Gravitational Radiation:

2. Astrophysical and Cosmological Sources of Gravitational Waves, and the Information They Carry

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PDFs of lecture slides are available at
http://www.cco.caltech.edu/~kip/LorentzLectures/
each Thursday night before the Friday lecture

Outline

- **Introduction:** EM and Grav'l waves contrasted; four GW frequency bands - detector and source summaries
- **Review** of gravitational waves and their generation
- **Sources:** Their physics and the information they carry [delay most of astrophysics to next week, with detection]
  - Laboratory Sources
  - Binary systems with circular orbits: Newtonian, Post-Newton
  - EMRIs: Extreme Mass Ratio Inspirals
  - Black-hole (BH) dynamics (Normal modes of vibration)
  - BH/BH binaries: inspiral, collision, merger, ringdown
  - BH/NS (neutron star) binaries: inspiral, tidal disruption [GRBs]
  - NS/NS binaries: inspiral, collision, ... [GRBs]
  - NS dynamics (rotation, vibration) [Pulsars, LMXBs, GRBs, Supernovae]
  - Collapse of stellar cores: [Supernovae, GRBs]
  - Early universe: GW amplification by inflation, phase transitions, cosmic strings, ...

Introduction

Electromagnetic and Gravitational Waves Contrasted

- **Electromagnetic Waves**
  - Oscillations of EM field propagating through spacetime
  - Incoherent superposition of waves from particles atoms, molecules
  - Easily absorbed and scattered

- **Gravitational Waves**
  - Oscillations of “fabric” of spacetime itself
  - Coherent emission by bulk motion of matter
  - Never significantly absorbed or scattered

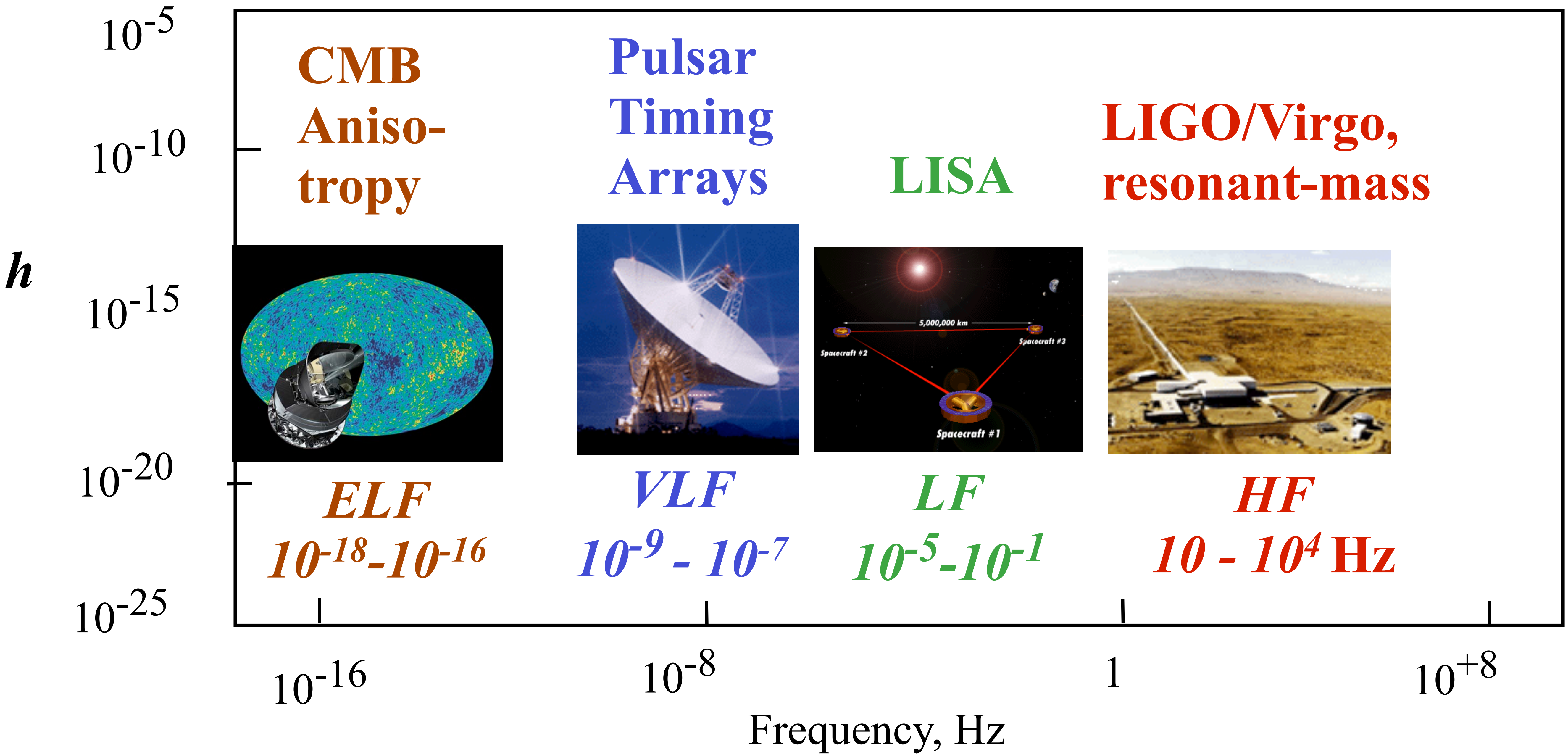
- **Implications**
  - Many GW sources won’t be seen electromagnetically
  - Surprises are likely
  - Revolution in our understanding of the universe, like those that came from radio waves and X-rays?

Electromagnetic and Gravitational Waves Contrasted

- **Electromagnetic Waves**
  - Usually observe time evolving spectrum (amplitude, not phase)
  - Most detectors very large compared to wavelength
    ⇒narrow field of view;
    good angular resolution, λ/D
  - Most sources very large compared to wavelength
    ⇒can make pictures of source

- **Gravitational Waves**
  - Usually observe waveforms $h_+(t)$ and $h_\times(t)$ in time domain (amplitude and phase)
  - Most detectors small compared to wavelength
    ⇒ see entire sky at once;
    poor angular resolution
  - Sources are not large compared to wavelength
    ⇒cannot make pictures;
    instead, learn about source from waveform (like sound)

Frequency Bands and Detectors



Some Sources in Our Four Bands

ELF	VLF	LF	HF
CMB Polarization	Pulsar Timing	LISA	Resonant-mass LIGO/VIRGO

The Big Bang Singularity (Planck era); Inflation

Exotic Physics in Very Early Universe: Phase transitions, cosmic strings, domain walls, mesoscopic excitations, … ?

Supermassive BH's (> one billion suns)

Massive BH's (300suns to 30 million suns),
EMRIs
Massive BH/BH

Binary stars

Soliton stars?

Naked singularities?

Small BH's (2 to 1000 suns),

Neutron stars

BH/BH, NS/BH, NS/NS binaries

Supernovae

Boson stars?

Naked singularities?

Review of GWs and their Generation

GWs: Review

- The gravitational-wave field, $h_{jk}^{\rm GW}$

  Symmetric, transverse, traceless (TT);
  two polarizations: +, x

- \+ Polarization

$$h_{xx}^{\rm GW} = h_+(t - z/c) = h_+(t - z)$$

$$h_{yy}^{\rm GW} = -h_+(t - z)$$

Lines of force

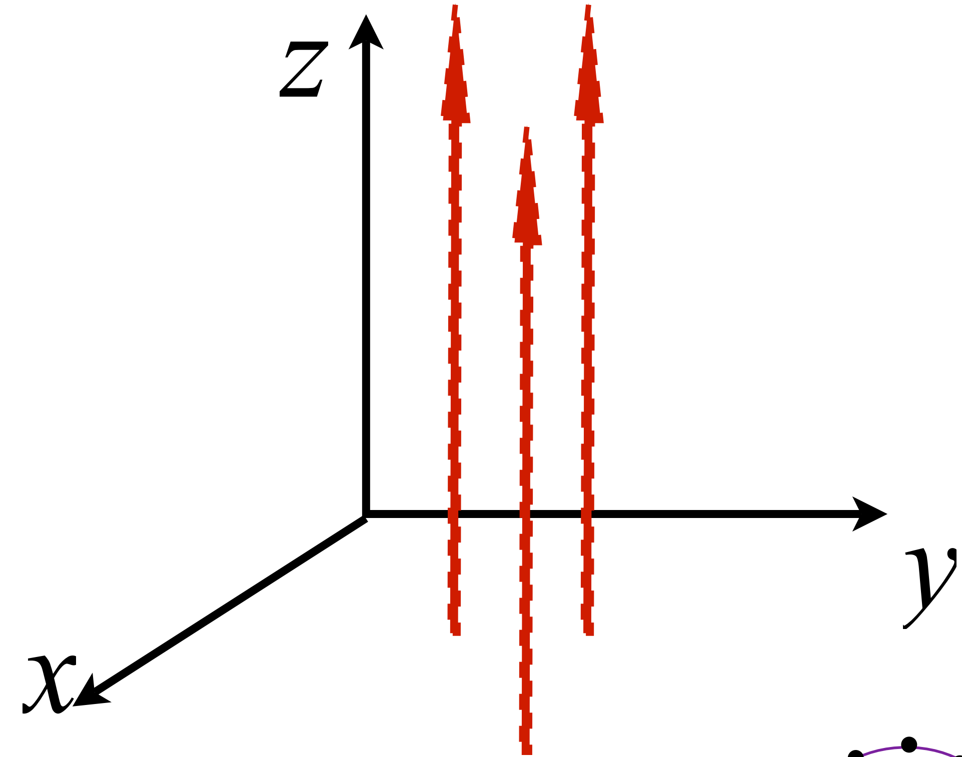
$$\ddot{x}_j = \frac{1}{2}\ddot{h}_{jk}^{\rm GW} x_k$$

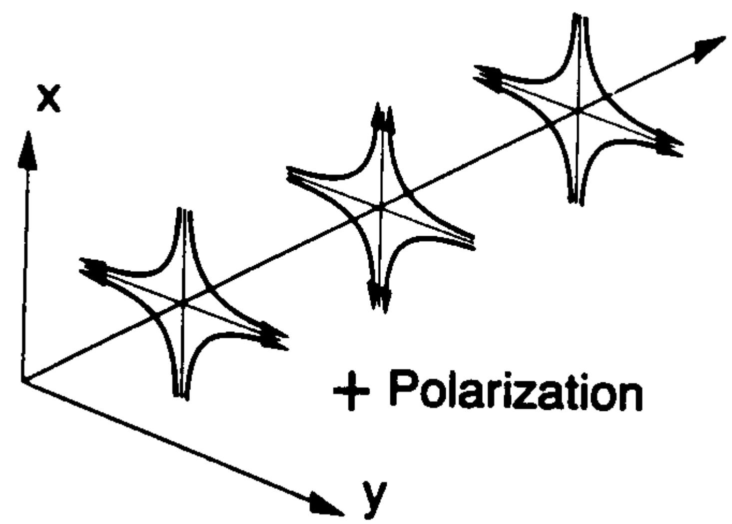
$$\ddot{x} = \ddot{h}_+ x$$

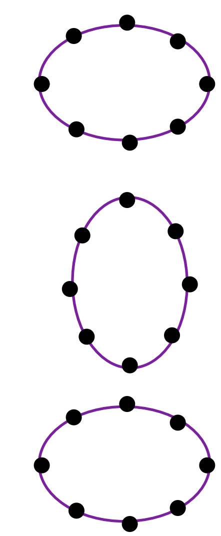
$$\ddot{y} = -\ddot{h}_+ y$$

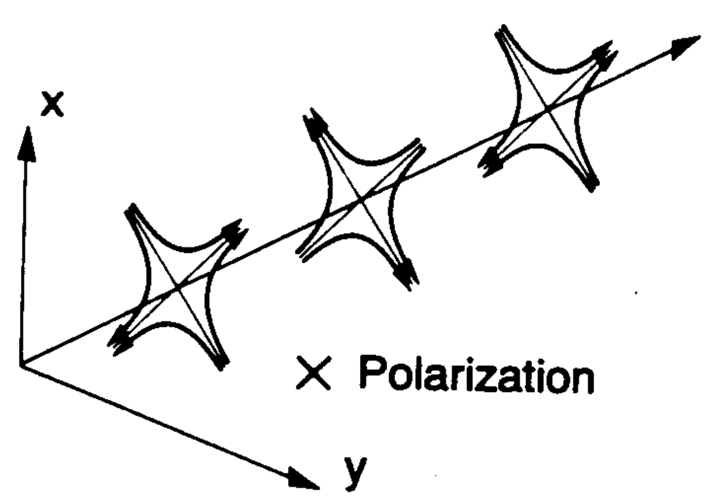
- x Polarization

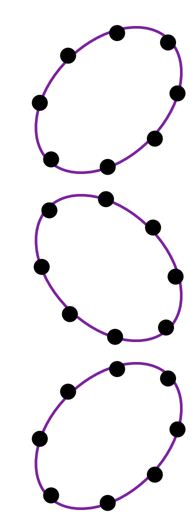
$$h_{xy}^{\rm GW} = h_{yx}^{\rm GW} = h_\times(t - z)$$











