The colloquium *From Planck to Escher* on June 14 was an introduction to the issues to be discussed in the series of lectures:

From Cosmological Observations to Fundamental Physics

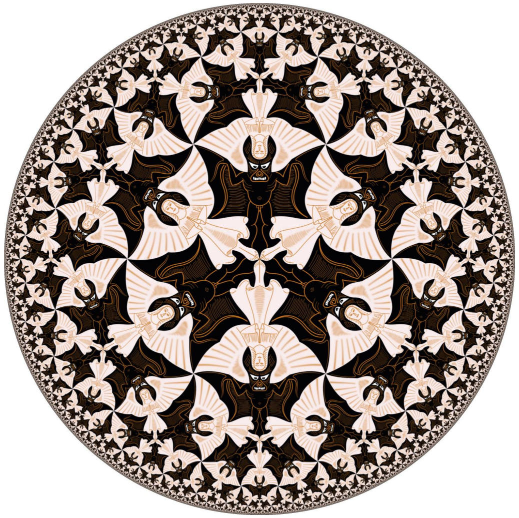
Thursday June 15 @ 13:45:
Andrei Linde "Inflationary cosmology"

Thursday June 22 @ 13.45:
Renata Kallosh, "De Sitter vacua in string theory and supergravity"

Monday June 26 @ 13:45:
Renata Kallosh and Andrei Linde, "Alpha-attractor models and B-mode targets"

Lecture II: Escher's part

1. Following an introduction to inflationary cosmology, I will describe a recently developed class of inflationary models, motivated by maximal supersymmetry and string theory, that gives specific targets for future B-mode experiments. These models are based on the hyperbolic geometry of the Poincare disk, which is beautifully represented by Escher's picture Circle Limit IV.



2. De Sitter vacua in string theory and supergravity

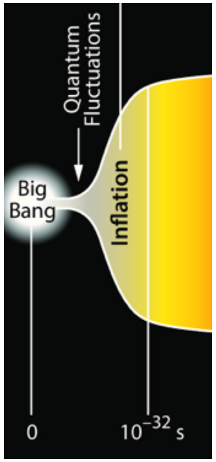
Glossary

N : the number of e-folds N left to the end of inflation, defined as

$$a = a_f e^{-N}$$

$$e^{55} \approx 10^{23}$$

where a is the scale factor and a_f is its value at the end of inflation. The spectrum of fluctuations observed in CMB corresponds to



$47 < N < 57$

n_s: Tilt of the spectrum of inflationary perturbations

$$n_s \approx 0.965$$

r : Level of B-modes, r=T/S. Not detected

$$r < 0.07$$

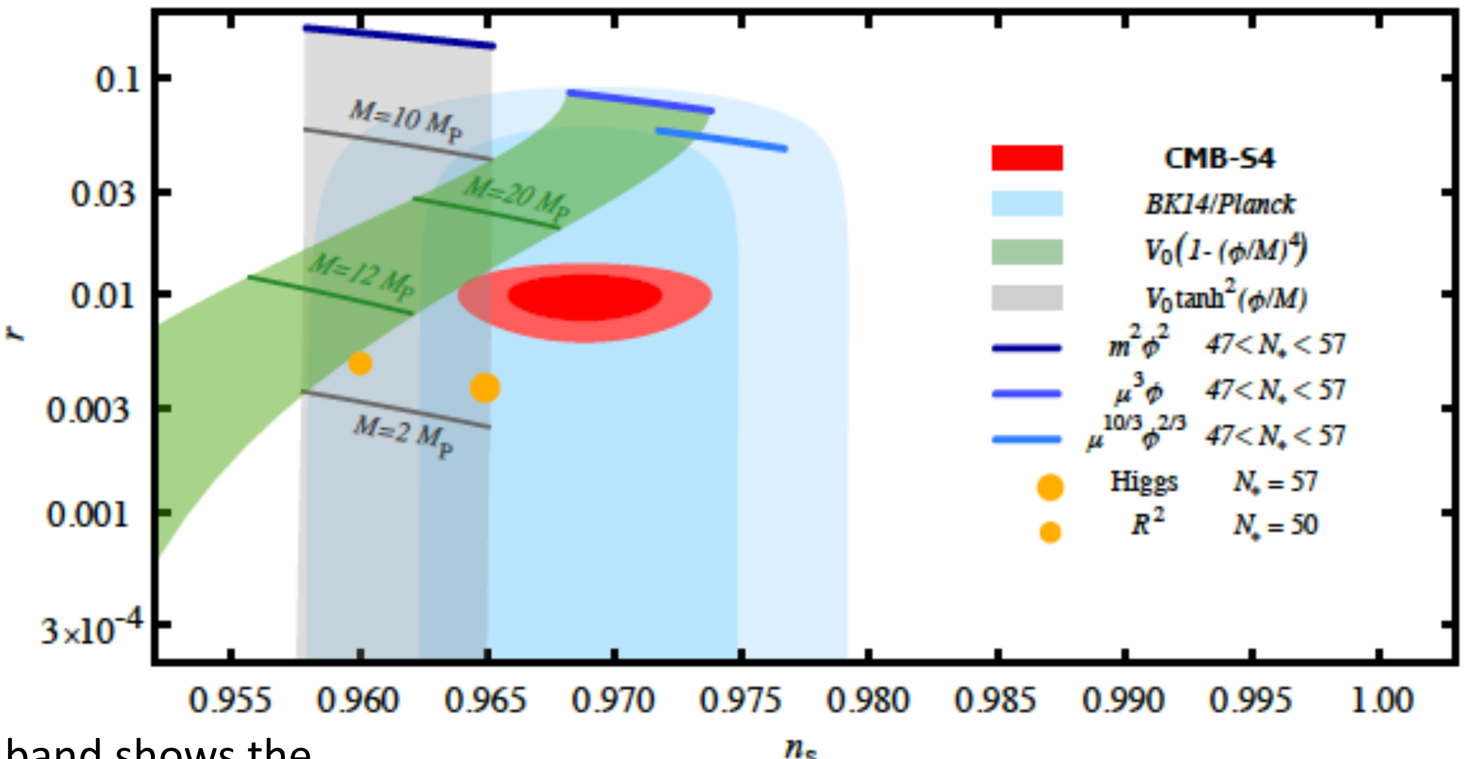
$\mathcal{N}$: is the number of supersymmetries, Majorana spinors. The maximal number for theories with highest spin 2 is

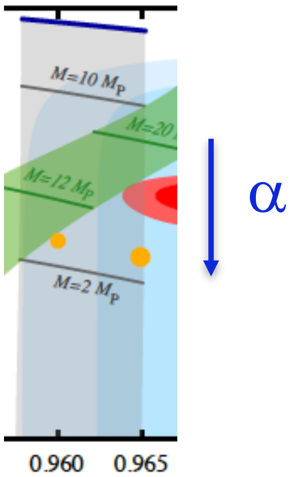
$$\mathcal{N} = 8$$

The minimal number which we will consider here is

$$\mathcal{N} = 1$$

Alpha-Attractors and B-mode Targets





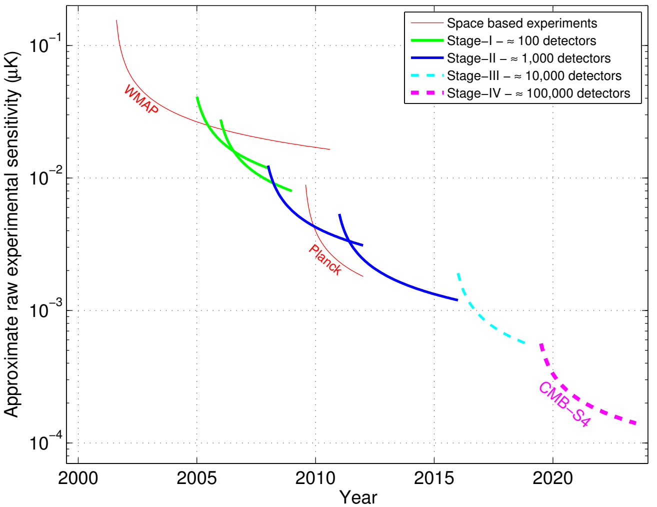
Gray band shows the prediction of a sub-class of α-attractors

$$n_s = 1 - \frac{2}{N}\,, \qquad r = \alpha\,\frac{12}{N^2}$$

Primordial Gravity Waves

CMB-S4 Science Book

October 2016

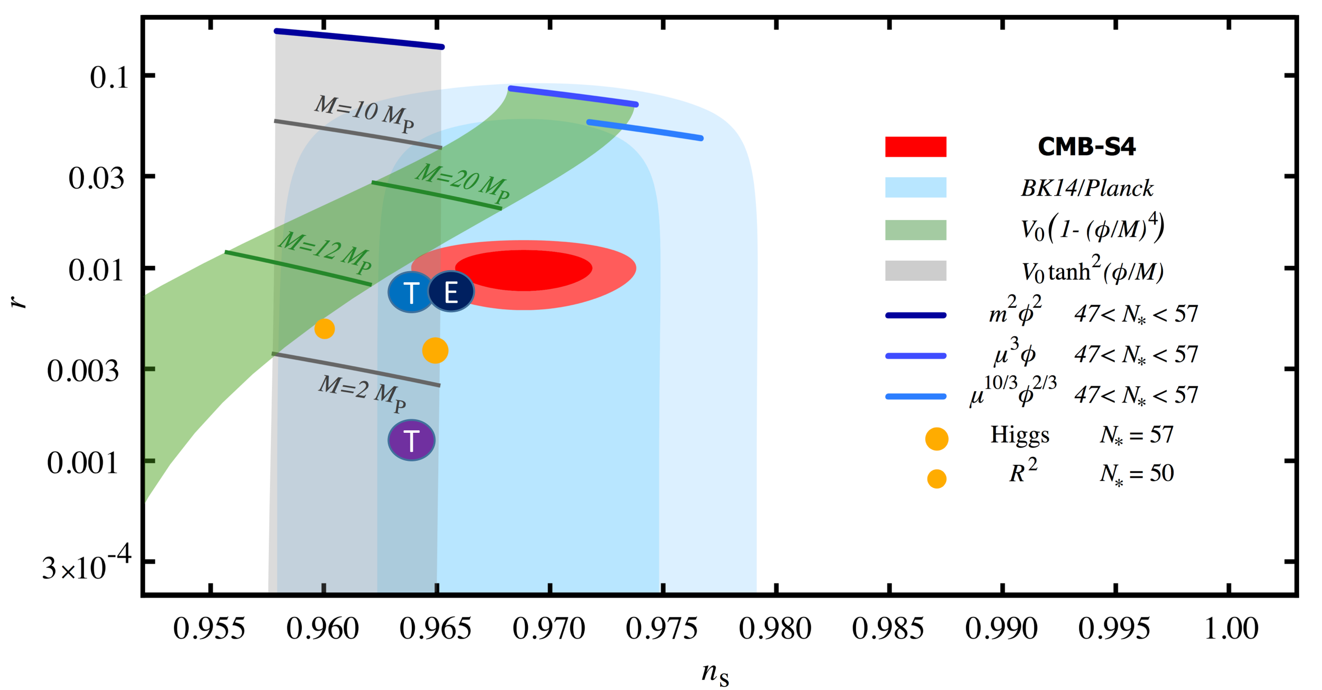


Ferrara, RK, 2016,
RK, Linde, Wrase, Yamada, 2017
RK, Linde, Roest, Yamada, 2017
α–attractor models

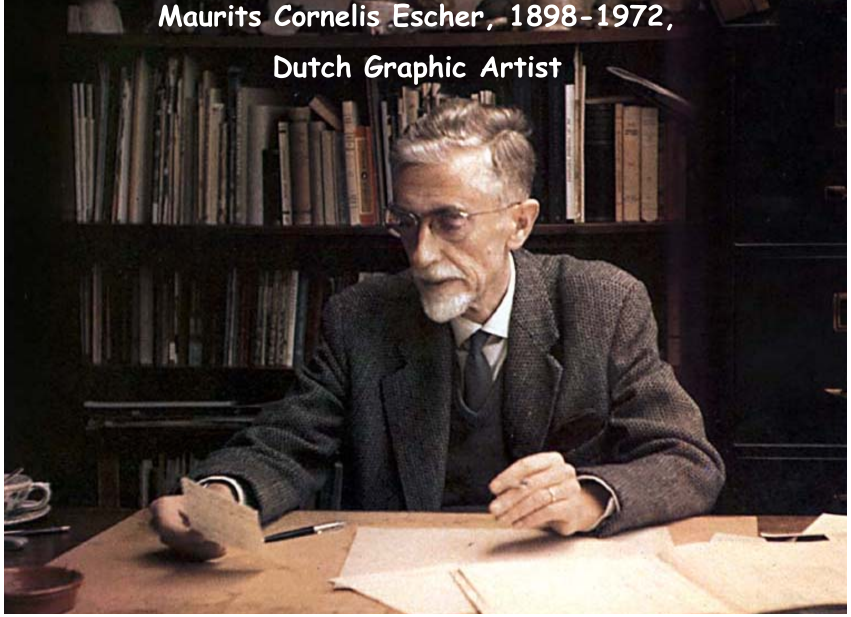
Well motivated new models originating in string theory, M-theory, maximal supergravity

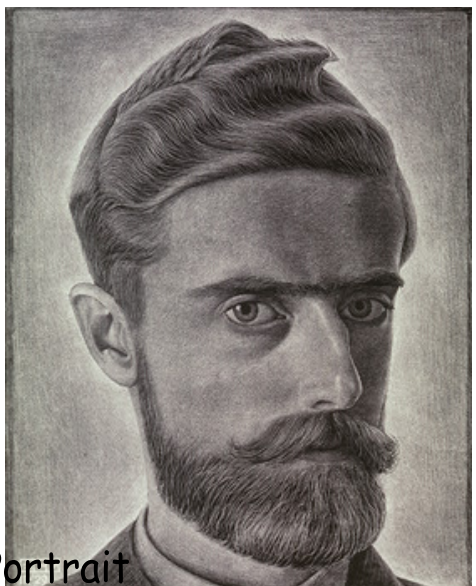
Ground based experiments

Future B-mode satellite missions



Why do we find it useful to talk about the size of the Escher's disks in discussions of the CMB future B-mode targets ?





