{2{,}45}{3{,}14} = 16380$, so $K_{11} - K_{22} = 0{,}819.10^5$.

In a zero magnetic field the resistance $\frac{K_{11} + K_{22}}{2}$ at $-70^0 = 1{,}22\,10^5$.

If we estimate the increase of resistance in a field of 6000 c. g. s. at $-70°$ 50 perCt. (at 100^0 it is 7 perCt., at 16^0 24 perCt.) $\frac{K_{11} - K_{22}}{2}$ becomes $1{,}83\,10^5$ and we would find:

$$K_{11} = \mathbf{2{,}24}.10^5. \qquad K_{22} = \mathbf{1{,}42}.10^5.$$

For the plates N^{os}. 2 and 3 the proportionality between

dissymmetry and increase of resistance, as we see above, remains true tolerably well between 16^0 and 100^0.

In order to display still better the variation of the dissymmetry with temperature in plate N^o. 2, we multiply the numbers of the last series by 4, to allow for the larger resistance of the secondary circuit, and reduce the numbers 10,59 and 2,87, in a field of 5500, to 12,40 and 3,36 in a field of 6000.

We find then for a same magnetic field of 6000:

T	100^0	16^0	-70^0
D	3,36	12.40	48,40.

Variation of the mean HALL-*current in bismuth with temperature for different values of magnetic force.*

4. During the experiments on variation of dissymmetry with temperature in plate N^o. 1 it was observed at the same time, that between the limits of temperature occurring thereby (10^0 and 100^0) the mean HALL-current did not decrease sensibly with heating. This looked contrary to the results of LEBRET, who found $\frac{S_{100}}{S_{20}} = 0{,}668$ in bismuth I and $= 0{,}656$ in bismuth II, always with quadratic plates; the round ones he used for the experiments on dissymmetry only.

The experiments spoken of were made in a field of 8600.

Later experiments in a field of 5500 however *gave* a decrease of mean HALL-current with rise of temperature.

I have investigated then whether the strength of the magnetic field exercised influence on the variation with temperature of the HALL-effect and obtained the following results.

Round plate N°. 1.		N°. 2.		N°. 3.	
M	$\frac{S_{100}}{S_{20}}$	M	$\frac{S_{100}}{S_{20}}$	M	$\frac{S_{100}}{S_{20}}$
7600	0,893	8600	0,728	7600	0,706
5500	0,783	5500	0,680	4800	0,700
500	0,658	1400	0,582	1400	0,575

These numbers are calculated from single observations, so the accuracy is not great. If we are allowed to draw conclusions from experiments with round plates in the same manner as with quadratic plates, they show clearly, that the relative *increment* of the HALL-effect with *fall* of temperature *increases*, when the magnetic field *grows weaker*. This may be expressed also as follows: At higher temperatures the HALL-current *increases more rapidly* with magnetisation than at lower temperatures.

Almost the same we observe with the electric conductivity of bismuth.

The *increment* of conductivity with *fall* of temperature *increases*, when the magnetic field *grows weaker*.

At higher temperatures the conductivity *decreases less rapidly* with magnetisation than at lower temperatures.

In the fields of 7600 and 1400 the ratio $\frac{r_{100}}{r_{20}}$ (r = resistance) is 1,10 and 1,34 respectively.

If we derive from the above tables the values in the fields of 7600 and 1400 for the three plates, and multiply the thus obtained numbers by 1,10 and 1,34 we find:

M	N°. 1.	N°. 2.	N°. 3.
	$\frac{S_{100}}{S_{20}} \times \frac{r_{100}}{r_{20}}$		
7600	0,982	0,781	0,777
1400	0,904	0,780	0,770

It appears from the experiments of HENDERSON, that in strong magnetic fields the conductivity of bismuth has a maximum value between 0° and 100°, which maximum for weaker fields wanders to lower temperatures. Very likely there will be also a maximum conductivity in a field of 6000 c. g. s., but at temperatures *below* 0°. Also here then the similarity with the HALL-effect reveals itself; this too reaches a maximum value at low temperatures [1]).

A variation in the rate of decrease with temperature

[1]) LEBRET reached this maximum with bismuth II only. My experiments at low temperatures however indicate for bismuth I a maximum at about — 30°.

of the mean HALL-current with altered magnetisation is observed also by CLOUGH and HALL in nickel [1]).

If this variation is found back also in quadratic plates, it will be possible perhaps to state a relation between the conductivity and the variation of the HALL-effect in bismuth [2]). To this and other questions I hope to revert at a later period.

Preliminary communication on the HALL-*effect in antimony.*

5. The preliminary results of the experiments on a round and a quadratic plate, cast by means of the same method as used for bismuth, but at a temperature of the mould still far below the melting point of antimony ($\pm$ 450°), are:

Coefficient of HALL R = 0,22 in a field of 5450.

Dissymmetry too small to be stated without doubt.

Decrease of R from 13° to 200° in the ratio of 1 to 0,66.

Observations during or soon after a cooling give always a too small value for R. The same phenomenon

[1]) LEBRET, Diss. p. 35, 36. CLOUGH and HALL, Proceed. of the Amer. Acad. 20, p. 189, 1893.

[2]) A later experiment with the quadratic plate spoken of in 2 has given for $\frac{S_{100}}{S_{20}}$ in fields of 950, 1500 and 4600 resp. 0,528, 0,578 and 0,654.

occurred in a very high degree in the experiments of DRUDE and NERNST [1]).

The small dissymmetry may be a consequence of the structure of the tested plates, which was still rather irregular, may however also be caused by a very low value of $K_{11} - K_{22}$. For the whole increase of resistance was found by VON ETTINGSHAUSEN in a field of 10600 only 1,1 perCt., by LENARD in a field of 6600 1.2 perCt.

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[1]) LEBRET, Diss. p. 39. Wied. Ann. 42 p. 568. 1891.