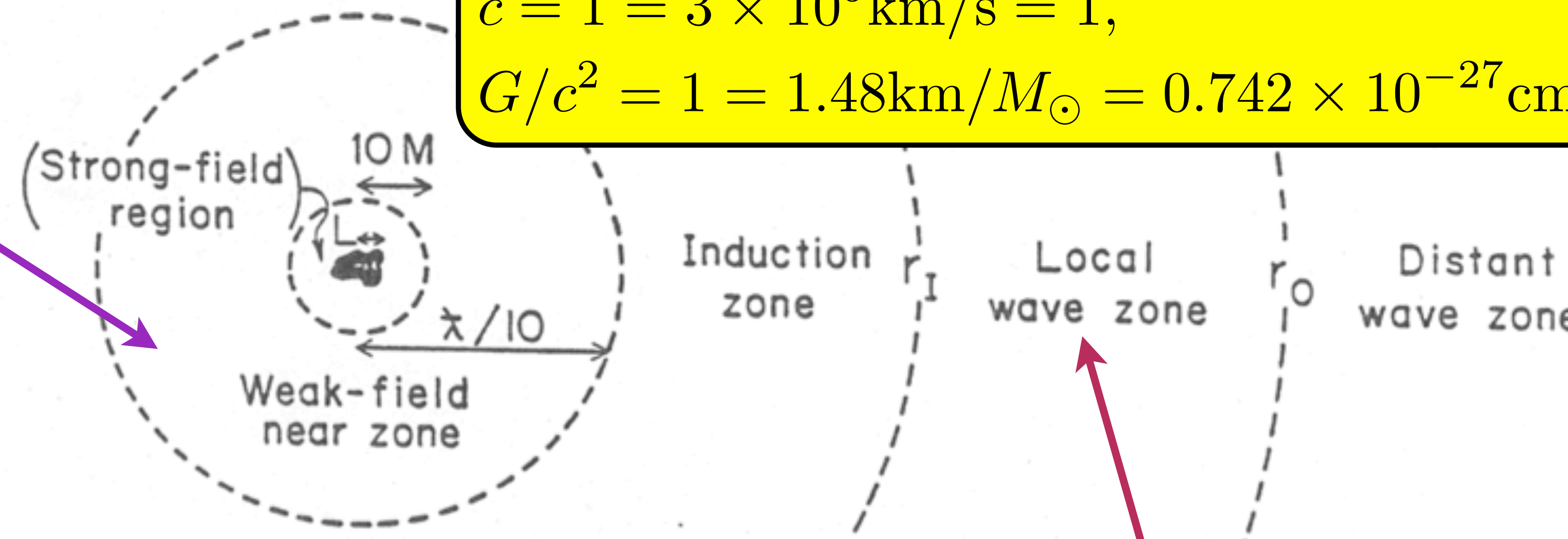
GW Generation: Slow-Motion Sources

$c = 1 = 3 \times 10^5 \mathrm{km/s} = 1,$

$G/c^2 = 1 = 1.48 \mathrm{km}/M_\odot = 0.742 \times 10^{-27} \mathrm{cm/g}$

$$\Phi = -\frac{M}{r} - \frac{3}{2}\frac{\mathcal{I}_{jk} n_j n_k}{r^3} + \ldots$$

quadrupole moment



for Newtonian source $\mathcal{I}_{jk}^{\mathrm{GW}} = \int \rho \left(x^j x^k - \frac{1}{3} r^2 \delta_{jk} \right) d^3 x$

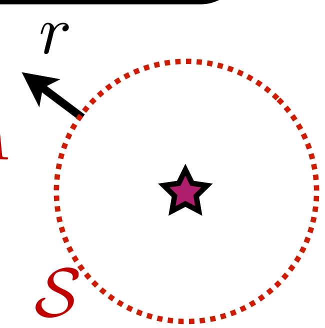
$$h_{jk}^{\mathrm{GW}} = 2 \left(\frac{\ddot{\mathcal{I}}_{jk}(t-r)}{r} \right)^{\mathrm{TT}}$$

$$h_{jk}^{\mathrm{GW}} \sim h_+ \sim h_\times \sim \frac{E_{\mathrm{kin}}^{\mathrm{quad}}/c^2}{r} \sim 10^{-21} \left(\frac{E_{\mathrm{kin}}^{\mathrm{quad}}}{M_\odot c^2} \right) \left(\frac{100\mathrm{Mpc}}{r} \right)$$

$$h_+ = \frac{\ddot{\mathcal{I}}_{\theta\theta} - \ddot{\mathcal{I}}_{\varphi\varphi}}{r}$$

$$h_\times = \frac{2\ddot{\mathcal{I}}_{\theta\varphi}}{r}$$

$$\frac{dM}{dt} = -\frac{dE_{\mathrm{GW}}}{dt} = -\oint_{\mathcal{S}} T_{\mathrm{GW}}^{tr} dA = -\frac{1}{16\pi} \oint_{\mathcal{S}} \langle \dot{h}_+^2 + \dot{h}_\times^2 \rangle dA$$



$$\frac{dE_{\mathrm{GW}}}{dt} = \frac{1}{5} \dddot{\mathcal{I}}_{jk} \dddot{\mathcal{I}}_{jk}$$

$$\frac{dE_{\mathrm{GW}}}{dt} \sim \mathcal{P}_o \left(\frac{\mathcal{P}^{\mathrm{quad}}}{\mathcal{P}_o} \right)^2$$

Internal power flow in quadrupolar motions

$$\mathcal{P}_o = \frac{c^5}{G} = 3.6 \times 10^{52} \mathrm{W} = 3.6 \times 10^{59} \mathrm{erg/s} \sim 10^{27} L_\odot \sim 10^7 L_{\mathrm{EM\ universe}}$$

Sources: Their Physics and The Information they Carry

Laboratory Sources of GWs

- **Me waving my arms**

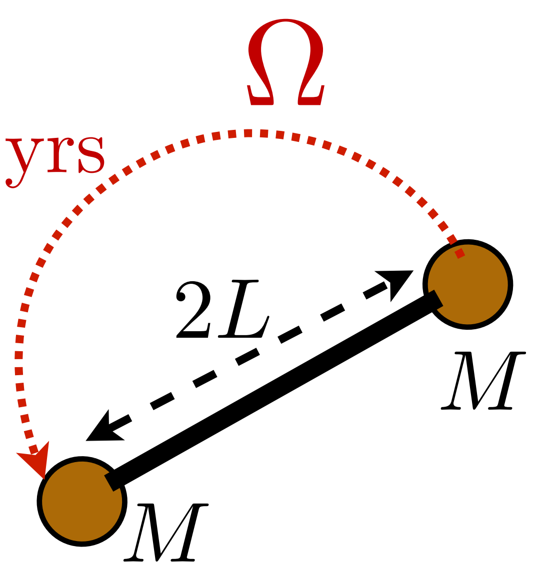
$$\mathcal{P}^{\rm quad} \sim \frac{(10\ \text{kg})(5\ \text{m/s})^2}{1/3\ \text{s}} \sim 100\text{W}$$

$$\frac{dE_{\rm GW}}{dt} \sim 4 \times 10^{52}\ \text{W} \left(\frac{100\ \text{W}}{4 \times 10^{52}\ \text{W}}\right)^2 \sim 10^{-49}\ \text{W}$$

Each graviton carries an energy

$$\hbar\omega = (7 \times 10^{-34}\text{joule s})(2Hz) \sim 10^{-33}\text{joule}$$

I emit 10^{-16} gravitons $\text{s}^{-1} \sim 3$ gravitons each 1 billion yrs



- **A rotating two tonne dumb bell**

$$M = 10^3\text{kg}, \quad L = 5\text{m}, \quad \Omega = 2\pi \times 10/s$$

$$\mathcal{P}^{\rm quad} \sim \Omega\, M (L\Omega)^2 \sim 10^{10}\ \text{W}$$

$$\frac{dE_{\rm GW}}{dt} \sim 4 \times 10^{52}\ \text{W} \left(\frac{10^{10}\ \text{W}}{4 \times 10^{52}\ \text{W}}\right)^2 \sim 10^{-33}\ \text{W} \quad \hbar\,(2\Omega) \sim 10^{-32}\text{joule}$$

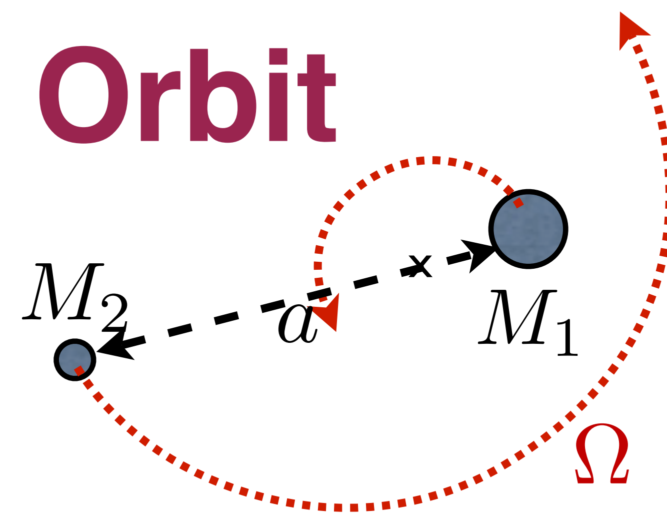
1 graviton emitted each 10 s

At $r = (1\ \text{wavelength}) = 10^4\ \text{km},\ h_+ \sim h_\times \sim 10^{-43}$

Generation and detection of GWs in lab is hopeless

Binary Star System: Circular Orbit

$$I_{jk} = \int \rho x_j x_k d^3x \qquad \text{trace} = \mu a^2 = \text{const}$$



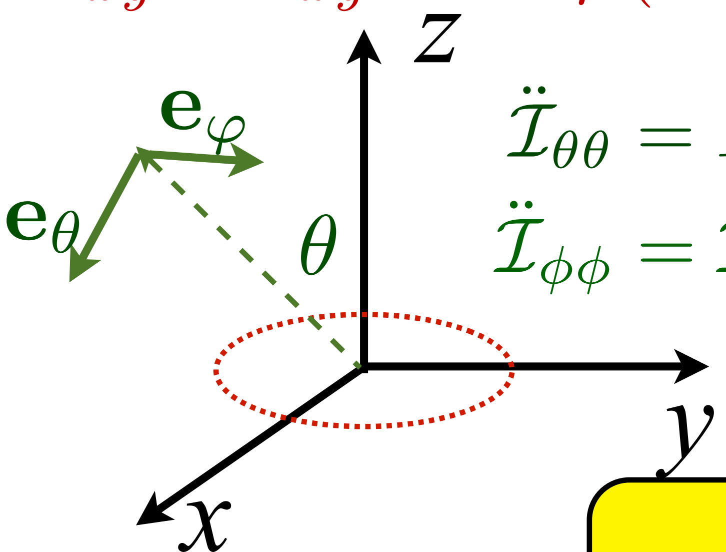
$$I_{xx} = \mu a^2 \cos^2 \Omega t, \; I_{yy} = \mu a^2 \sin^2 \Omega t, \; I_{xy} = \mu a^2 \cos \Omega t \sin \Omega t$$

$$\ddot{\mathcal{I}}_{xx} = \ddot{I}_{xx} = -2\mu(a\Omega)^2 \cos 2\Omega t, \; \ddot{\mathcal{I}}_{yy} = \ddot{I}_{yy} = 2\mu(a\Omega)^2 \cos 2\Omega t$$

$$\ddot{\mathcal{I}}_{xy} = \ddot{I}_{xy} = -2\mu(a\Omega)^2 \sin 2\Omega t$$

$$M = M_1 + M_2$$

$$\mu = \frac{M_1 M_2}{M}, \Omega = \sqrt{M/a^3}$$



$$\ddot{\mathcal{I}}_{\theta\theta} = \ddot{I}_{xx} \cos^2 \theta + \ddot{I}_{zz} \sin^2 \theta \quad (\ddot{I}_{zz} \to 0)$$

$$\ddot{\mathcal{I}}_{\phi\phi} = \ddot{\mathcal{I}}_{yy}, \; \ddot{\mathcal{I}}_{\theta\phi} = \ddot{\mathcal{I}}_{xy} \cos \theta$$

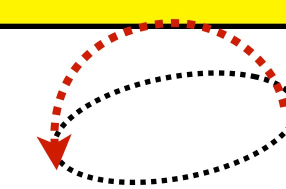
$$f = \frac{2\Omega}{2\pi} = \frac{1}{\pi}\sqrt{\frac{M}{a^3}} = 200\text{Hz}\left(\frac{10M_\odot}{M}\right)\left(\frac{10M}{a}\right)^{3/2}$$

$= 10^{-4}$ to 10^{-3} Hz for EM-observed compact binaries

$= 30$ to 3000 Hz for final stages of NS/NS, BH/BH

$$h_+ = 2\frac{\ddot{\mathcal{I}}^{\rm TT}_{\theta\theta}}{r} = \frac{(\ddot{\mathcal{I}}_{\theta\theta} - \ddot{\mathcal{I}}_{\varphi\varphi})}{r} = -2(1+\cos^2\theta)\frac{\mu(a\Omega)^2}{r}\cos 2\Omega(t-r)$$

$$h_\times = 2\frac{\ddot{\mathcal{I}}^{\rm TT}_{\theta\phi}}{r} = -4\cos\theta\frac{\mu(a\Omega)^2}{r}\sin 2\Omega(t-r)$$



- Angular dependence comes from TT projection
- As seen from above, θ=0, circular polarized: $h_+ = A\cos 2\Omega t, \; h_\times = A \sin 2\Omega$
- As seen edge on, θ=π/2, linear polarized: $h_+ = A\cos 2\Omega t, \; h_\times = 0$

Binary Star System: Circular Orbit

- **Energy Loss ⇒Inspiral; frequency increase: "Chirp"**

$$\frac{dE_{\rm GW}}{dt} = \frac{1}{5}\left[(\dddot{\mathcal{I}}_{xx})^2 + (\dddot{\mathcal{I}}_{yy})^2 + 2(\dddot{\mathcal{I}}_{xy})^2\right] = \frac{32}{5}\mu^2 a^4 \Omega^6 = \frac{32}{5}\frac{\mu^2 M^3}{a^5}$$

$$\frac{dE_{\rm binary}}{dt} = \frac{d}{dt}\left(\frac{-\mu M}{2a}\right) = -\frac{32}{5}\frac{\mu^2 M^3}{a^5}$$