Self Portrait

1929

1922, 1936: visits Alhambra in Granada, Spain

Islamic tiling

The Moors were past master of this. They decorated the walls and floors, particularly in the Alhambra in Spain, by placing congruent, multicoloured pieces of majolica together without leaving any spaces between.

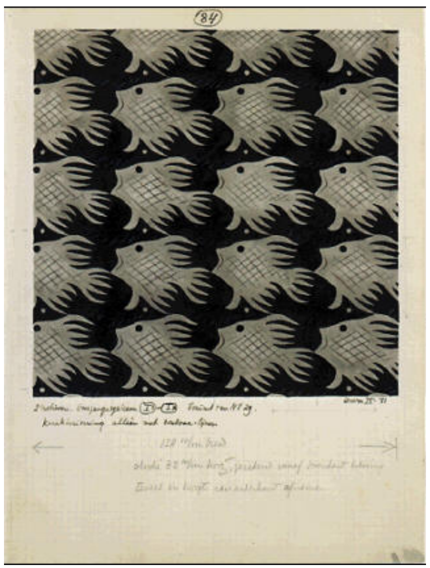
Landscapes to Mindscapes

Escher after Alhambra

This is the richest source of inspiration that I have ever struck…

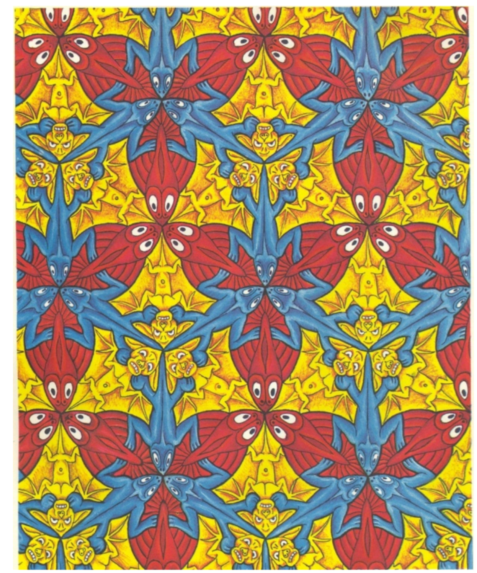
A tessellation of a flat surface is the tiling of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps.

Euclidean geometry



Low symmetry

Intermediate symmetry

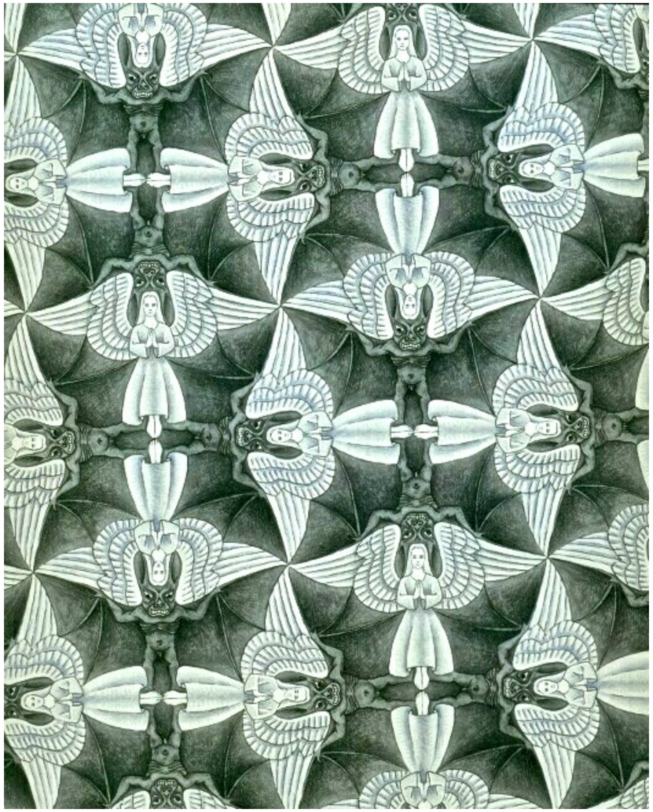


High symmetry

1952

Escher struggled to capture infinity

It was after the 1936 Alhambra visit that his obsession with plane-filling designs really took hold, and he began to produce designs like *Angels and Devils*



For *Angels and Devils* Escher worked in ink rather than wood-cut. It has two ‘tiles’ that meet exactly and form a repeating design that could be extended indefinitely.

Such drawings hint at infinity, in that the patterns could in principle be extended and repeated forever, but no finite diagram can actually show the whole tiling.

Escher found this frustrating, and wanted a better solution to represent infinity.

Coxeter and Escher

In 1954 the International Congress of Mathematicians was located in Amsterdam. To coincide with the Congress, a major exhibition of Escher’s work was held at the Stedelijk Museum in Amsterdam

In this way the well known geometer H. S. M. Coxeter first became acquainted with Escher's art. Three years later Coxeter asked Escher if he could include some of his tesselations pictures as illustrations of symmetry. Coxeter sent Escher a reprint of his lecture.

Figure 7 of the Coxeter article resonated strongly with one of Escher's own interests, the problem of producing **an infinitely repeating pattern in a finite figure**

Coxeter→Escher (1958)

Coxeter, H. S. M., "Crystal symmetry and its generalizations," *Royal Society of Canada*(3), **51** (1957), 1-13.

Figure 7: A tesselation of hyperbolic plane by 30°-45°-90° triangles (Poincare Disk Model)

Escher wrote to Coxeter, "some of the text illustrations and especially figure 7 gave me quite a shock"

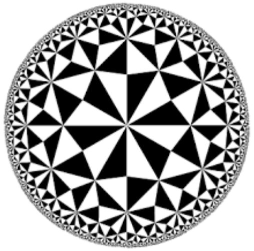
The figure's hyperbolic tiling, with triangular tiles diminishing in size and repeating (theoretically) infinitely within the confines of a circle, was exactly what Escher had been looking for in order *to capture infinity in a finite space*.

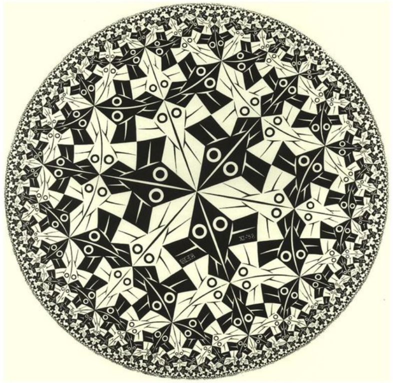
To a mathematician, Coxeter's figure represents a **non-Euclidean analogue of a periodic tiling of the Euclidean plane**

1958

Escher was quite excited by that figure, since it showed him how to solve a problem that he had wondered about for a long time: how to create a repeating pattern within a limiting circle, so that the basic subpatterns or motifs became smaller toward the circular boundary.

A Tale Both **Shocking** and **Hyperbolic**





1939

Circle Limit I, M.C. Escher (1958)

NON-EUCLIDEAN GEOMETRIES

Given any straight line and a point not on it,

there exists ***one and only one straight line*** which passes through that point and never intersects the first line => **Euclidean Geometry**

$\mathcal{R} = 0$

there exists ***no line*** which passes through that point and never intersects the first line
=> **Elliptic Geometry** (e.g. Spherical geometry)

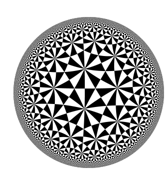
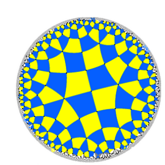
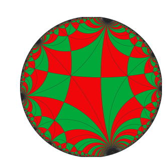
$\mathcal{R} > 0$

there "exists ***more than one straight line*** which passes" through that point and never intersects the first line => **Hyperbolic Geometry**

$\mathcal{R} < 0$

Escher knew almost no mathematics

Hyperbolic Coxeter groups are visualized via Poincaré disk model in Escher's art


Coxeter proved later that Escher's art is mathematically perfect

How to capture infinity ?

We modify the kinetic term of scalars according to hyperbolic geometry

$$\frac{\partial Z \partial \bar{Z}}{(1 - \frac{Z\bar{Z}}{3\alpha})^2}$$

$$\frac{1}{2}R - \frac{1}{2}\frac{\partial\phi^2}{\left(1 - \frac{\phi^2}{6\alpha}\right)^2} - \frac{1}{2}m^2\phi^2$$

Switch to canonical variables

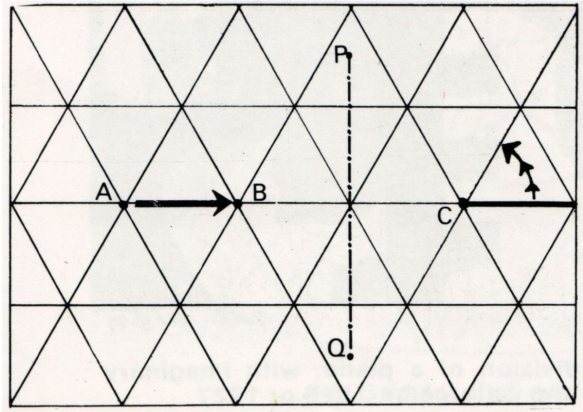
$$\phi = \sqrt{6\alpha}\,\tanh\frac{\varphi}{\sqrt{6\alpha}}$$

The potential becomes

$$V = 3\alpha\, m^2 \tanh^2\frac{\varphi}{\sqrt{6\alpha}}$$

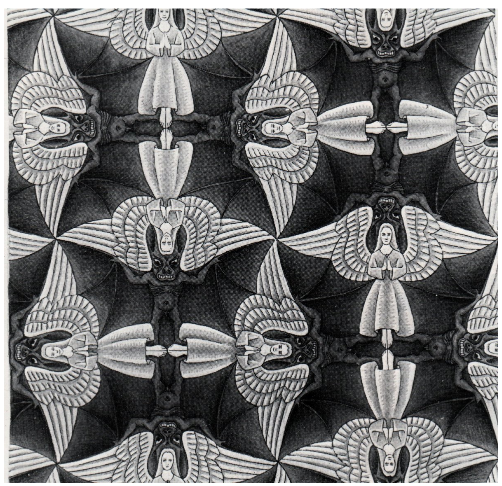
$$\phi^2 < 6\alpha \qquad -\infty < \varphi < \infty$$

Principles of Plane Tessellations

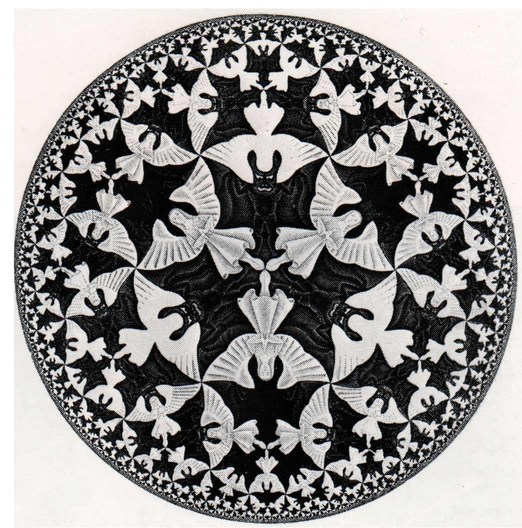


The whole surface is covered with equilateral triangles. If we shift the whole plane over the distance **AB**, it will cover the underlying pattern once again. This is a **translation** of the plane. We can also turn the duplicate through 60 degrees about the point **C**, and we notice that again it covers the original pattern exactly. This is a **rotation**. Also if we do a **reflection** about the line **PQ**, the pattern remains the same.