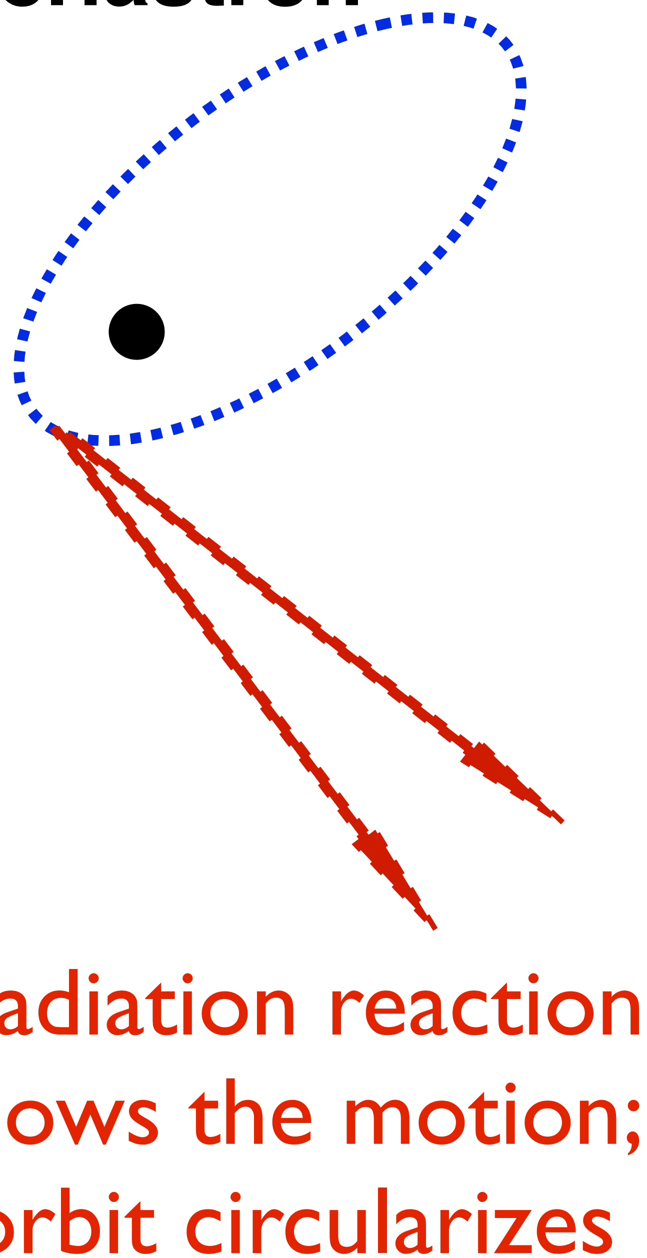
$$a = a_o(1 - t/\tau)^{1/4}, \qquad \tau_o = \frac{5}{256}\frac{a_o^4}{\mu M^2} = \frac{5\mathcal{M}}{256(\mathcal{M}\Omega)^{8/3}}$$

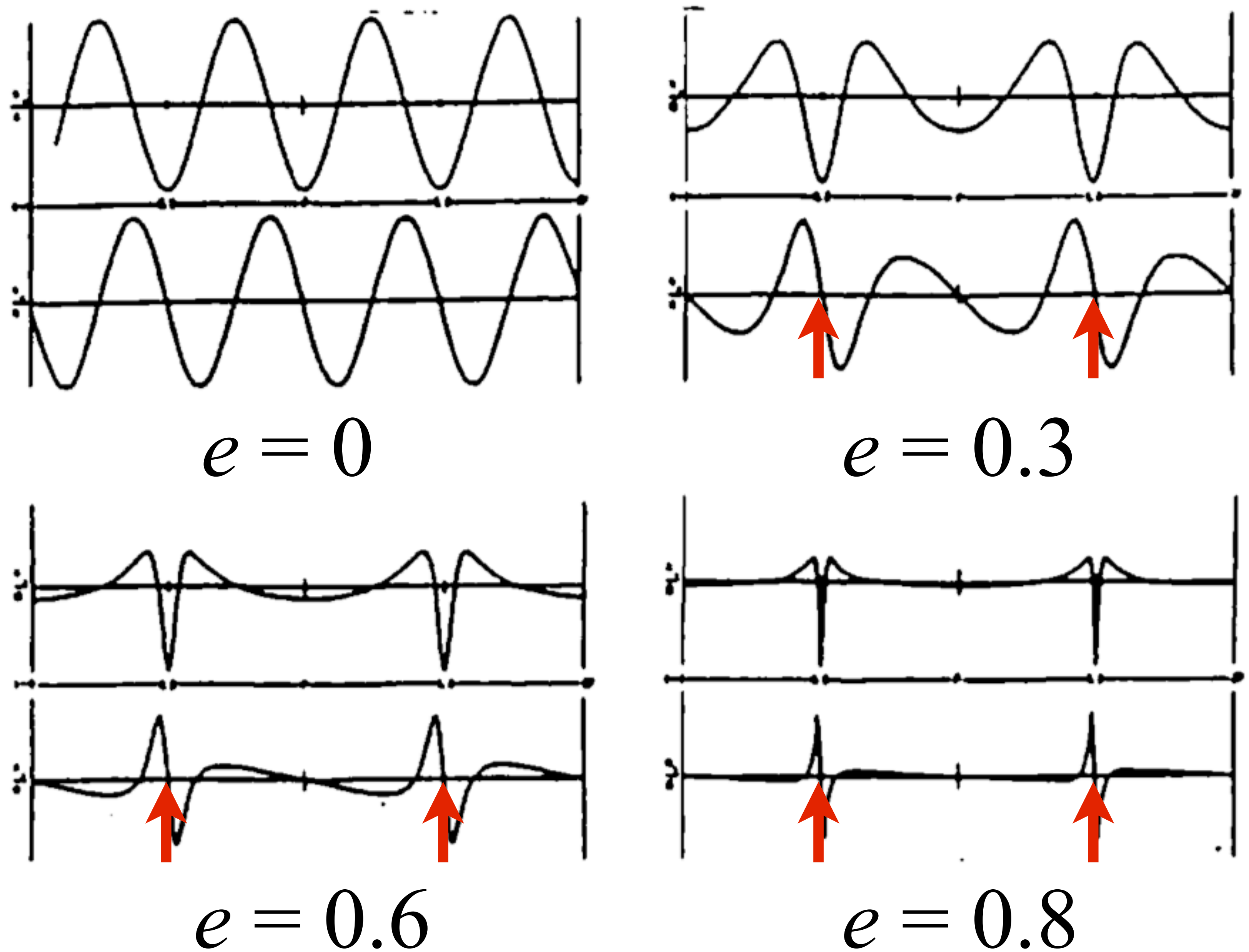
$$\mathcal{M} = \text{chirp mass} = \mu^{3/5} M^{2/5}$$

- **Observables:**
  - from h_+ & $h_\times \sim \cos(2\Omega t\text{+phase})$: **GW frequency** f=Ω/π
  - from df/dt = -3f/8τ_o: **Time to merger τ_o and chirp mass $\mathcal{M}$**
  - from GW amplitudes $h_+^{\rm amp} = -2(1+\cos\theta)\frac{\mu(a\Omega)^2}{r} = -2(1+\cos\theta)\frac{\mathcal{M}(\pi\mathcal{M}f)^{2/3}}{r}$ and $h_\times^{\rm amp} = -4\cos\theta\frac{\mathcal{M}(\pi\mathcal{M}f)^{2/3}}{r}$: **Orbital inclination angle θ and distance to source *r***
- **At Cosmological Distances:** $\mathcal{M}(1+z)$, **Luminosity distance**
  - complementary to EM astronomy, where z is the observable

Why Are Circular Orbits Expected?

- **For elliptical orbits: GWs emitted most strongly at periastron**





Late Inspiral of Binary: Post-Newtonian Theory

- **As binary shrinks, it becomes more relativistic:** $v \simeq \Omega a \simeq \sqrt{M/a}$ **increases toward** $1 = c$
  - Post-Newtonian corrections become important
- **High-precision waveforms are needed in GW data analysis**
  - Compute via "PN expansion" - expand in $v \simeq \Omega a \simeq \sqrt{M/a}$
  - Example: Equation of Motion $d^2 x_1^i = a_1^i$ *without spins!*

$$a_1^i = -\frac{Gm_2 n_{12}^i}{r_{12}^2}$$

Leading term: Newtonian gravity.

$$+\frac{1}{c^2}\left\{ \left[\frac{5G^2 m_1 m_2}{r_{12}^3} + \frac{4G^2 m_2^2}{r_{12}^3} + \frac{Gm_2}{r_{12}^2}\left(\frac{3}{2}(n_{12}v_2)^2 - v_1^2 + 4(v_1 v_2) - 2v_2^2\right)\right] n_{12}^i + \frac{Gm_2}{r_{12}^2}\left(4(n_{12}v_1) - 3(n_{12}v_2)\right) v_{12}^i \right\}$$

1PN Corrections: $\sim (v/c)^2$

Slide adapted from Scott Hughes

Late Inspiral of Binary: Post-Newtonian Theory

2PN Corrections

~(v/c)[4]

$$
\begin{aligned}
&+\frac{1}{c^4}\Bigg\{\Bigg[-\frac{57G^3m_1^2m_2}{4r_{12}^4}-\frac{69G^3m_1m_2^2}{2r_{12}^4}-\frac{9G^3m_2^3}{r_{12}^4}\\
&\quad+\frac{Gm_2}{r_{12}^2}\Bigg(-\frac{15}{8}(n_{12}v_2)^4+\frac{3}{2}(n_{12}v_2)^2v_1^2-6(n_{12}v_2)^2(v_1v_2)-2(v_1v_2)^2+\frac{9}{2}(n_{12}v_2)^2v_2^2\\
&\qquad+4(v_1v_2)v_2^2-2v_2^4\Bigg)\\
&\quad+\frac{G^2m_1m_2}{r_{12}^3}\left(\frac{39}{2}(n_{12}v_1)^2-39(n_{12}v_1)(n_{12}v_2)+\frac{17}{2}(n_{12}v_2)^2-\frac{15}{4}v_1^2-\frac{5}{2}(v_1v_2)+\frac{5}{4}v_2^2\right)\\
&\quad+\frac{G^2m_2^2}{r_{12}^3}\left(2(n_{12}v_1)^2-4(n_{12}v_1)(n_{12}v_2)-6(n_{12}v_2)^2-8(v_1v_2)+4v_2^2\right)\Bigg]n_{12}^i\\
&\quad+\Bigg[\frac{G^2m_2^2}{r_{12}^3}\left(-2(n_{12}v_1)-2(n_{12}v_2)\right)+\frac{G^2m_1m_2}{r_{12}^3}\left(-\frac{63}{4}(n_{12}v_1)+\frac{55}{4}(n_{12}v_2)\right)\\
&\qquad+\frac{Gm_2}{r_{12}^2}\Bigg(-6(n_{12}v_1)(n_{12}v_2)^2+\frac{9}{2}(n_{12}v_2)^3+(n_{12}v_2)v_1^2-4(n_{12}v_1)(v_1v_2)\\
&\qquad\quad+4(n_{12}v_2)(v_1v_2)+4(n_{12}v_1)v_2^2-5(n_{12}v_2)v_2^2\Bigg)\Bigg]v_{12}^i\Bigg\}\\
&+\frac{1}{c^5}\Bigg\{\left[\frac{208G^3m_1m_2^2}{15r_{12}^4}(n_{12}v_{12})-\frac{24G^3m_1^2m_2}{5r_{12}^4}(n_{12}v_{12})+\frac{12G^2m_1m_2}{5r^3}(n_{12}v_{12})v_{12}^2\right]n_{12}^i\\
&\quad+\left[\frac{8G^3m_1^2m_2}{5r_{12}^4}-\frac{32G^3m_1m_2^2}{5r_{12}^4}-\frac{4G^2m_1m_2}{5r_{12}^3}v_{12}^2\right]v_{12}^i\Bigg\}
\end{aligned}
$$

2.5 PN Corrections

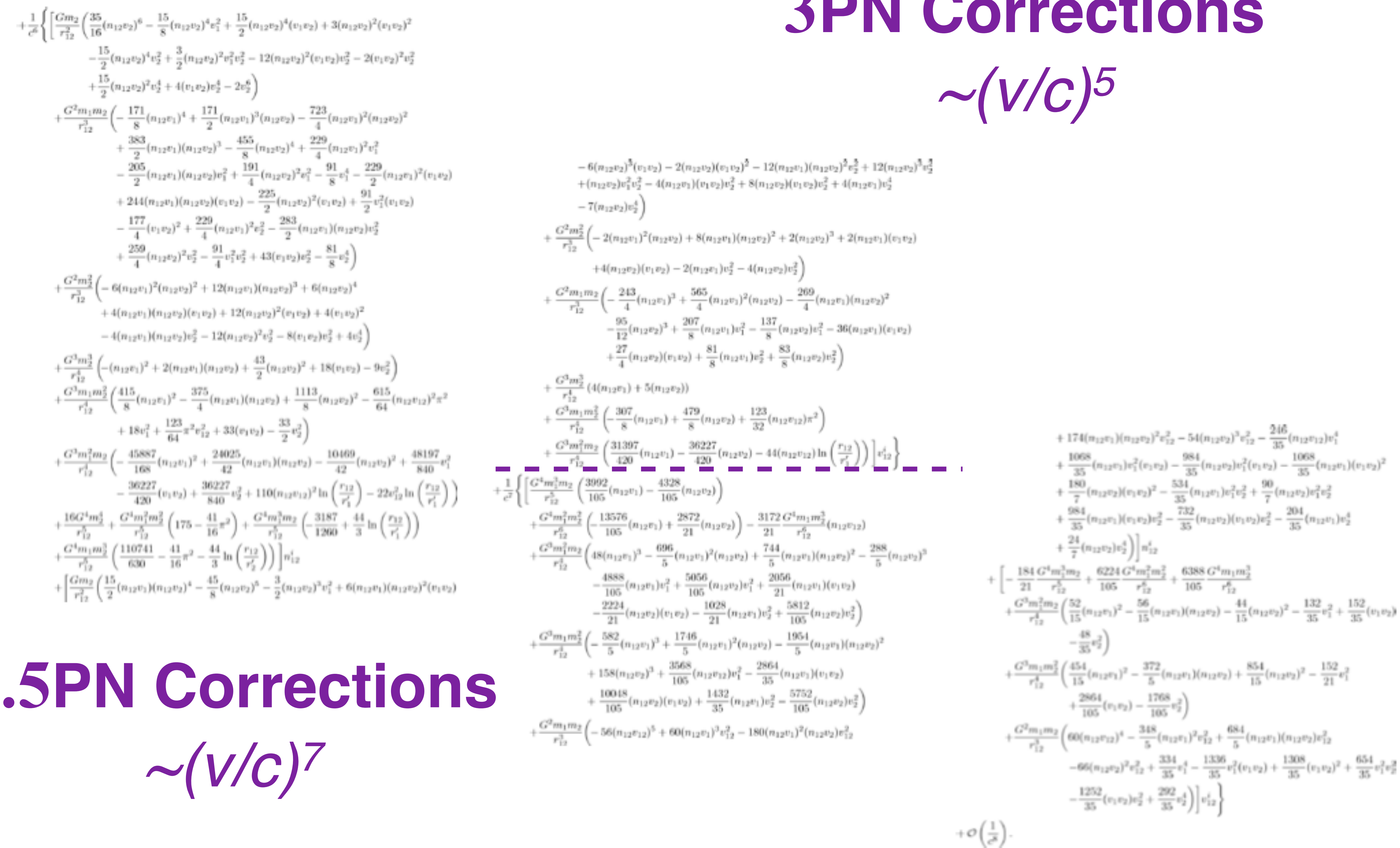
~(v/c)[5] rad'n reaction

Slide adapted from Scott Hughes

Late Inspiral of Binary: Post-Newtonian Theory

3PN Corrections

$\sim(v/c)^5$



3.5PN Corrections

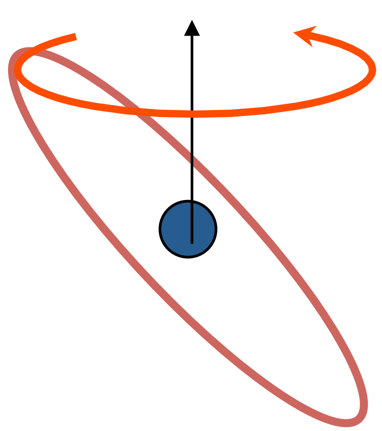
$\sim(v/c)^7$

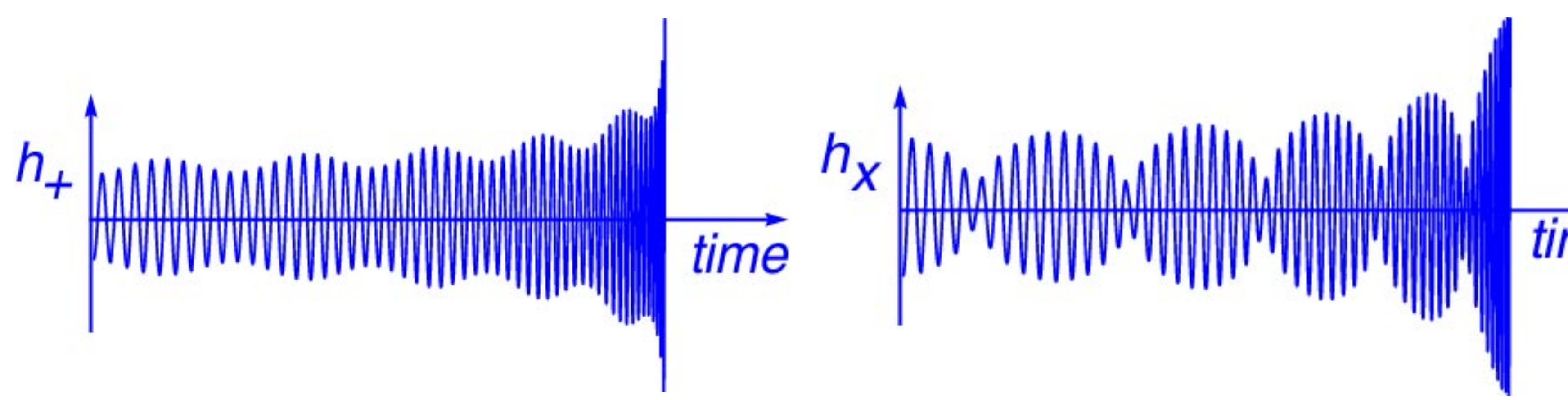
[Blanchet 2006, Living Reviews in Relativity, **9**, 4, Eq. (168)]

Slide adapted from Scott Hughes

Late Inspiral of Binary: Post-Newtonian Waveforms

- **PN Waveforms now known to 3.5PN order**
  - adequate for LIGO/VIRGO GW data analysis up to $v \simeq c/3$, $a \simeq 10M$
  - for black-hole binaries with $M_1 \simeq M_2$ about 10 orbits (20 cycles) of inspiral left
  - Thereafter: Numerical Relativity must be used
- **PN Waveforms carry much information:**
  - Mass ratio M_1/M_2, and thence, from Chirp mass: individual masses
  - Holes' vectorial spin angular momenta [Drag inertial frames; cause orbital precession, which modulates the waves]

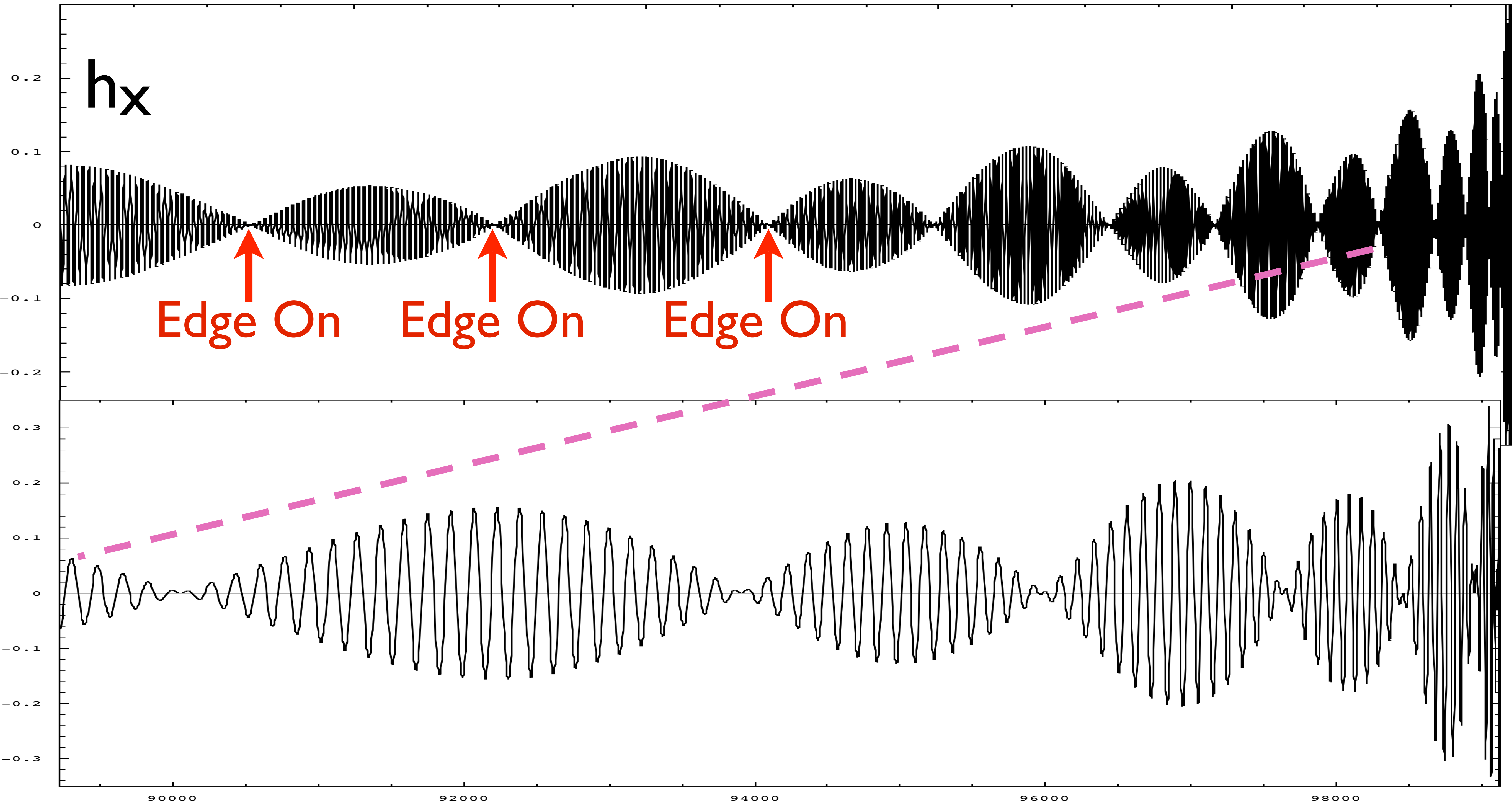




Late Inspiral of Binary: Post-Newtonian Waveforms

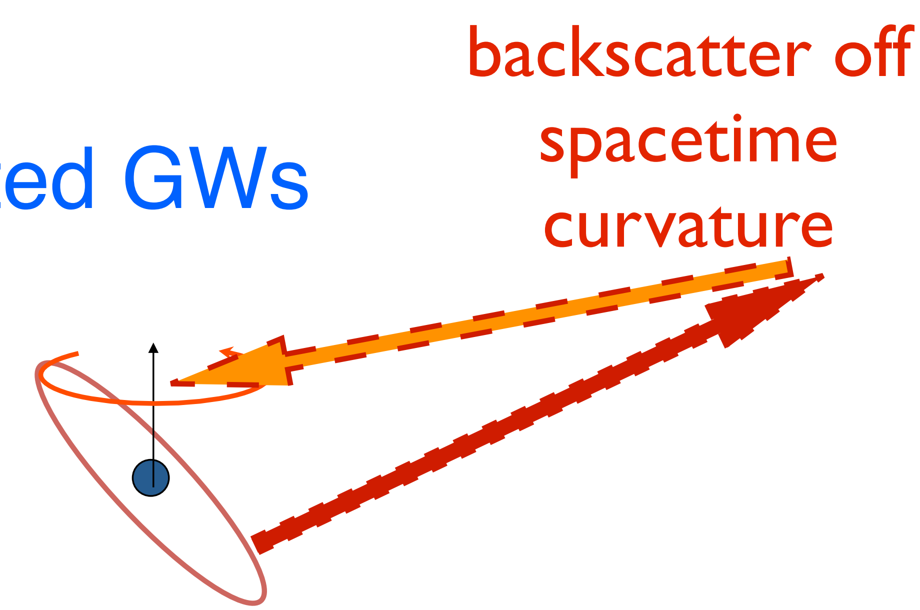
Frame Dragging

Example: Last ~10 secs for 1Msun/10Msun NS/BH binary



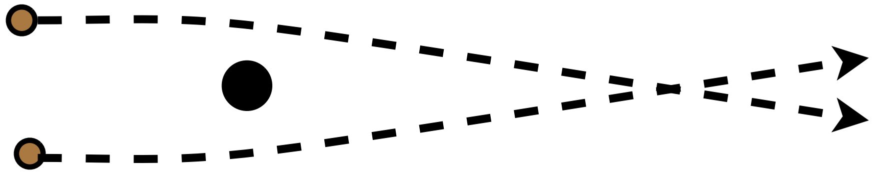
Late Inspiral of Binary: Post-Newtonian Waveforms

- **PN Waveforms also carry details of the transition from Newtonian gravity (at early times) to full general relativistic gravity (at late times)**
  - **High-precision observational studies of the transition: tests of general relativity; e.g.**
  - Periastron shift (familiar in solar system)
  - Frame dragging (see above)
  - Radiation reaction in source
  - PN corrections to all of these
  - Radiation reaction due to tails of emitted GWs and tails of tails
  - ...

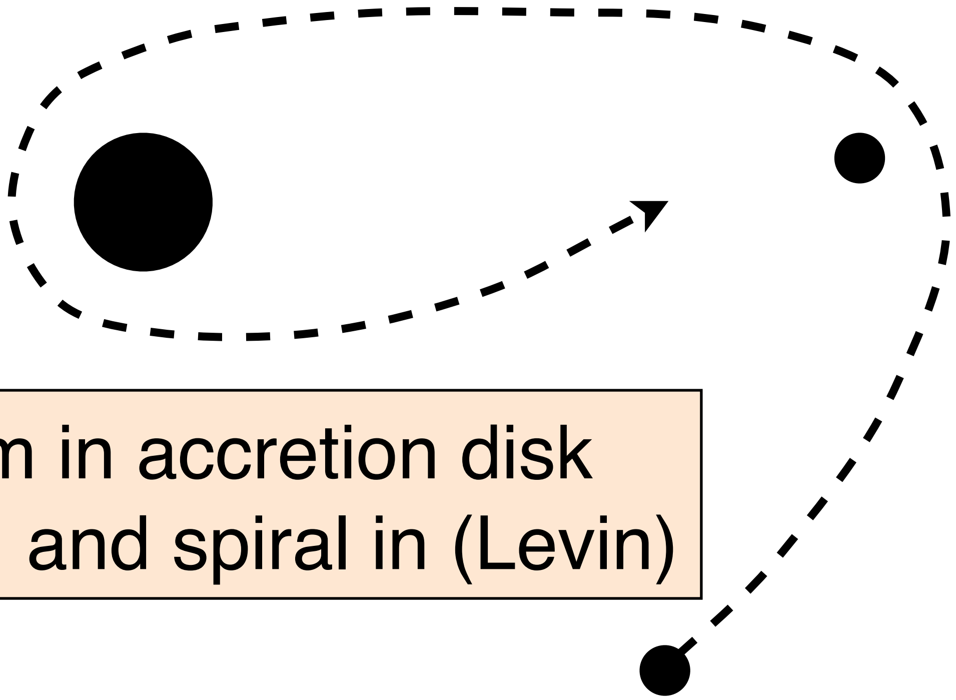


Extreme Mass Ratio Inspiral (EMRI)

- **The context:** stellar-mass black holes, neutron stars & white dwarfs orbiting supermassive black holes in galactic nuclei
  - more massive objects sink to center via "tidal friction"



  - objects occasionally scatter into highly elliptical orbits around SMBH
  - thereafter, radiation reaction reduces eccentricity to e ~ 0 to 0.8 at time of plunge into SMBH
  - typical numbers in LISA band: $M \simeq M_1 \sim 10^5$ to $10^7\ M_\odot$, $\mu \simeq M_2 \sim 1$ to $10\ M_\odot$,



BH may form in accretion disk around SMBH, and spiral in (Levin)