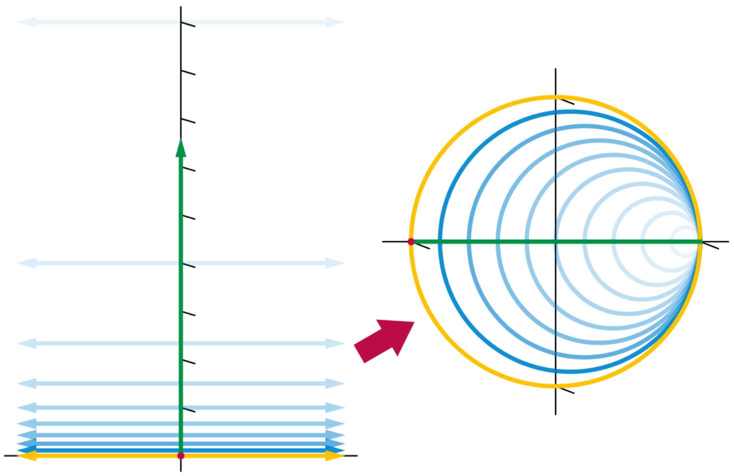
Periodic space filling for "Angels and Devils" (1941)

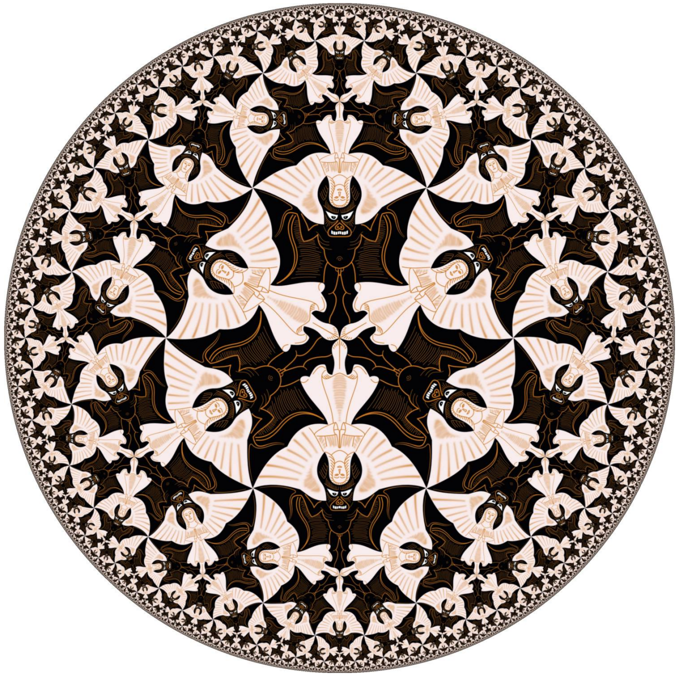


The tessellation and the final result for the hyperbolic tiling for "Angels and Devils", "Circle Limit IV" (1960)



From the disk to a half-plane (the Cayley transform)

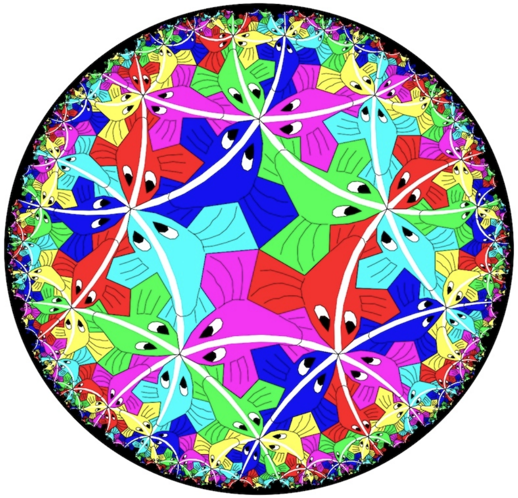




Isometries:

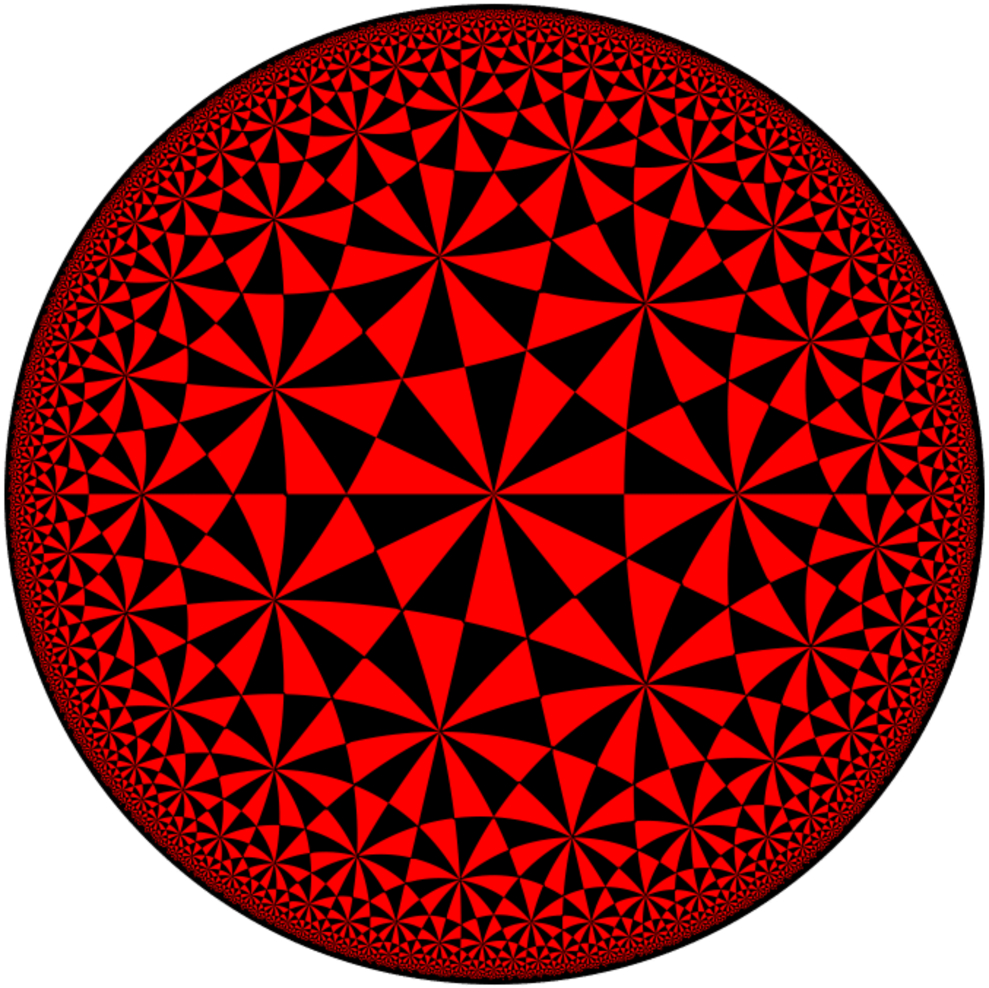
Mobius Transform

$$Z' = \frac{\beta Z + \gamma}{\bar{\gamma} Z + \bar{\beta}}$$



$$\tau' = \frac{a\tau + b}{c\tau + d}$$

Möbius transformations applied to hyperbolic tilings allowed to produce animations



$$K_{\rm disk} = -\frac{1}{2}\log\frac{(1-Z\bar{Z})^2}{(1-Z^2)(1-\bar{Z}^2)}\,.$$

Meaning of the measurement of the curvature of the 3d space

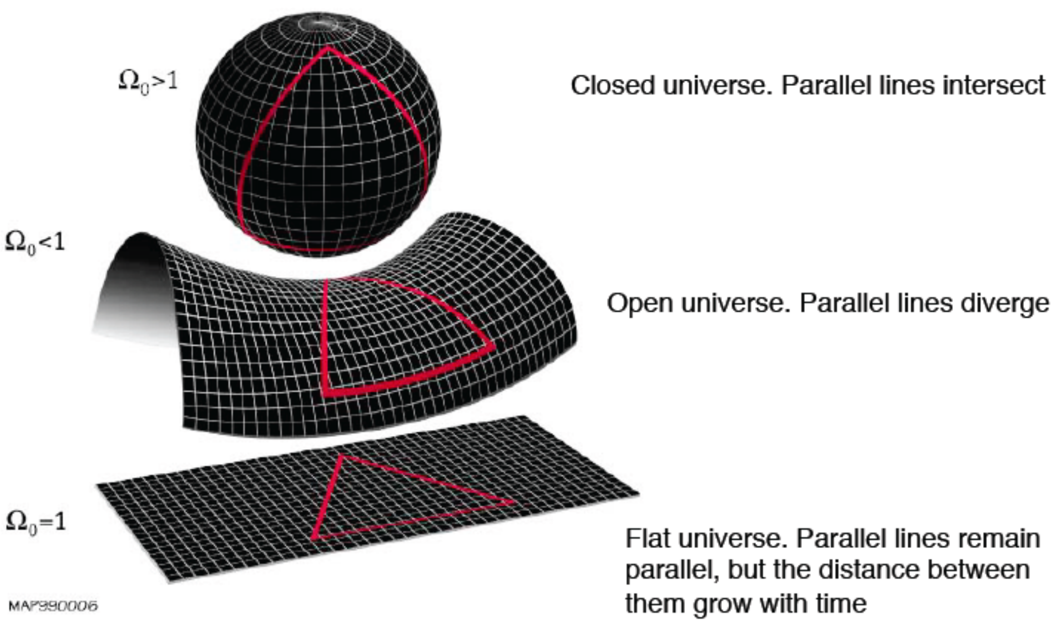
k=+1, k=-1, k=0

Spatial curvature parameter

$$ds^2 = -dt^2 + a(t)^2 \gamma_{ij} dx^i dx^j$$

$$\Omega_K = -0.0003 \pm 0.0026$$

Closed, open or flat universe



In the context of α-attractors cosmological models, measuring r means measuring the curvature of the hyperbolic geometry of the moduli space

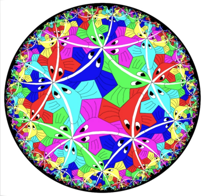
$$n_s = 1 - \frac{2}{N}\,, \qquad r = \alpha \frac{12}{N^2}$$

$$R_K = -\frac{2}{3\alpha}$$

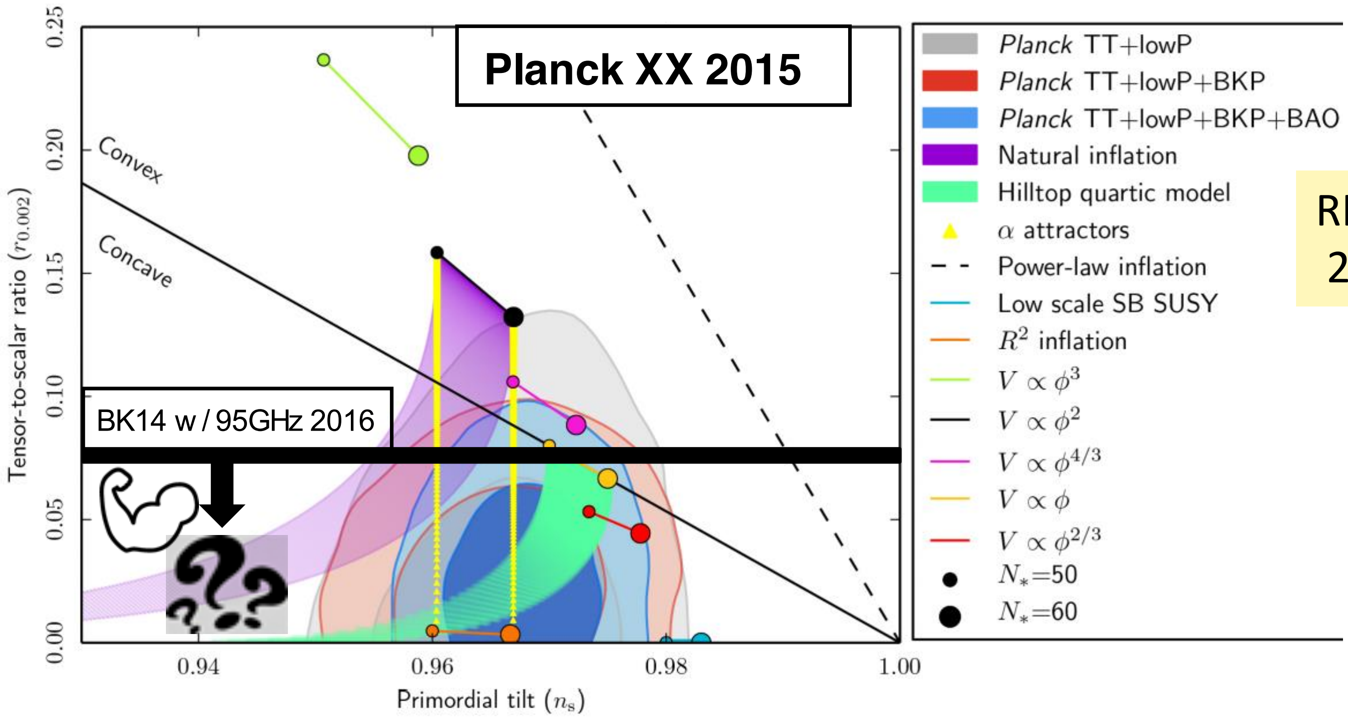
scalar fields are coordinates of the *Kahler geometry*