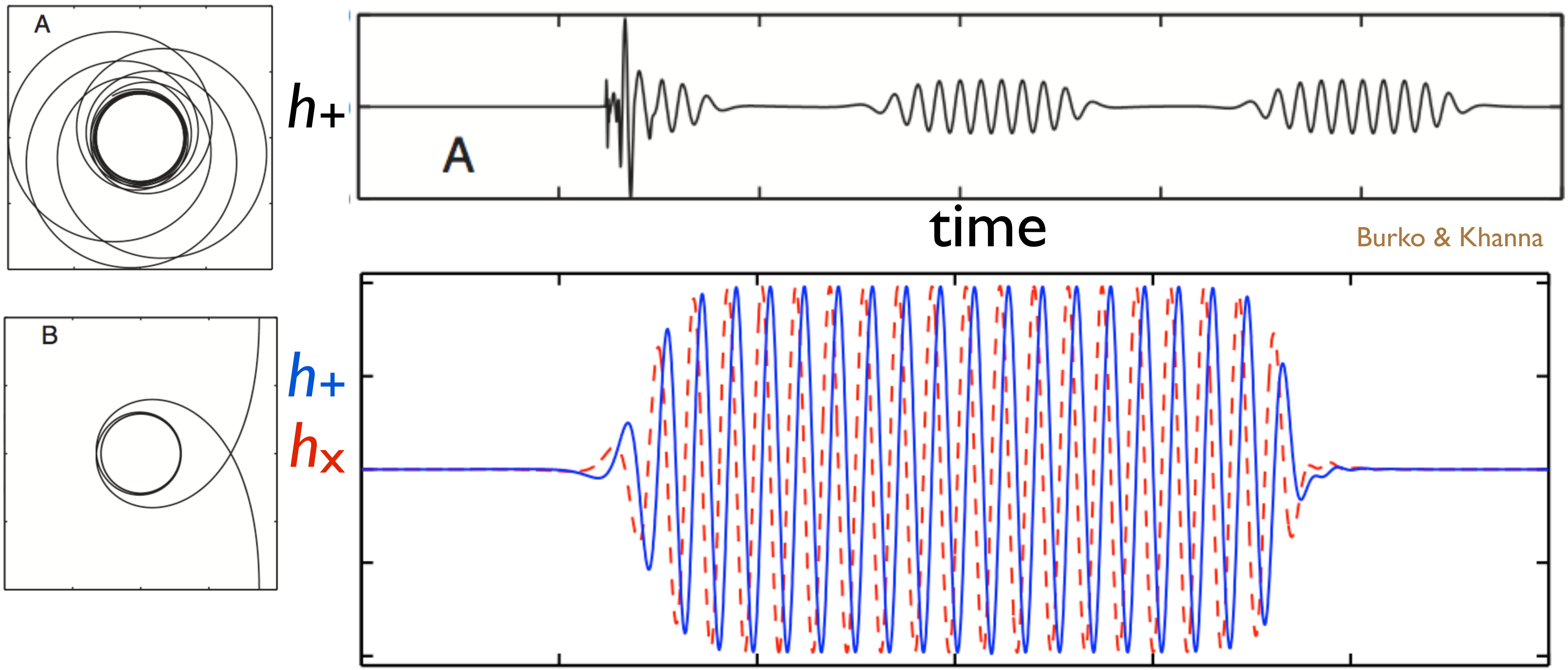
$$f \simeq \frac{1}{\pi}\sqrt{\frac{M}{a^3}} \sim 3\times 10^{-3}\ \mathrm{Hz}\ \left(\frac{M}{10^6 M_\odot}\right)\left(\frac{7M}{a}\right)^{3/2}$$

$$\tau \simeq \frac{5}{256}\frac{a^4}{\mu M^2} \sim 1\ \mathrm{yr}\ \left(\frac{M}{10^6 M_\odot}\right)^2\left(\frac{10 M_\odot}{\mu}\right)\left(\frac{a}{7M}\right)^4$$

$f\tau \sim 10^5$ cycles of waves in final year, all from $a \lesssim 7M \sim 4$ horizon radii

Extreme Mass Ratio Inspirals: EMRIs

- **Orbits around SMBH very complex, but integrable** (complete set of “isolating” constants of motion)
  - Equatorial examples:



  - Nonequatorial: Precession of orbit plane. If hole spins fast, then at small r, orbit spirals up and down like electron in a magnetic bottle

Extreme Mass Ratio Inspirals

- **If central body is not a black hole - but is axisymmetric** (e.g. boson star or naked singularity)
  - Numerical solutions of orbital (geodesic) equations: orbits almost always look integrable - not chaotic
  - "Fourth integral" appears to be quartic in momentum, $\mathcal{C} = C_{\alpha\beta\gamma\delta}(x^\mu)p^\alpha p^\beta p^\gamma p^\delta$
  - Progress toward proving so: J. Brink, Phys Rev D
- **Ryan's Mapping Theorem**
  - If orbit is indeed integrable, then waveforms carry:
    1. Full details of the orbit, and
    2. A full map of the central body's spacetime geometry
  - Has been proved only for equatorial orbits, but function counting suggests true for generic orbits

Black-Hole Dynamics

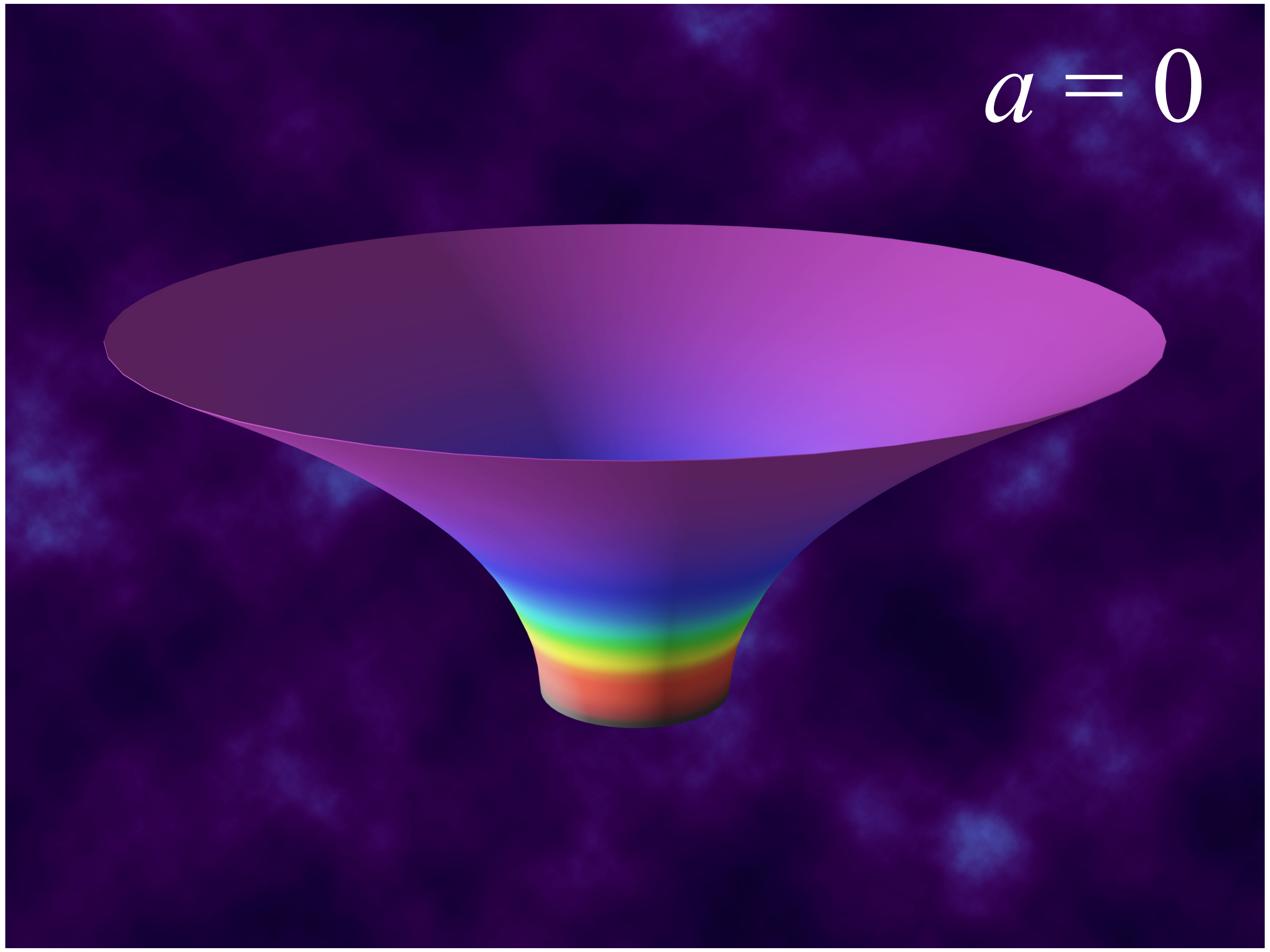
- **Two parameters: Mass M , spin angular momentum aM^2**
  - $0 \leq a \leq 1.$ In astrophysical universe: $0 \leq a \lesssim 0.998$
- **Kerr metric for quiescent black hole**

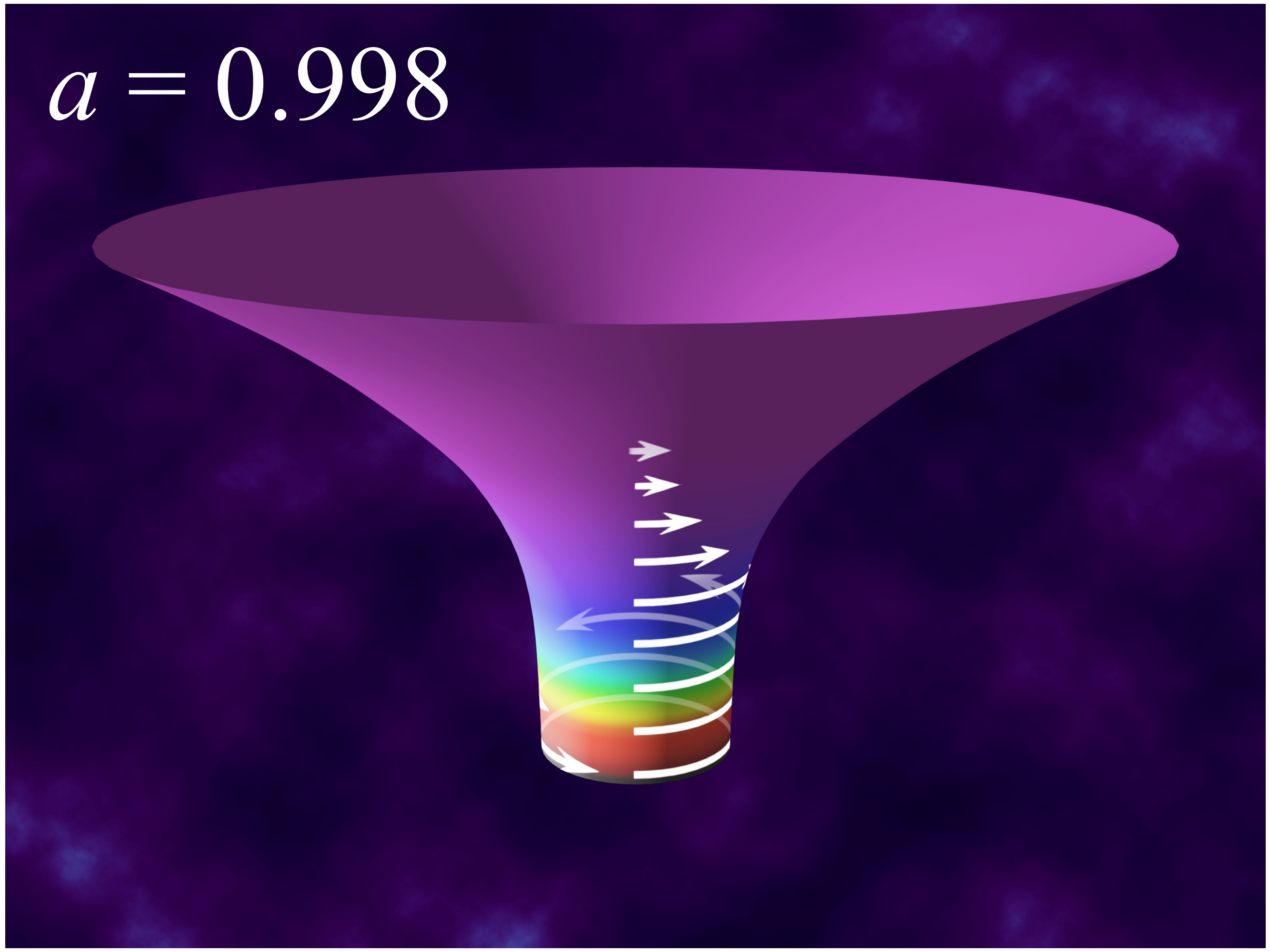
$$ds^2 = g_{rr} dr^2 + g_{\theta\theta}\, d\theta^2 + g_{\phi\phi}\,(d\phi - \omega\, dt)^2 - \alpha^2\, dt^2$$