Decreasing r, decreasing α, increasing curvature R_K

$$3\alpha = R^2_{\text{Escher}} \approx 10^3 r$$



Hyperbolic geometry of a Poincaré disk

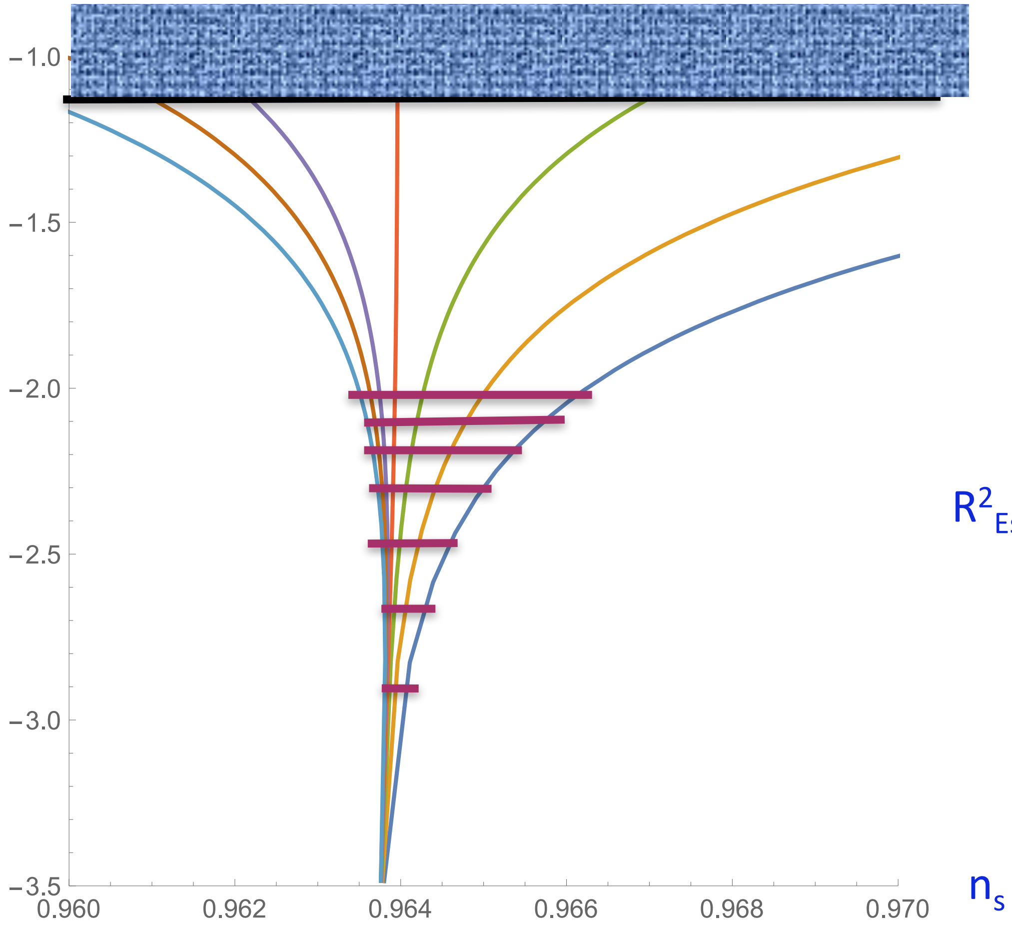


RK, Linde, Roest 2013

Next in CMB cosmology: **Relentless observation**

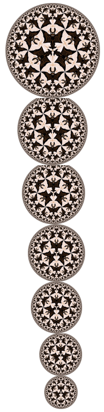
B-mode targets: from maximal supersymmetry to minimal supersymmetry

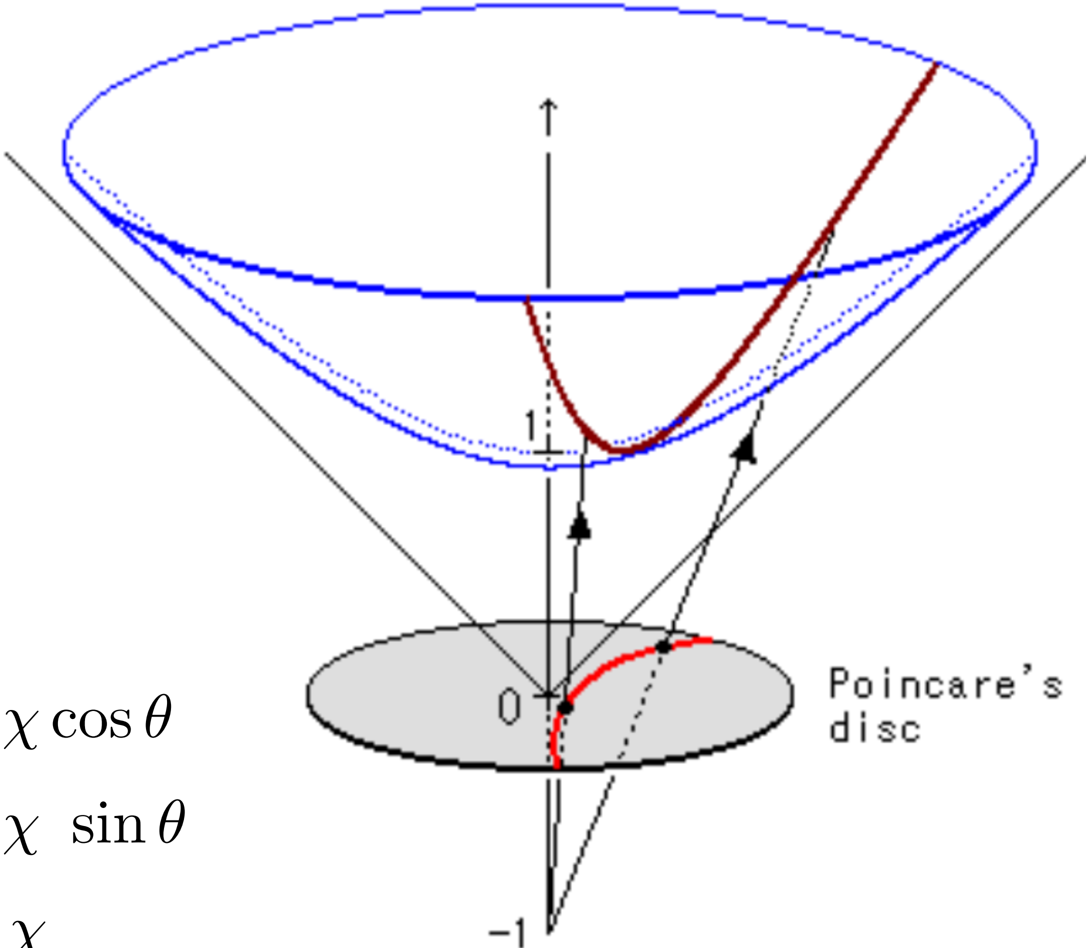
α-attractors $\log_{10} r$- n_s plane



$r < 0.07$

R^2_{Escher} $=3\alpha$ = 7,6,5,4,3,2,1





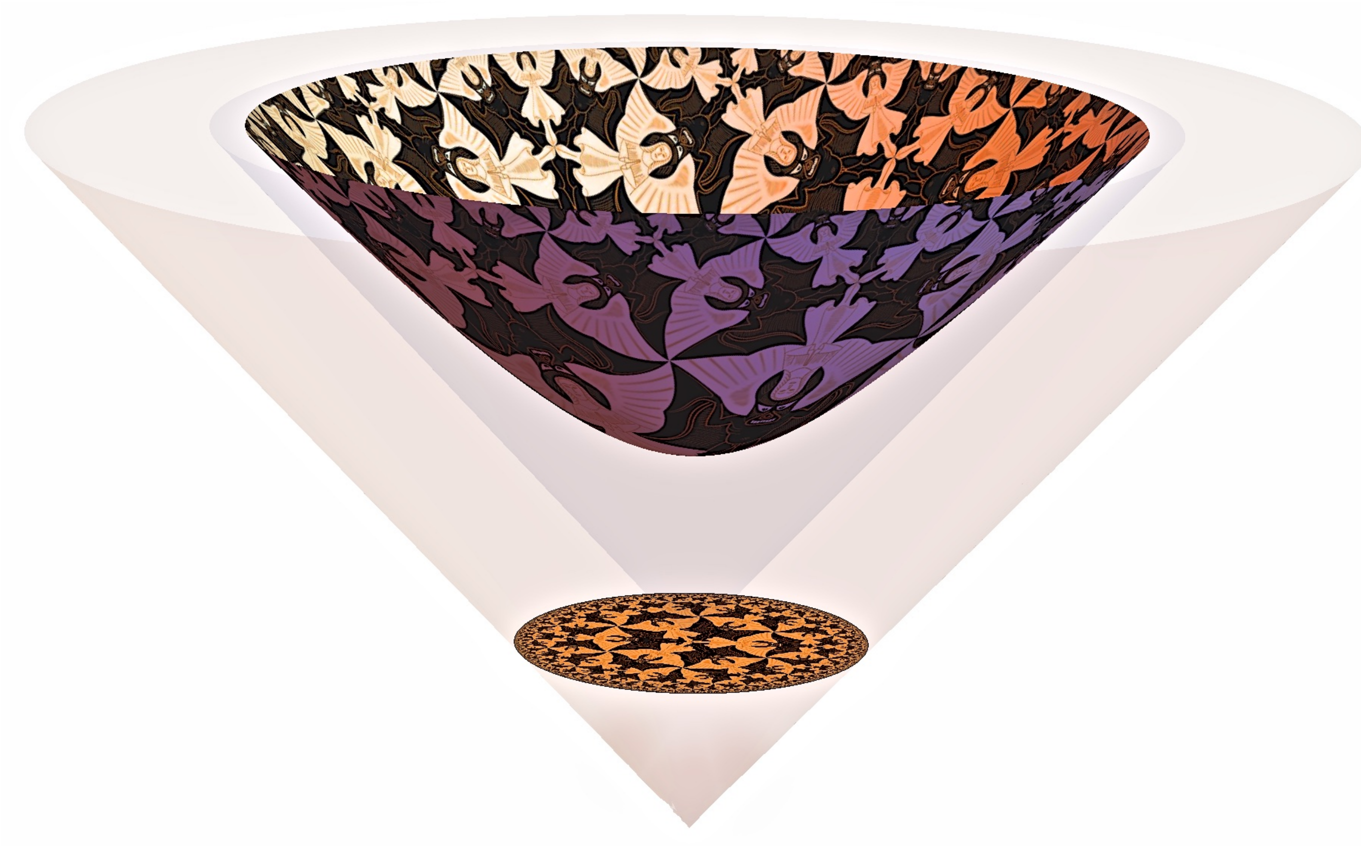
$$u = \sqrt{3\alpha}\sinh\chi\cos\theta$$