space curvature — ***3-metric***

space rotation — ***shift*** function

time warp — ***lapse*** function



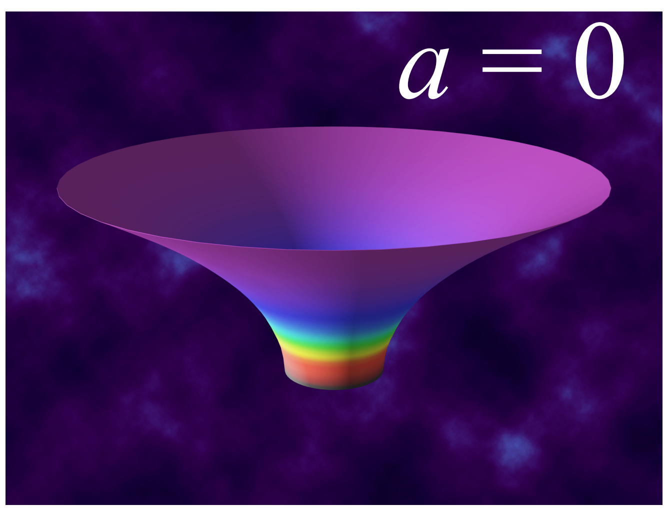


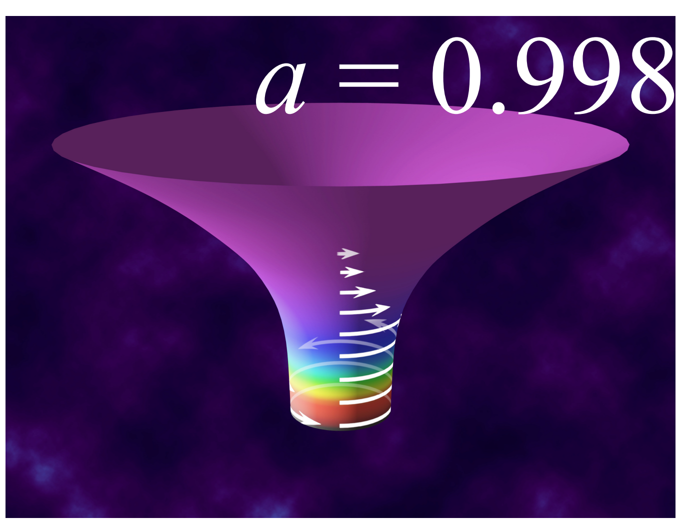
Black-Hole Dynamics

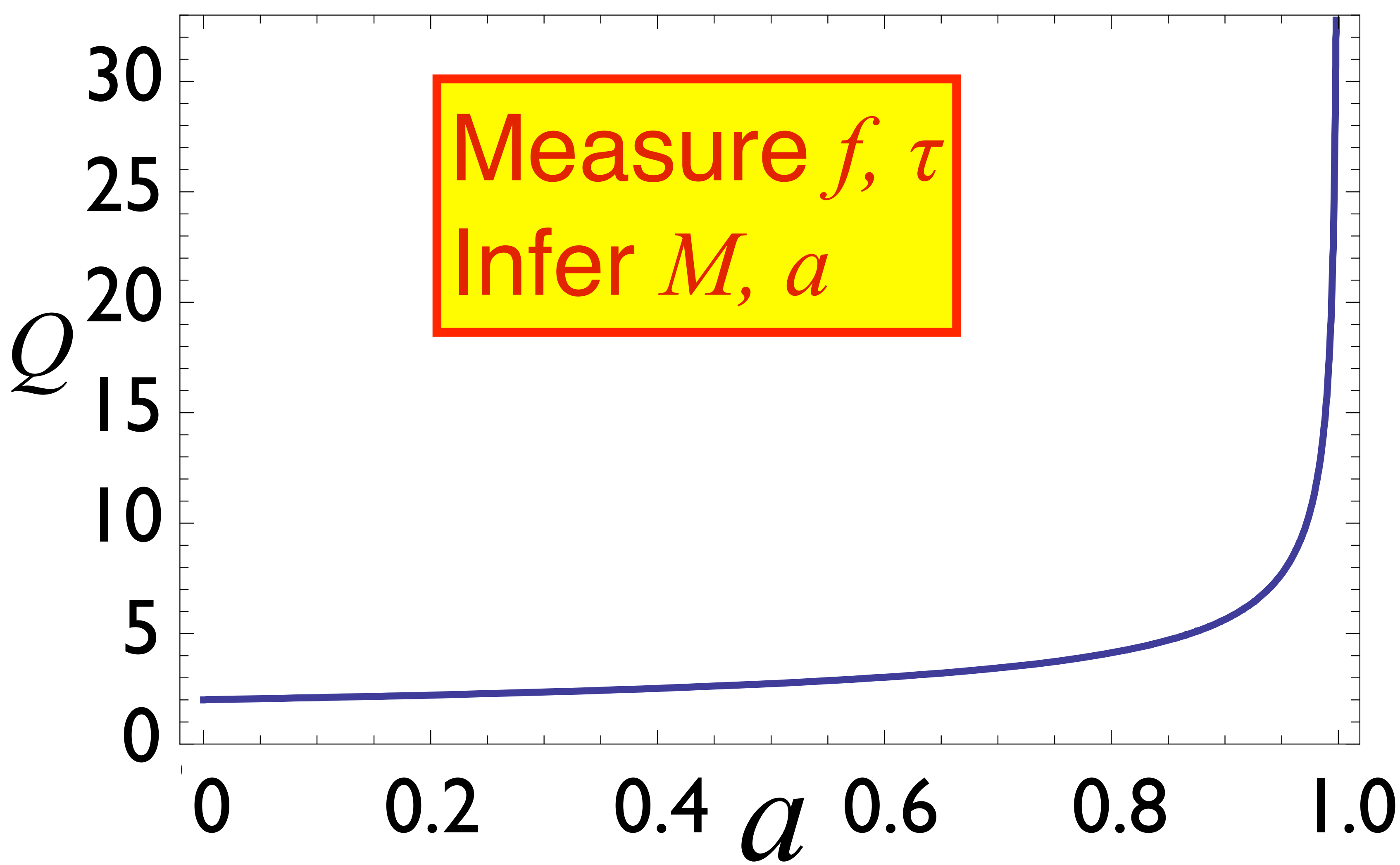
- **Black-hole vibrations** (analyzed via perturbation theory):
  - Rich spectrum of normal modes; but the most weakly damped, and usually the most strongly excited, is the fundamental quadrupole mode
  - GWs: h_+ & $h_\times \sim \sin[2\pi f(t\text{-}r)]\exp[-(t\text{-}r)/\tau)]$
  - $f \simeq (1/2\pi)[1\text{-}0.63(1\text{-}a)^{0.3}]$, $Q = \pi f \tau \simeq 2/(1\text{-}a)^{0.45}$

$$f \simeq 1.2 \text{ kHz} \left(\frac{10M_\odot}{M}\right) \text{ for } a = 0$$

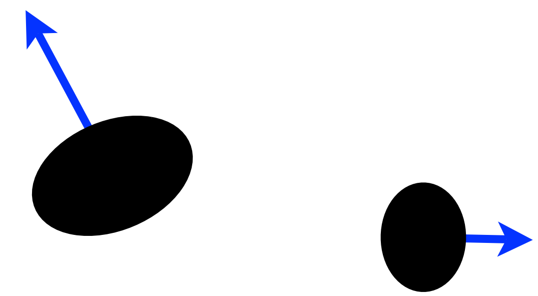
$$f \simeq 3.2 \text{ kHz for } a = 1$$







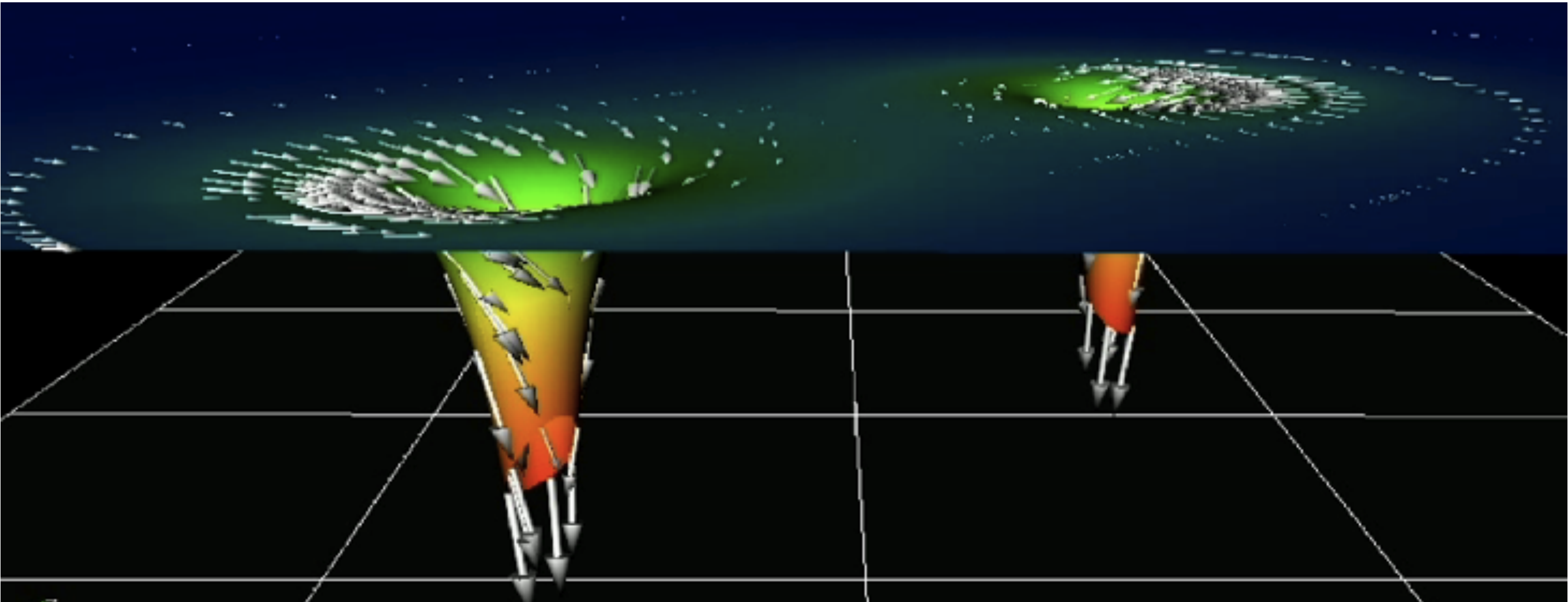
BH/BH Binary: Inspiral, Collision, Merger, Ringdown



- **Early Inspiral:** Post-Newtonian approximation
- **Late Inspiral,** $v \gtrsim c/3,\ a \lesssim 10M$: Numerical relativity
- **Collision, Merger, and Early Ringdown:** Numerical Relativity
- **Late Ringdown:** Black-hole Perturbation Theory
- **For GW data analysis (next week):** need cumulative phase accuracy 0.1 radians [LIGO/VIRGO searches], 0.01 radians [LIGO/VIRGO information extraction]; much higher for LISA
  - 0.01 has been achieved

BH/BH Binary: Inspiral, Collision, Merger, Ringdown

- **Numerical-Relativity simulations of late inspiral, collision, merger, and ringdown:**
  - My Ehrenfest Colloquium
  - Copy of slides on line at http://www.cco.caltech.edu/~kip/LorentzLectures/
  - Comparison with observed waveforms: Tests of general relativity in highly dynamical, nonlinear, strong-gravity regime. "Ultimate tests"



BH/NS Binaries: Inspiral, Tidal Disruption, ...

- **Early Inspiral:** Post-Newtonian Approximation
- **Late Inspiral, tidal disruption, ...:** Numerical Relativity
  - much less mature than for BH/BH binaries
  - details and waveform depend on masses, spins, and NS equation of state
  - example: $P=K\rho_o^2$, $M_{BH} = 3\ M_{NS}$, BH spin a=0.75 [Etienne, Liu, Shapiro, Baumgarte, PRD 79, 044024 (2009)]

BH/NS Binaries: Inspiral, Tidal Disruption, ...

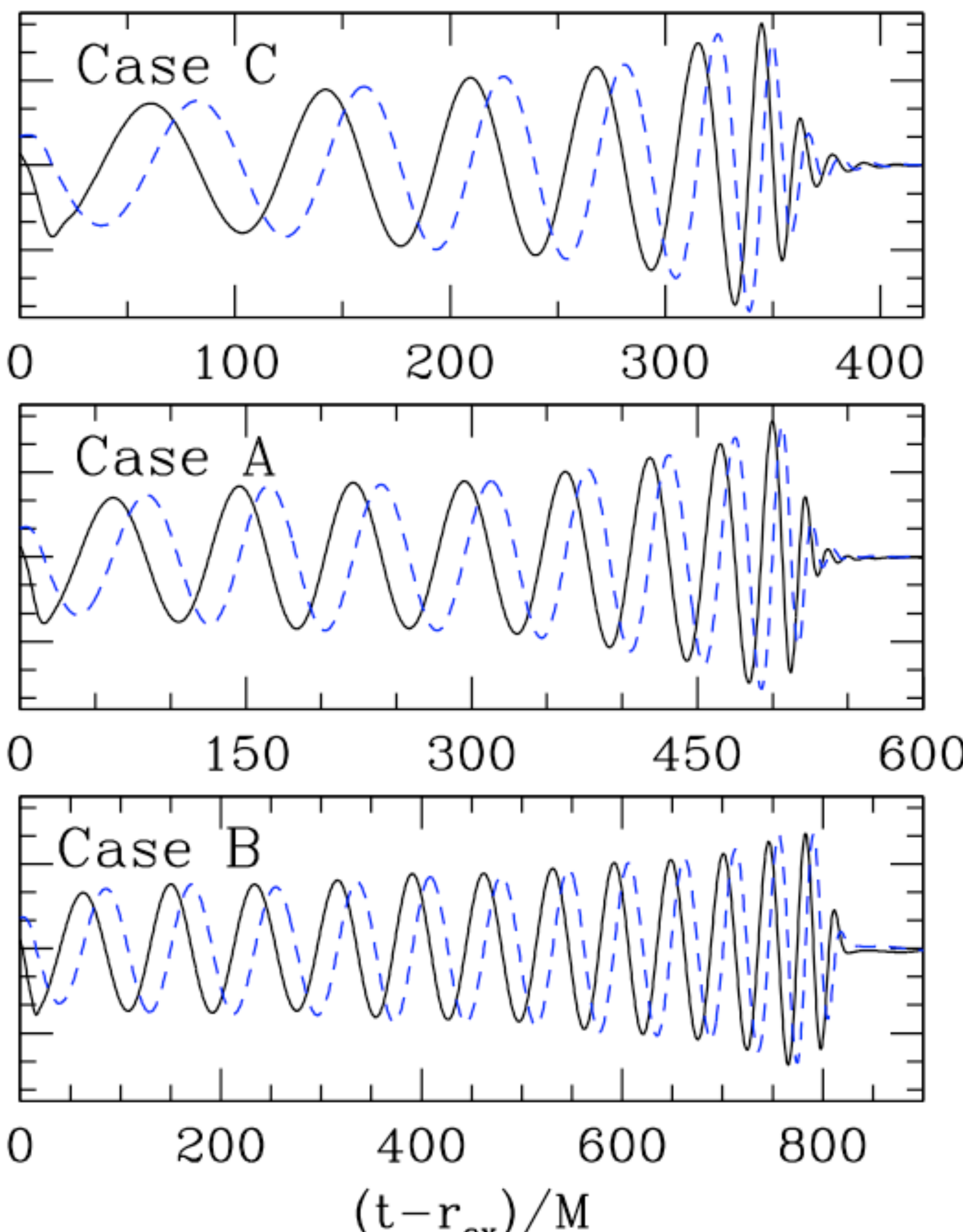
BH/NS Binaries: Inspiral, Tidal Disruption, ...