$$v = \sqrt{3\alpha}\sinh\chi \;\; \sin\theta$$

$$w = \sqrt{3\alpha}\cosh\chi$$

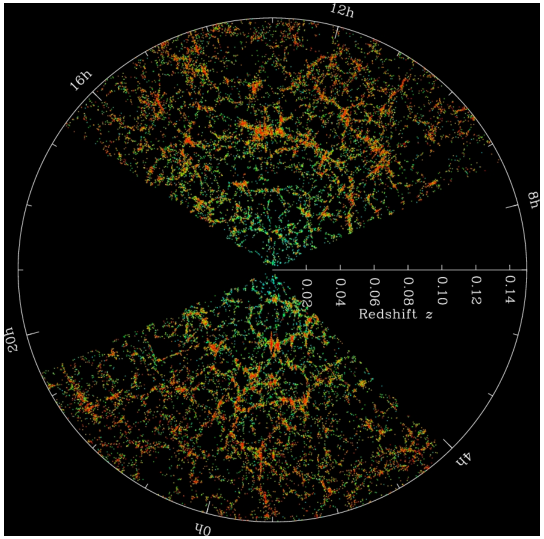
$$w^2 - u^2 - v^2 = 3\alpha$$

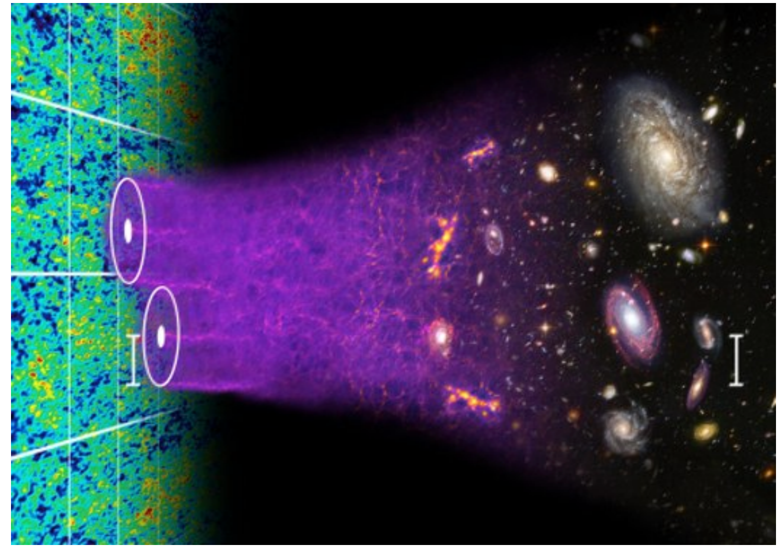
$$ds^2 = \frac{3\alpha}{(1 - Z\bar{Z})^2} dZ d\bar{Z} = \frac{3\alpha}{4}(d\chi^2 + \sinh^2\chi d\theta^2)$$



BOSS: Baryon Oscillation Spectroscopic Survey

2016

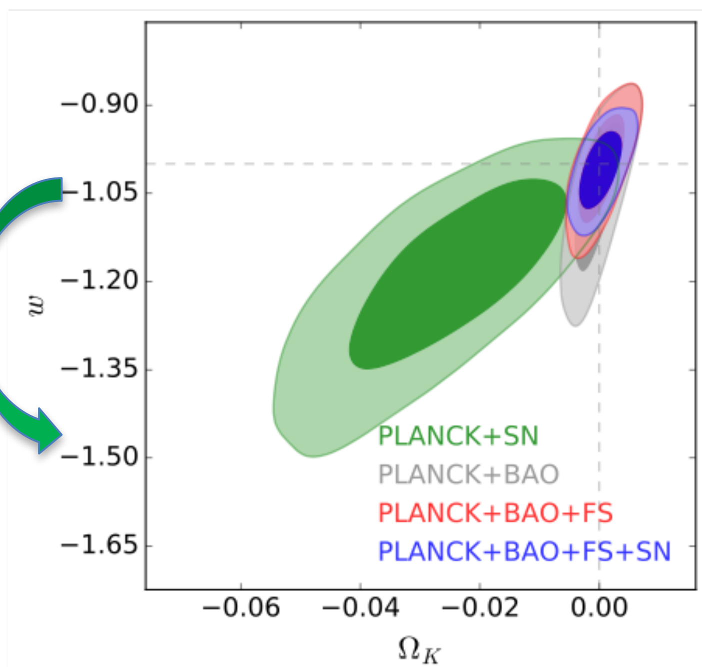


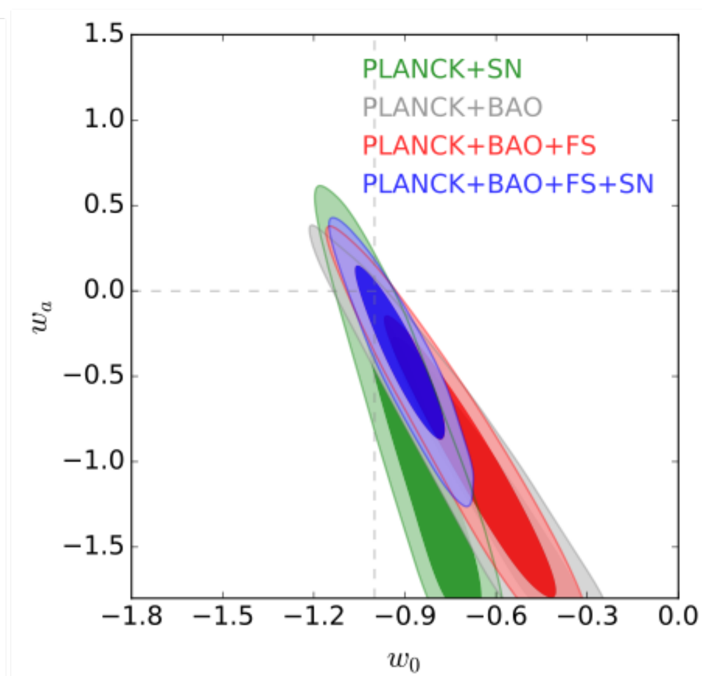


full shape (FS) measurements of galaxy clustering

Dark Energy and the Geometry of Space

Doomsday





Phantom Energy and Cosmic Doomsday

Feb 25 2003

Caldwell, Kamionkowski, and Weinberg

But what about $w < -1$? Matter with $w < -1$, dubbed "phantom energy" 1415 refs

Science News

Cosmic Doomsday Scenario: Phantom energy would trigger the Big Rip

BY

RON COWEN

11:00AM, MARCH 5, 2003

Cosmologists have long speculated about the fate of the universe. Will it expand forever or collapse in a Big Crunch? In the latest model, published online last week, the universe instead ends with a **Big Rip–every galaxy, star, planet, molecule, and atom torn asunder 21 billion years from now.**

Now this issue seem to become irrelevant:
Quantum consistency of these models is questionable!
Future dark energy probes?

Increasing experimental evidence that CC constant is a good fit for dark energy and current acceleration

1998 - 2017

$$w = \frac{p}{\rho} \approx -1 \qquad w'(z) \approx 0$$

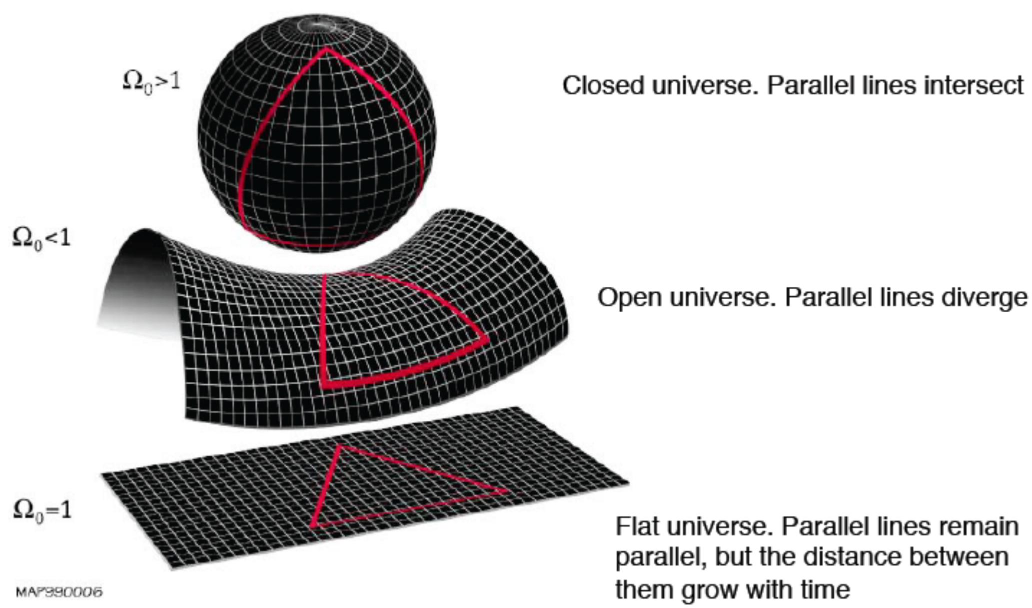
We need to construct de Sitter vacua:
space-times with positive cosmological constant

Increasing experimental evidence for early universe inflation

Spatial part of the space-time curvature

$$\Omega_K \approx 0$$

Closed, open or flat universe



1607.03155

Recent data (CMB and non-CMB)

BOSS collaboration

The clustering of galaxies in the completed SDSS-III Baryon Oscillation Spectroscopic Survey: cosmological analysis of the DR12 galaxy sample

2016

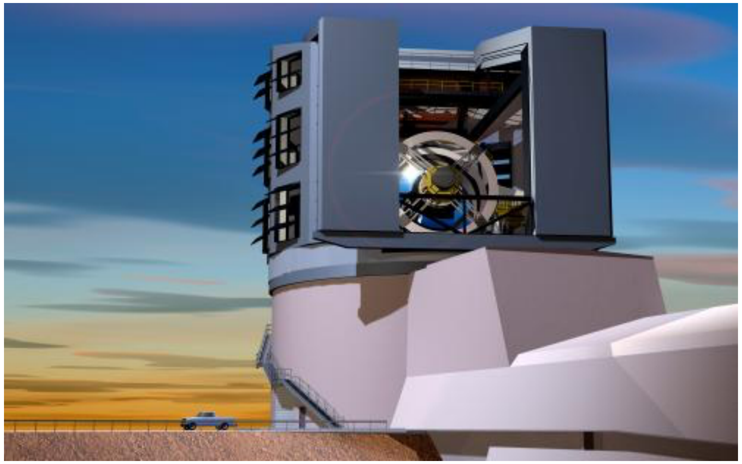
$$\Omega_K = 0.0003 \pm 0.0026$$

$$w = -1.01 \pm 0.06$$

strong affirmation of the spatially flat cold dark matter model with a cosmological constant (ΛCDM)

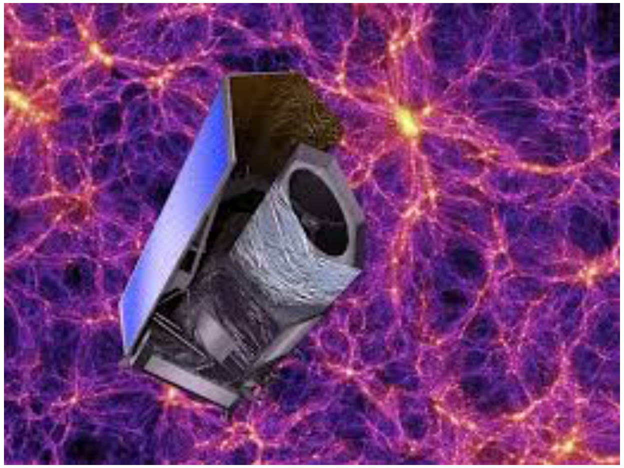
When combined with supernova Ia data, we find **$H_0 = 67.3 \pm 1.0$** even for our most general dark energy model, in tension with some direct measurements.

Future Dark Energy Probes



Operation from 2023 ?

In its deep wide-angle survey the LSST will be able to pin down the equation of state of dark energy to better than a few percent. Due to its wide coverage of the sky, the LSST is uniquely capable of detecting any variation in dark energy with direction. In turn, this will tell us something about physics at the earliest moments of our universe, and how it set the course for cosmic evolution.



Launch in 2020 ?

Euclid will:

1. Investigate the properties of the dark energy by accurately measuring both the acceleration as well as the variation of the acceleration at different ages of the Universe

Maximally symmetric spaces in GR

Case of positive curvature: de Sitter

$$R_{\mu\nu\alpha\beta} = H^2 \left(g_{\mu\nu}\, g_{\alpha\beta} - g_{\mu\beta}\, g_{\nu\alpha} \right) \ ,$$

$$R_{\mu\nu} = 3\, H^2\, g_{\mu\nu} \ ,$$

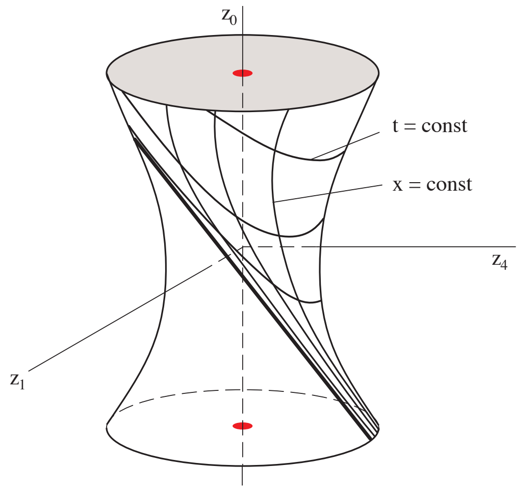
$$R = 12\, H^2 = 4V(\phi)|_{\phi_{\min}} = 4\Lambda$$

$$\begin{aligned} z_0 &= H^{-1} \sinh H\, t + \frac{1}{2} H\, e^{H\, t}\, \mathbf{x}^2 \ , \\ z_4 &= H^{-1} \cosh H\, t - \frac{1}{2} H\, e^{H\, t}\, \mathbf{x}^2 \ , \\ z_i &= e^{H\, t}\, x_i \ , \qquad i = 1, 2, 3 \ . \end{aligned}$$

This coordinate system spans the half of the hyperboloid with $z_0 + z_4 > 0$

a hyperboloid in a 5d Minkowski space

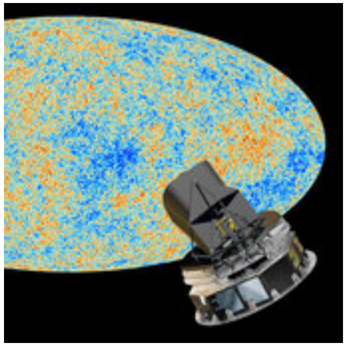
$$z_0^2 - z_1^2 - z_2^2 - z_3^2 - z_4^2 = -H^{-2}$$



The metric induced at 4d is a FRW metric of the expanding Friedman universe

$$ds^2 = dt^2 - a(t)^2 d\vec{x}^2$$

$$a(t) = e^{Ht}$$

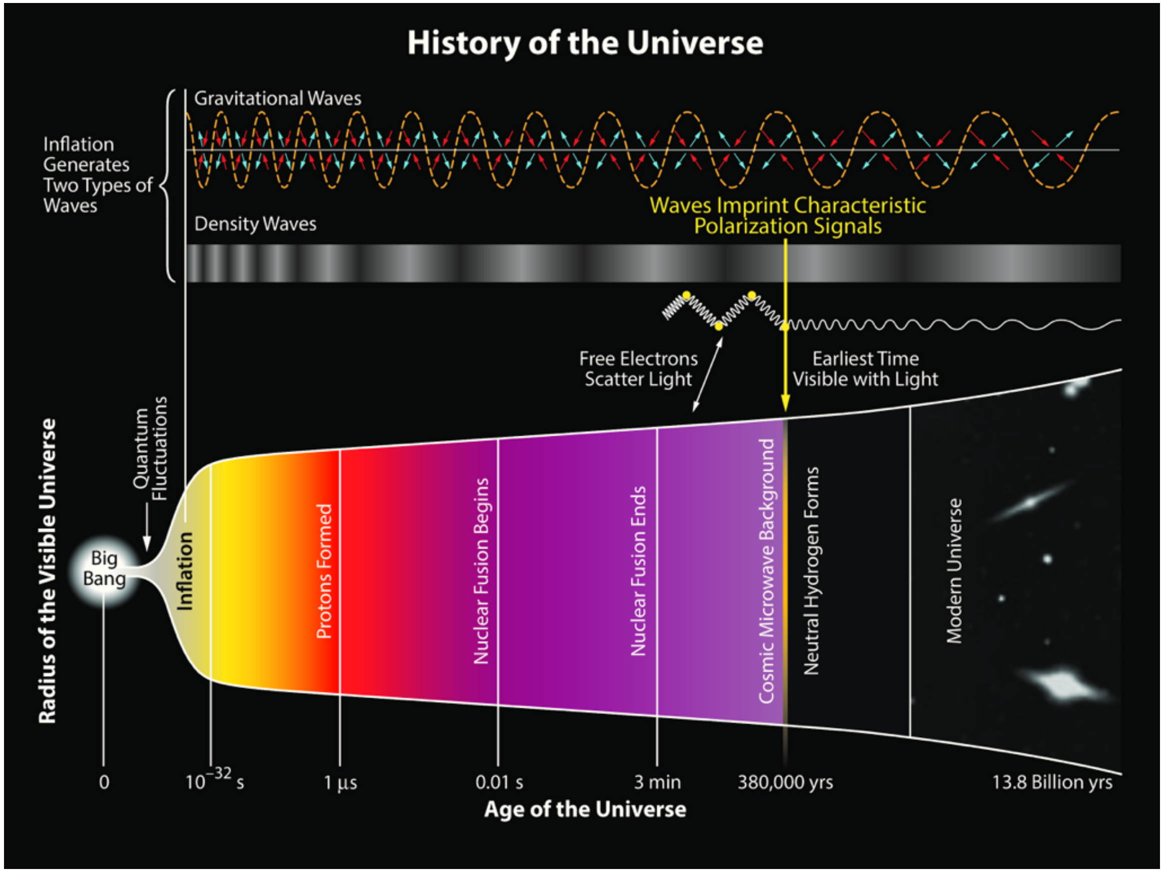


We see small perturbations around this geometry of the order 10^{-5}

$$10^{-10} M_P^4$$

Current acceleration during the last few billion years

$$\dot{a} = H e^{Ht} \qquad \ddot{a} = H^2 e^{Ht} > 0 \qquad 10^{-120} M_P^4$$



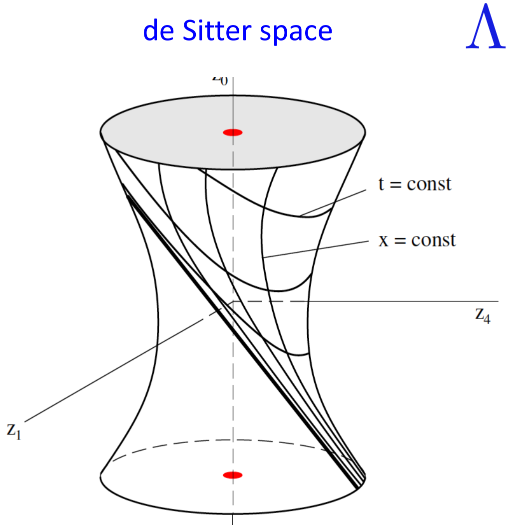
$$H^2 \approx 10^{-10}$$

During inflation the space has a nearly de Sitter geometry

Slow-roll parameters in inflation: deviation from exact de Sitter

$$H^2 \approx 10^{-120}$$

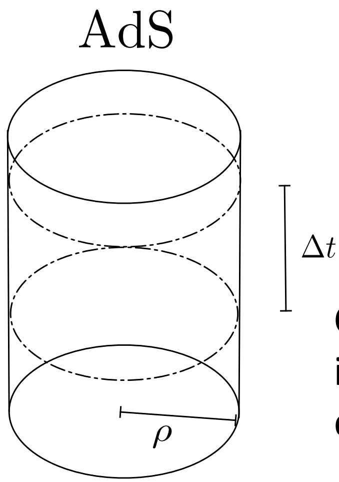
During current acceleration the space has a de Sitter geometry, assuming that $\Lambda = 3H^2$ is constant



$\Lambda > 0$

+ for AdS_4 +

$$z_0^2 - z_1^2 - z_2^2 - z_3^2 - z_4^2 = -H^{-2}$$



$\Lambda < 0$

Conformal symmetry in Minkowski space dimension d

Figure 2: This figure shows AdS in global coordinates. The center is at $\rho = 0$, while spatial infinity is approached as $\rho \to \pi/2$. The global time coordinate t runs from $-\infty$ to ∞.

$$X_A X^A \equiv X_0^2 + X_{d+1}^2 - \sum_{i=1}^{d} X_i^2 = R^2$$ manifest SO(2,d)

$$X_0 = R\frac{\cos t}{\cos \rho}$$
$$X_{d+1} = R\frac{\sin t}{\cos \rho}$$
$$X_i = R\tan\rho\,\hat{\Omega}_i$$

Global coordinates

$$ds^2 = \frac{1}{\cos^2\left(\frac{\rho}{R}\right)}\left(dt^2 - d\rho^2 - \sin^2\left(\frac{\rho}{R}\right) d\Omega_{d-1}^2\right)$$ isometry SO(2,d-1)

SO(1, d+1) in Euclidean case

For example, if $d = 3$ then we can write

$$d\Omega^2 = d\theta^2 + \sin^2\theta\, d\phi^2$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = 0$$

$$R = 4\,\Lambda$$

The sign of Λ is coordinate independent

The AdS/CFT correspondence

The AdS/CFT correspondence arose as a duality between Type IIB string theory in the background geometry $AdS_5 \otimes S^5$ and the maximally supersymmetric quantum field theory in $D = 4$ dimensions, the $\mathcal{N} = 4$ super-Yang-Mills (SYM) theory

Most concrete results have been obtained in a low energy limit in which the string theory is well approximated by classical supergravity, initially D = 10 Type IIB supergravity.

Applications have broadened greatly; as practiced now, the subject includes a general correspondence between theories of gravity (plus other fields) in D + 1 spacetime dimensions and quantum field theories without gravity in D dimensions. Tractable calculations in a classical approximation on the gravity side yield information about quantum systems in a strong coupling limit for which the traditional techniques of quantum field theory are inadequate. It is truly surprising how much information about D-dimensional quantum systems can be captured by classical gravity in D + 1 dimensions. Field theory also leads to new insights into gravity.