- **Gravitational Waveforms,** dependence on BH spin: for $P=K\rho_o^2$, $M_{BH} = 3\ M_{NS}$ [Etienne, Liu, Shapiro, Baumgarte, PRD 79, 044024 (2009)]

a = -0.50

a = 0

a = +0.75



>99% of NS mass swallowed, <1% in disk

96% of NS mass swallowed, 4% in disk

80% of NS mass swallowed, 20% in disk

Just beginning to explore influence of equation of state

NS/NS Binaries: Inspiral, Collision, Merger

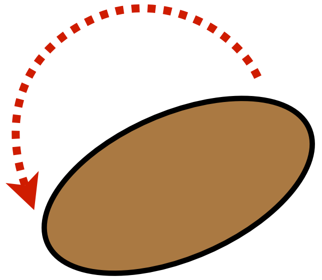
- **The collision and merger radiate at frequencies f $\gtrsim$2000 Hz; too high for LIGO/VIRGO**
    - by contrast, BH/NS tidal disruption can be at f ~ 500 - 1000 Hz, which is good for LIGO/VIRGO

Neutron-Star Dynamics

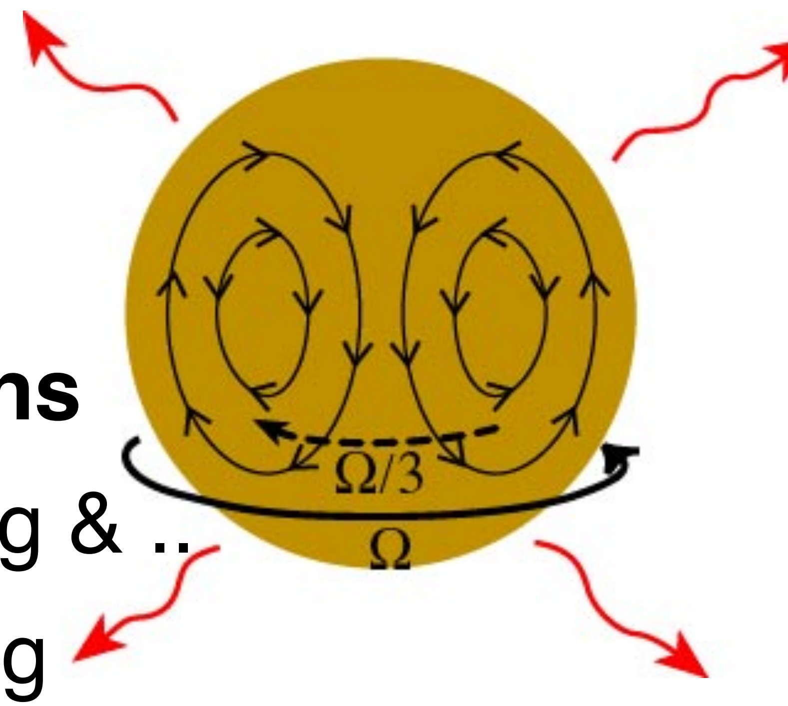
- **Structure** depends on poorly known equation of state of bulk nuclear matter at densities ρ~(nuclear density)~$2\mathrm{x}10^{14}$ g/cm^3 to ~10x(nuclear density).
  - e.g., for M=1.4Msun, NS radius is as small as $R \simeq 8$km for softest equations of state, and R≃16km for stiffest equations of state
- **Solid Crust** can support deformations from axisymmetry, with (quadrupole moment)/(star's moment of inertia) $\equiv \varepsilon < 10^{-5}$
- **Internal magnetic fields'** pressure ➪ $\varepsilon \sim 10^{-6}$ if B~10^{15}G
- **Pulsars & other spinning NSs:** If NS rotates with angular velocity Ω around a principal axis of its moment of inertia tensor, it radiates primarily at angular frequency $\omega=2\Omega$; otherwise it precesses and may radiate strongly at $\omega= \Omega+ \Omega_{prec}$.
- In a star quake, the GW frequency and amplitude of these two "spectral lines" may change suddenly

$$h_+ \sim h_\times \sim 2\varepsilon \frac{\omega^2 I}{r} \sim 3 \times 10^{-23} \left(\frac{\varepsilon}{10^{-6}}\right) \left(\frac{f}{1 \text{ kHz}}\right)^2 \left(\frac{10 \text{ kpc}}{r}\right)$$

Neutron-Star Dynamics



- **Tumbling "Cigar":**
  - If NS spins fast enough (e.g. when first born), it may deform into a triaxial ellipsoid that tumbles end over end, emitting GWs at $\omega = 2\Omega_{\text{tumble}}$
- **Vibrational normal modes:**
  - A neutron star has a rich spectrum normal modes, that will radiate GWs when excited
  - Especially interesting are R-modes (analogs of Rossby Waves in Earth's atmosphere and oceans): supported by Coriolis force
    - R-mode emits GWs at $\omega = 2(\Omega - \Omega/3) = 4\Omega/3$
    - **Radiation reaction** pushes wave pattern backward (in its direction of motion as seen by star), so **amplifies the oscillations**
    - Oscillations damped by mode-mode mixing & ..
    - Not clear whether R-modes are ever strong enough for their GWs to be seen



Current quadrupole rad'n, not mass quadrupole

Neutron-Star Dynamics

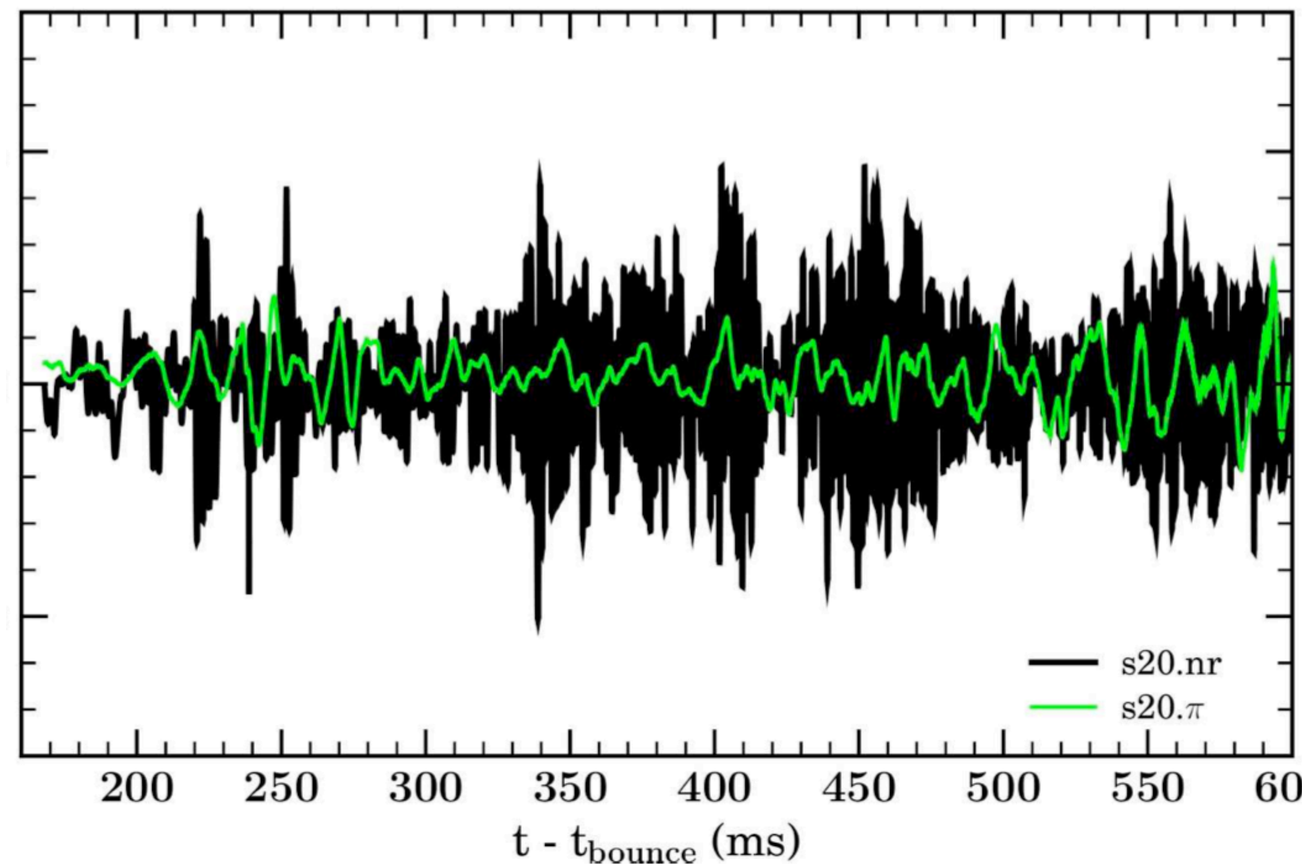
- There is a rich variety of ways that a NS can radiate GWs.
- The emitted waves will carry rich information about NS physics and nuclear physics.
- Coordinated GW & EM observations have great potential

Collapse of Stellar Cores: Supernovae

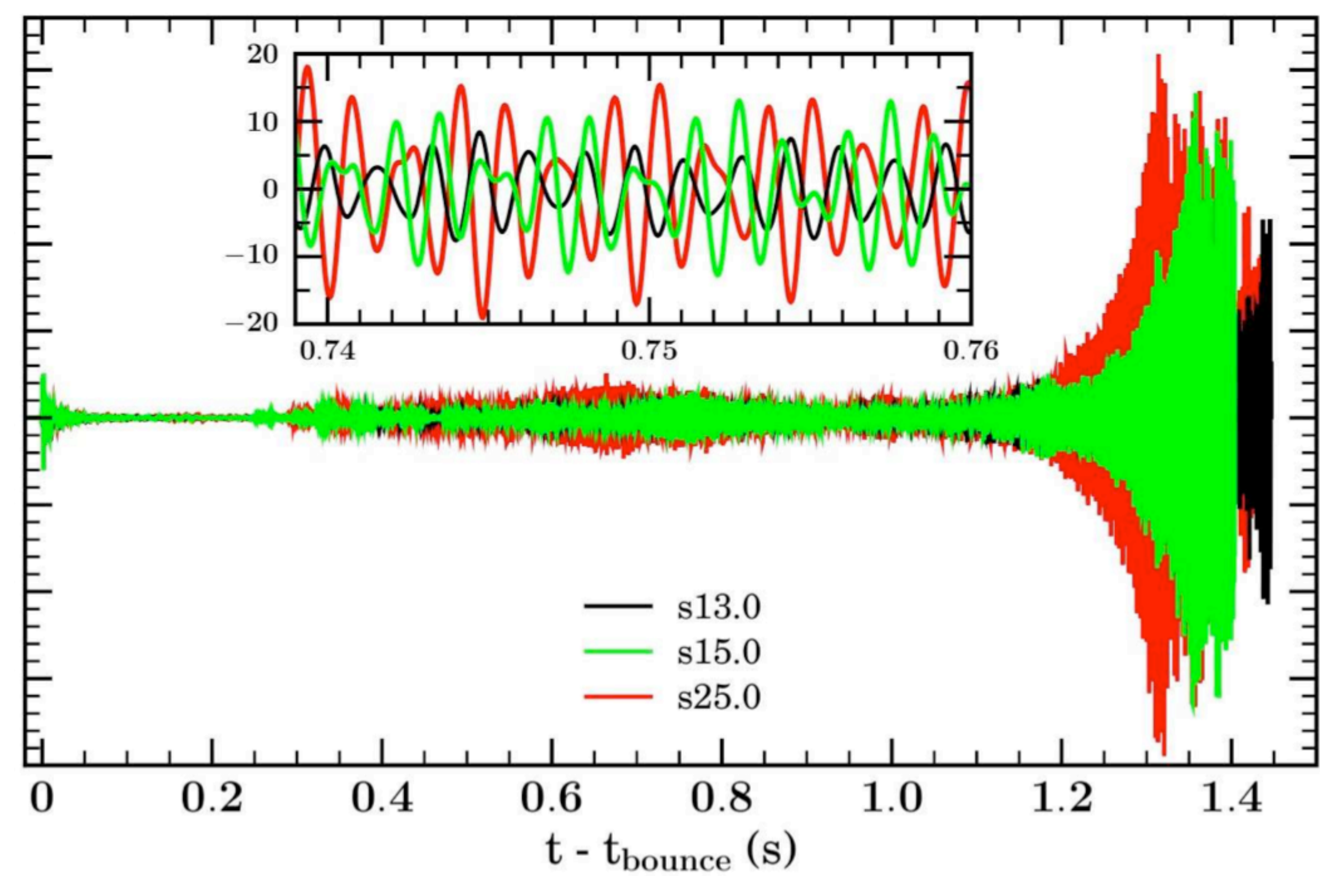
- **Original Model (Colgate et al, mid 1960s):**
  - Degenerate iron core of massive star (8 to 100 Msun) implodes.
  - Implosion halted at ~ nuclear density (forms proto-neutron star); creates shock at PNS surface
  - Shock travels out through infalling mantle and ejects it.
- **Improved simulations:** Shock stalls; cannot eject mantle.
- **Today: three competing mechanisms for explosion.**
  - each mechanism produces a characteristic GW signal [C. Ott]

Collapse of Stellar Cores: Supernovae

- **Neutrino Mechanism**
  - Convection in PNS dredges up hot nuclear matter from core. It emits few x 10^{52} ergs of neutrinos in ~ 1 sec, of which 10^{51} ergs get absorbed by infalling mantle, creating new shock that ejects mantle
  - Convection → Stochastic GWs