Important applications in condensed matter physics

Here in Leiden

J. Zaanen, K. Schalm, …

In dS/CFT the representations of conformal group are not unitary, major issue

Attempts to build holographic cosmology

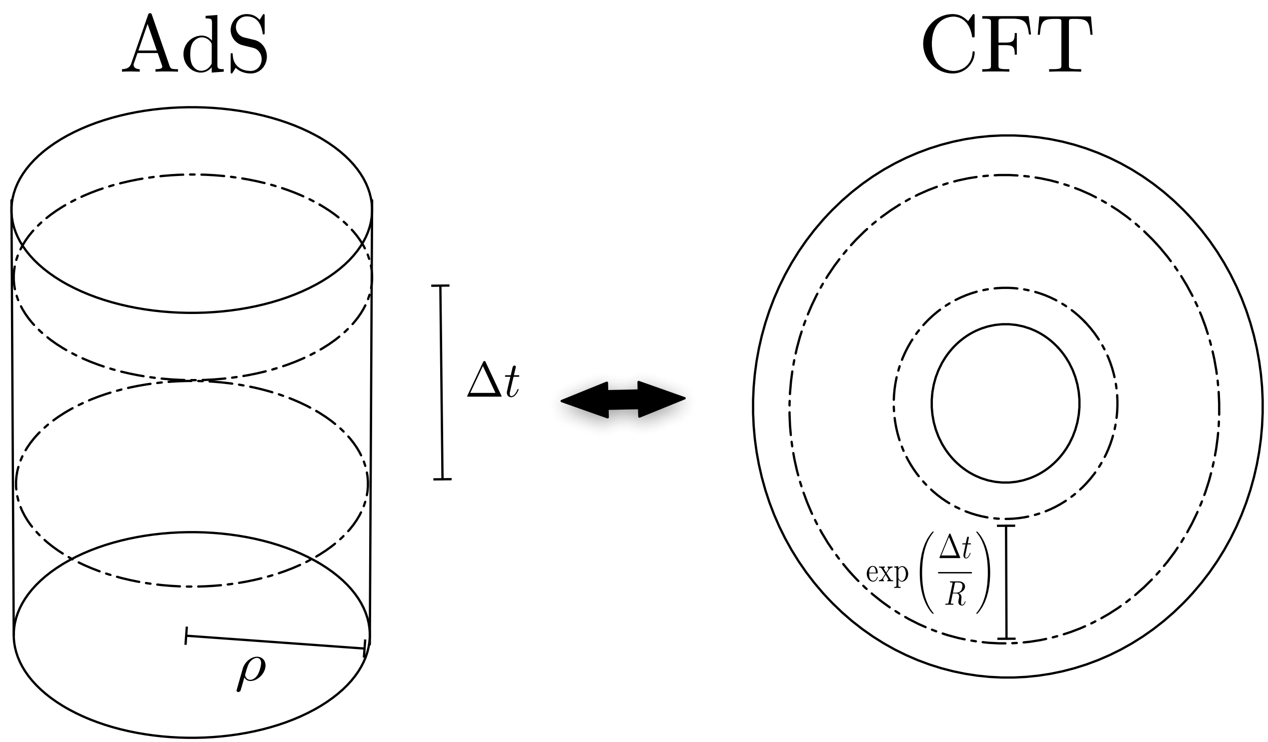
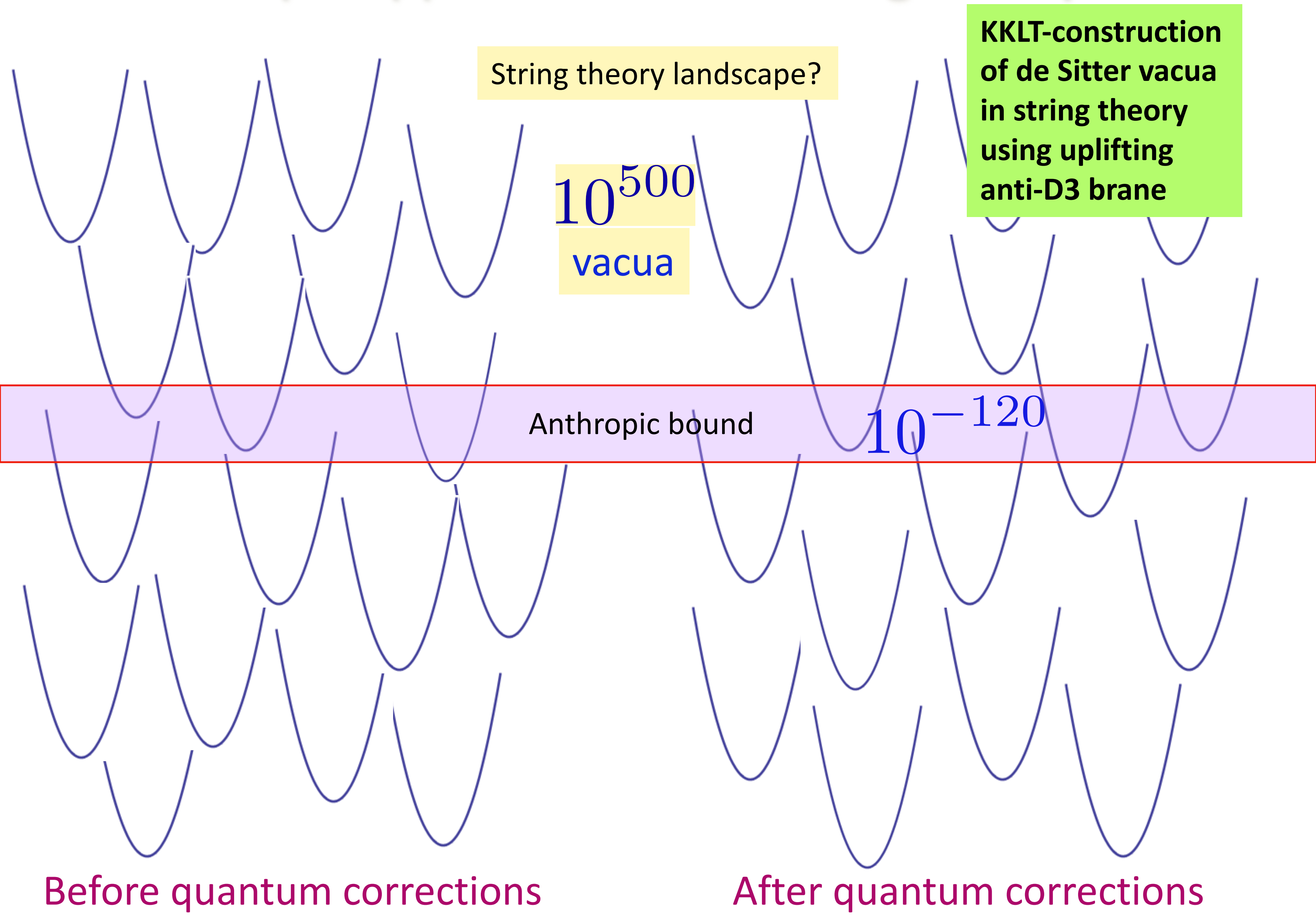
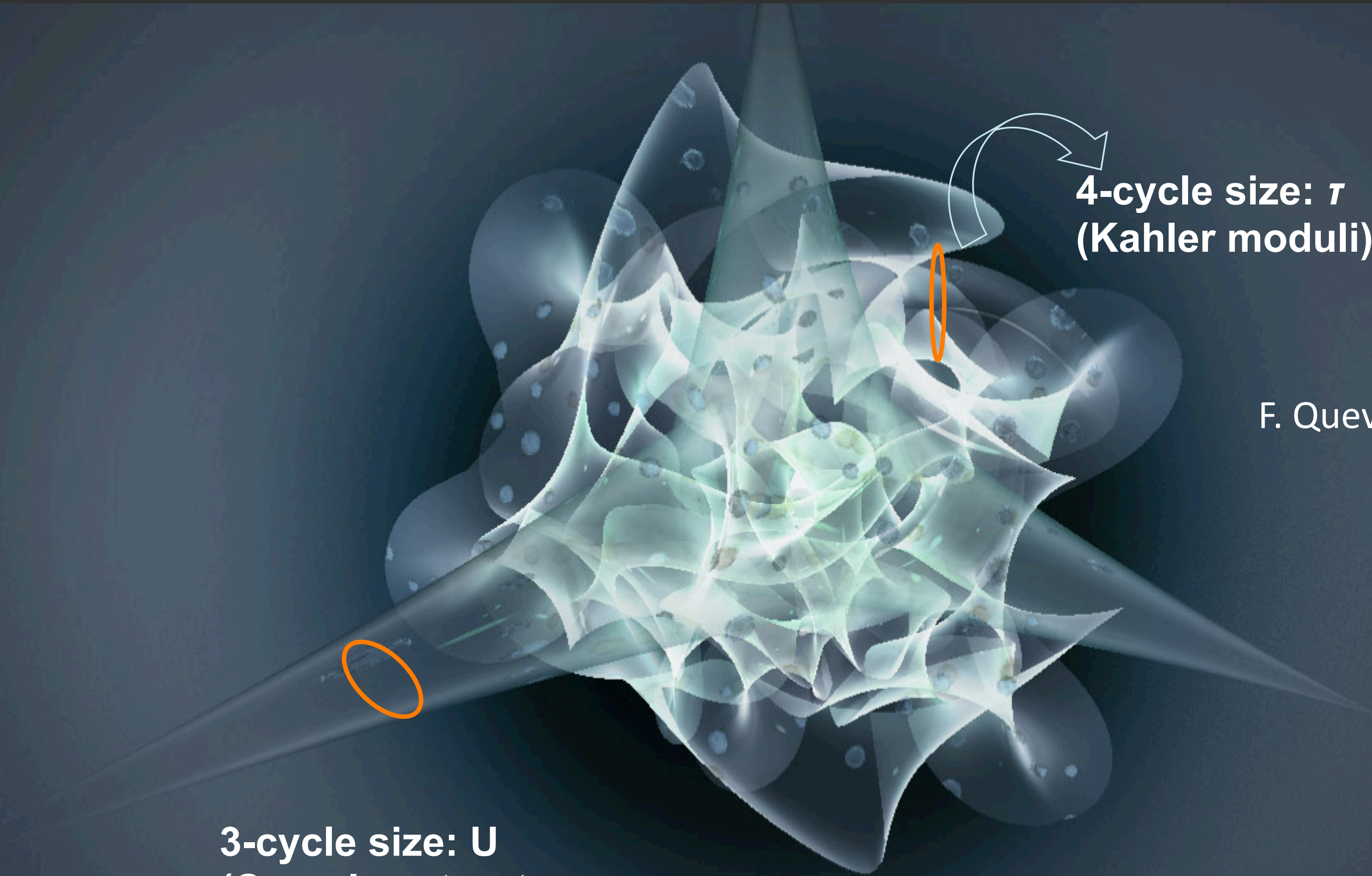


Figure 1: This figure shows how the AdS cylinder in global coordinates corresponds to the CFT in radial quantization. The time translation operator in the bulk of AdS is the Dilatation operator in the CFT, so energies in AdS correspond to dimensions in the CFT. We make this mapping very explicit in section 5.2.2.

Anthropic approach to Λ in string theory:



IIB MODULI STABILISATION



F. Quevedo

String landscape picture: many moduli, one has to stabilize many scalars to produce the (metastable) de Sitter vacua with positive CC

KKLT construction, 2003 **Anti-D3-brane** in Giddings-Kachru-Polchinski background

De Sitter vacua in string theory
Kachru, RK, Linde, Trivedi 2410 refs.

Towards inflation in string theory
Kachru, RK, Linde, Maldacena, McAllister, Trivedi 1015 refs.

Supergravity approximation: starting 2002, how to construct **de Sitter vacua inspired by string theory**.

Many groups working
Here in Leiden, Achucarro, Schalm et al using Khaler function
in Germany, in Italy, in Japan, in US

Lorentz Center workshop on Theoretical Approaches to Cosmic Acceleration (3–7 July, 2017)

New ideas starting 2014

String theory and supergravity prefer AdS or Minkowski vacua with unbroken supersymmetry

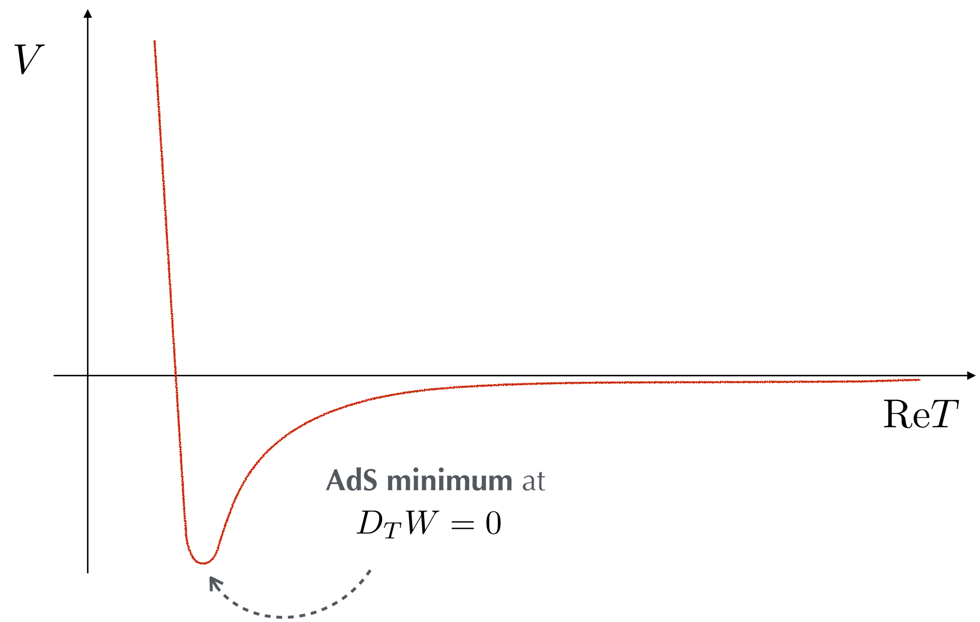
KKLT

Kachru, Kallosh, Linde, Trivedi 2003

$$K = -3\log(T+\bar{T})$$

$$W = W_0 + A\exp(-aT)$$

$$V = V_T$$

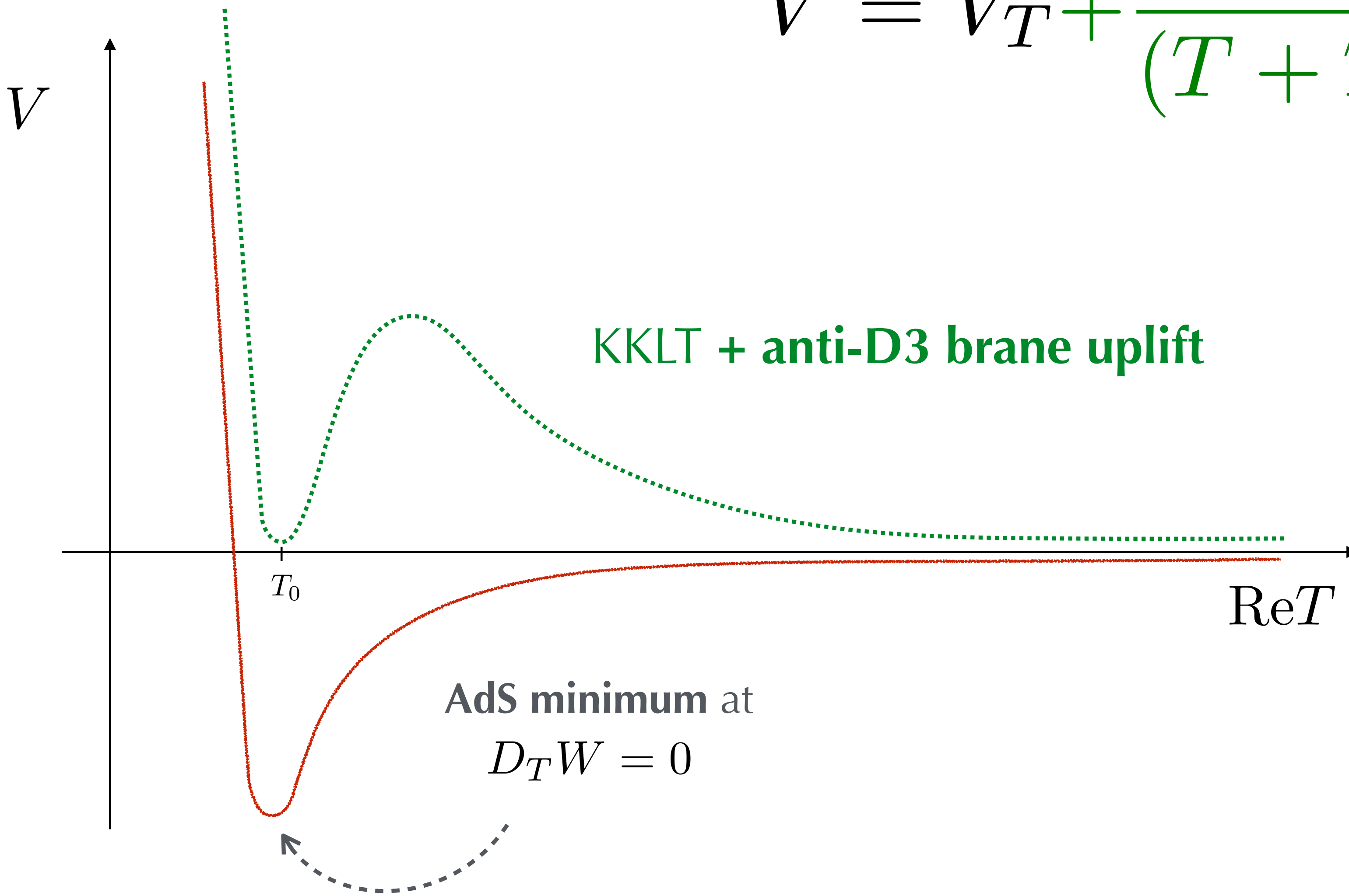


Stabilization of the volume of the extra six dimensions (Calabi-Yau manifold)

Negative CC

Still 2003, positive energy from the anti-D3 brane

$$V = V_T + \frac{\mu^4}{(T+\bar{T})^2}$$



D=4 Supergravity Language

$$K = -3\log(T + \bar{T} - S\bar{S})$$

$$W = W_0 + A\exp(-aT) + \mu^2 S$$

The nilpotent superfield

Represents anti-D3 brane

$$S(x,\theta) = s(x) + \sqrt{2}\lambda(x)\theta + F(x)\theta^2$$

$$S^2(x,\theta) = 0$$

Volkov, Akulov 1972,1973
Rocek; Ivanov, Kapustnikov 1978
Lindstrom, Rocek 1979
Casalbuoni, De Curtis, Dominici, Feruglio, Gatto 1989
Komargodski, Seiberg 2009

$$\Downarrow$$

$$S(x,\theta) = \frac{\lambda\lambda}{2F} + \sqrt{2}\lambda\theta + F\theta^2$$

no scalar! **just fermions!**

The scalar is a bilinear of a goldstino fermions, not a fundamental field

Supersymmetric uplift!

Standard linear SUSY

1 Majorana fermion 1 complex scalar

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu A)^2 - \tfrac{1}{2}(\partial_\mu B)^2 - \tfrac{1}{2}i\bar{\psi}\gamma^\mu\partial_\mu\psi - \tfrac{1}{2}m^2A^2 - \tfrac{1}{2}m^2B^2 - \tfrac{1}{2}im\bar{\psi}\psi - gmA(A^2+B^2) - \tfrac{1}{2}g^2(A^2+B^2)^2 - ig\bar{\psi}(A-\gamma_5 B)\psi\,.$$

Wess-Zumino, 1974: minimal SUSY with a Majorana fermion and a complex scalar

Gravity NO-GO for de Sitter

AdS/CFT studies

$$\sqrt{|g|}\,\Lambda \leq 0$$

LHC, as of June 2017

No SUSY partners yet

Non-linear SUSY

1 Majorana fermion 2 Majorana fermions

$$\mathcal{L} = -f^2 \det(1 + ig^2\psi^\mu\hat{\partial}_\mu\bar{\psi})$$

Volkov, Akulov, 1972 Non-linearly realized supersymmetry: only fermions are present

In pure supergravity, de Sitter vacua were constructed in 2015

Bergshoeff, Freedman, RK, Van Proeyen;
Hasegawa, Yamada;
Kuzenko

$$\sqrt{|g|}\,\Lambda = \sqrt{|g|}\,f^2 > 0$$

The State of SUSY

PASCOS 2017
July 19-23

Nathaniel Craig
UC Santa Barbara

Why Supersymmetry?

- ☑ Hierarchy problem
- ☑ Gauge coupling unification
- ☑ WIMP dark matter

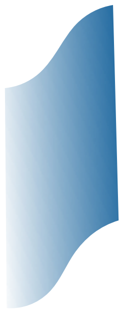
Conclusions

Non-observation of strongly-produced SUSY liberates us from tyranny of traditional motivations de Sitter

$V > 0$

String Theory

Anti-D3 Brane



Supergravity

Nilpotent Superfield

KKLT construction of de Sitter vacua in string theory: Positive energy from anti-D3 brane

Basic idea: D-brane and anti-D-brane are extended objects in superstring theory. Like strings, they have various possible descriptions.

1. They are solutions of d=10 supergravity

2. They have their own world-volume action

Anti-D-brane: **Volkov-Akulov beautiful geometric construction**

$$E = dX - \bar{\theta}\Gamma^m d\theta$$

ultimate spontaneously broken supersymmetry: Majorana goldstino

$$S^{\overline{\rm D3}} = -2T_3 \int d^4\sigma \, \det E$$

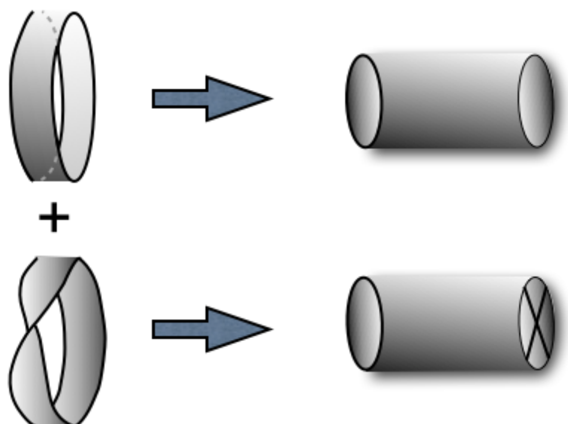
The action of the kappa-symmetric Dp-brane when all fields but fermions are truncated: the goldstino independent constant is positive!

String Theory Realizations of the Nilpotent Goldstino

RK, Quevedo, Uranga 2015

A technical tool for the string theory landscape construction and for inflationary model building

$$S^2(x, \theta) = 0$$



The one-loop open string annulus and Moebius strip diagrams turn into closed string channel diagrams describing tree level exchange of NSNS and RR states between two boundaries (branes or antibranes), or between one boundary and one crosscap (O3-plane)

Non-linear supersymmetry: not of the kind that was not found at LHC

Volkov-Akulov, 1972

Allows de Sitter vacua in supergravity without scalars

String theory D-branes: supersymmetric KKLT

RK, Wrase, 2014
Bergshoeff, Dasgupta, RK, Wrase, Van Proeyen 2015
RK, Quevedo, Uranga 2015
RK, Vercnocke , Wrase 2016

Cosmological Models with nilpotent stabilizer

Antoniadis, Dudas, Ferrara and Sagnotti, 2014
Ferrara, RK, Linde 2014 Dall'Agata, Zwirner 2014
RK, Linde, Scalisi, 2014
Carrasco, RK, Linde, Roest, 2015
McDonough, Scalisi, 2016
Ferrara, RK, 2016
RK, Linde, Wrase,Yamada, 2017

The positive contribution to the vacuum energy which converts the AdS minimum into dS minimum due to the presence of the non-perturbative anti-D3 brane can be effectively described in supergravity by the presence of the nilpotent superfield S

$$S^2 = 0$$

RK, Linde, Roest, Yamada, 2017

Anti-D3 Induced Geometric Inflation

On complex **Kahler Geometry**

A. Moroianu
2004

1.Kahler structures were introduced by Erich Kahler in his article in **1933** with the following motivation. Given any Hermitian metric on a complex manifold, we can express the fundamental two–form Ω in local holomorphic coordinates as follows:

$$\Omega = i\mathcal{G}_{\alpha\bar{\beta}} dz^{\alpha} d\bar{z}^{\bar{\beta}}$$

He then noticed that the condition $d\Omega = 0$ is equivalent to the local existence of some function $\mathcal{G}$

$$\mathcal{G}_{\alpha\bar{\beta}} = \frac{\partial^2 \mathcal{G}}{\partial z^{\alpha} \partial \bar{z}^{\bar{\beta}}}$$

In other words, the whole metric tensor is defined by a unique function! This remarkable (bemerkenswert) property of the metric allows one to obtain simple explicit expressions for the Christoffel symbols and the Ricci and curvature tensors, and **“a long list of miracles occur then”.** The function $\mathcal{G}$ is called **Kahler function**

(Kahler potential K and superpotential W versus a Kahler function $\mathcal{G}$)

Example of Kahler geometry: Poincare disk of unit size, hyperbolic 2d geometry

$$ds^2 = \frac{dx^2 + dy^2}{(1 - x^2 + y^2)^2} = \frac{dZ d\bar{Z}}{(1 - Z\bar{Z})^2}$$

If we assume that scalars in cosmology are coordinates of the Poincare disk, and use in in the context of inflationary model building, we find the universal formula

$$n_s = 1\text{-}2/N$$

which fits the data!

If we use the Poincare disk of the size $R^2=3\alpha$

$$ds^2 = 3\alpha \frac{dZ d\bar{Z}}{(1 - Z\bar{Z})^2}$$

we find the universal formula for

$$r = 3\alpha \;\; 12/N^2$$

which is waiting for the data on B-modes!

Kahler Geometry and Supersymmetry

One can have Kahler geometry in the bosonic theory, for example Calabi-Yau complex manifolds, which often show up in compactification of string theory from d=10 to d=4

Kahler geometry is a necessary condition for supersymmetry, but in application to cosmology the most relevant point is that the kinetic term of scalar field is defined by Kahler geometry.

Maximal superconformal supersymmetry requires a unit size Poincare disk geometry for the complex scalar in this theory

First constructed here in the Netherlands

Extended Conformal Supergravity E. Bergshoeff, M. de Roo, B. de Wit **Nucl. Phys. (1981)**

Conformal Invariance In Supergravity Eric Arnold Bergshoeff **Ph.D. Thesis (1983)**

Matter Coupling in N=4 Supergravity M. de Roo **Nucl. Phys. (1985)**

Related work by de Wit, van Holten, Van Proeyen

$$ds^2 = \frac{dZ d\bar{Z}}{(1 - Z\bar{Z})^2}$$

Relevant inflationary models with

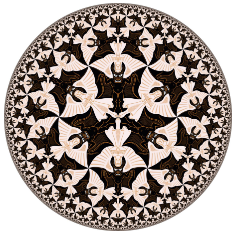
$$ds^2 = 3\alpha \frac{dZ d\bar{Z}}{(1 - Z\bar{Z})^2}$$

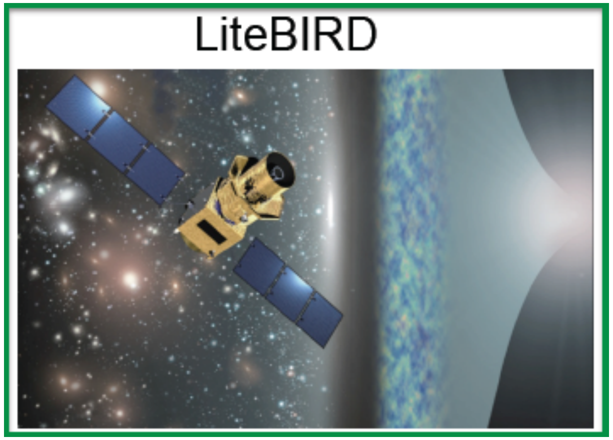
Prediction for **primordial gravity waves**

Escher in the Sky

$\alpha = 1/3 \quad r \approx 10^{-3}$

$\mathcal{N}$=4 supergravity,
maximal superconformal $\mathcal{N}$=4 model,
maximal supergravity $\mathcal{N}$=8





CMB mission target, 2025-

$r = 10^{-3}$

Published and forecasted r constraints