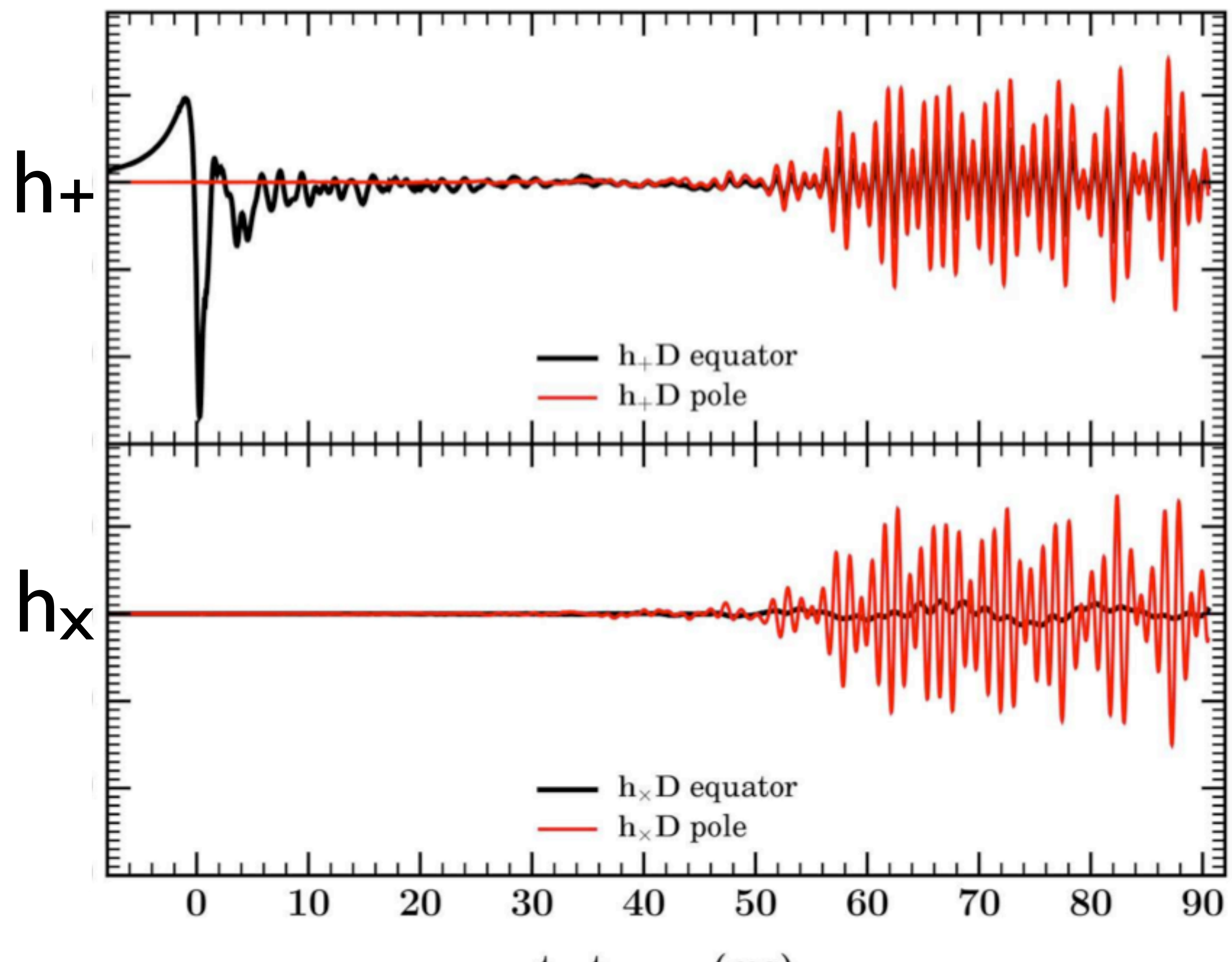


- **Acoustic Mechanism**
  - After ~300 ms, convective turbulence drives dipolar and quadrupolar oscillations of PNS. Oscillations send sound waves into mantle. They steepen, shock, and eject mantle.
  - Pulsations → Quasiperiodic GWs



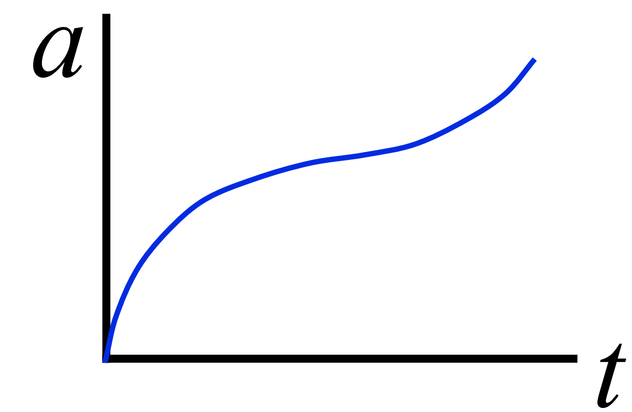
Collapse of Stellar Cores: Supernovae

- **Magneto-Rotational Mechanism**
  - Core of pre-supernova star spins fast (~1 rotation/s). Its collapse is halted by centrifugal forces (~ 1000 rotations/s); sharp bounce. PNS differential rotation (shear) feeds a "bar-mode" instabilities ("tumbling cigars") at 50ms. Differential rotation stretches magnetic field, amplifies it; magnetic stresses drive polar outflows (jets).
  - Bounce → sharp GW burst
  - Tumbling → narrow-band, periodic GWs (two modes)



Early-Universe GW Sources

- **GW Propagation in Expanding Universe** (geometric optics)
  - Metric for expanding universe: $ds^2 = -dt^2 + a^2(t)[dx^2 + dy^2 + dz^2]$
  - Primordial plasma at rest, (x,y,z)=const
  - Set $dt = a\, d\eta$, so $ds^2 = a^2(\eta)[-d\eta^2 + dx^2 + dy^2 + dz^2]$
  - Rays: $(x,y) = const;\ z = \eta.$
    cross sectional area of bundle of rays: $A = a^2 \Delta x \Delta y$
  - GW fields in geometric optics limit: h_+ and $h_\times$ are constant along ray except for amplitude fall-off ~ $1/\sqrt{A}$ ~ $1/a$



$$h_+ = \frac{Q_+(x,y,\eta - z)}{a}, \quad h_\times = \frac{Q_\times(x,y,\eta - z)}{a}$$

  - Monochromatic waves: $h \sim \frac{\exp(-i\phi)}{a} = \frac{\exp[-i\sigma(\eta - z)]}{a}$

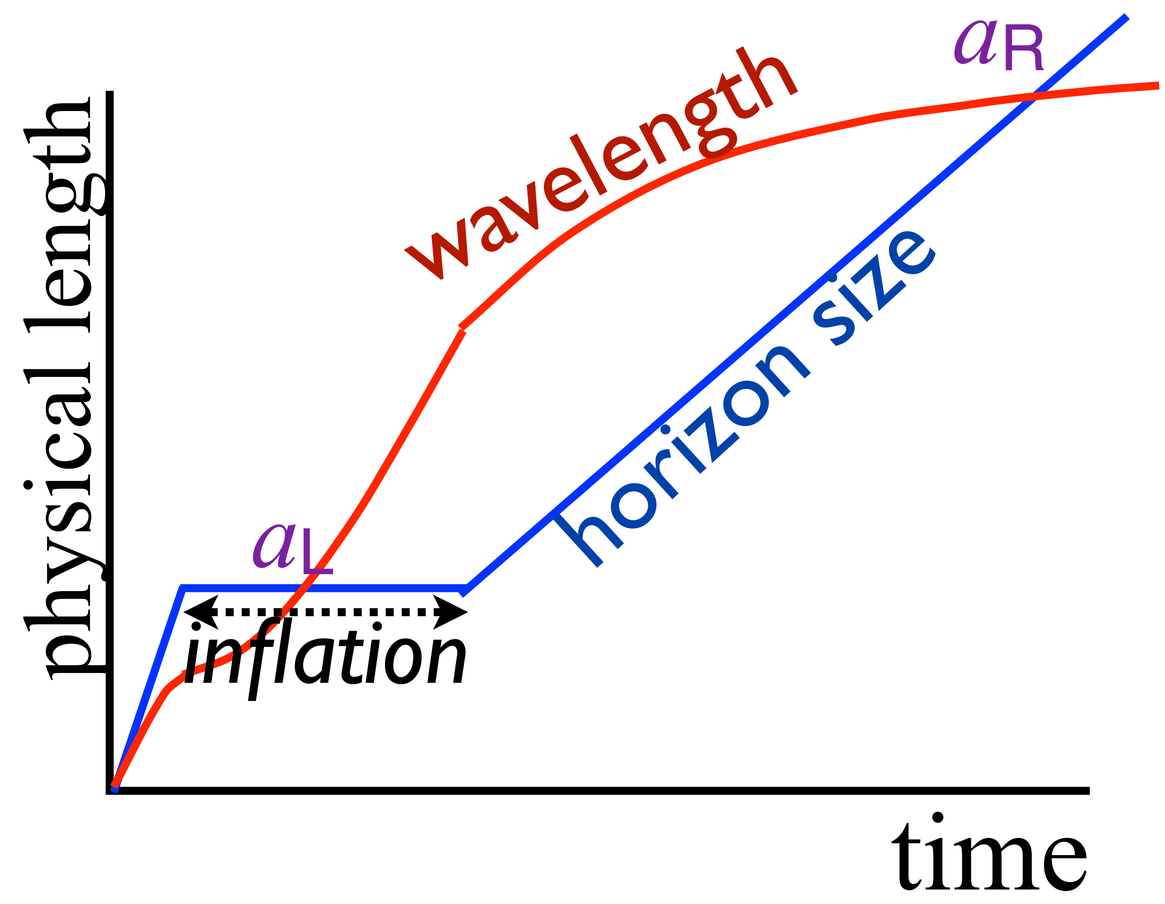
$$\text{angular frequency } \omega = \frac{d\phi}{dt} = \sigma \frac{d\eta}{dt} = \frac{\sigma}{a}, \text{ so wavelength } \lambda = 2\pi \frac{c}{\omega} \propto a$$

  - Number of gravitons conserved:

$$N_{\text{graviton}} \propto (ah_+)^2 + (ah_\times)^2 = \text{constant along rays}$$

Early-Universe GW Sources

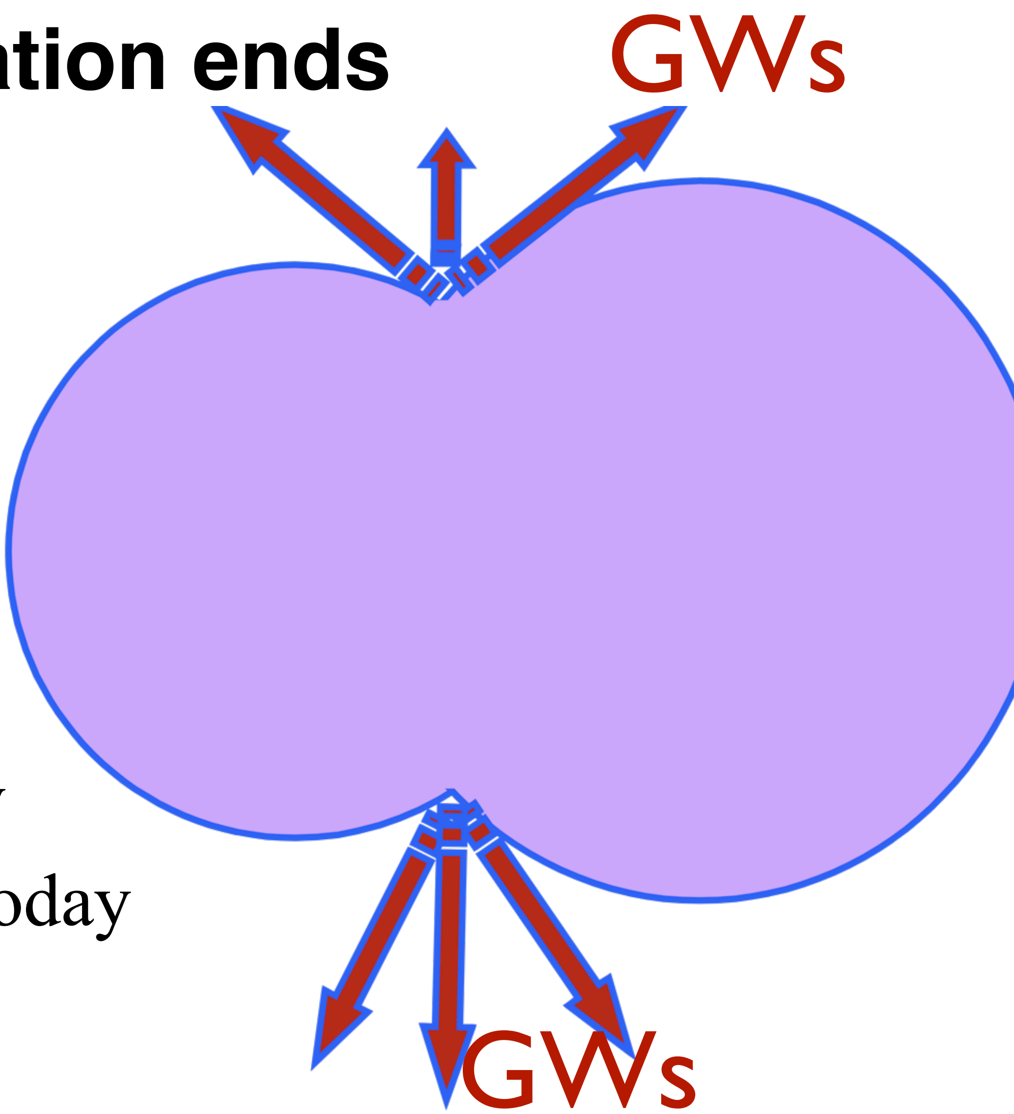
- **Amplification of GWs by Inflation:**
  - During inflation $a \sim \exp(t/\tau)$, where $\tau \sim 10^{-34}$ sec; and cosmological horizon has fixed size, $c\tau$
  - GW wavelength $\lambda \sim a$ is stretched larger than horizon at some value a_L. Wave no longer knows it is a wave (geometric optics fails). Wave stops oscillating and its amplitude freezes: h_+ and $h_\times$ become constant.
  - After inflation ends, horizon expands faster than wavelength. At some value a_R wavelength reenters horizon, wave discovers it is a wave again and begins oscillating.

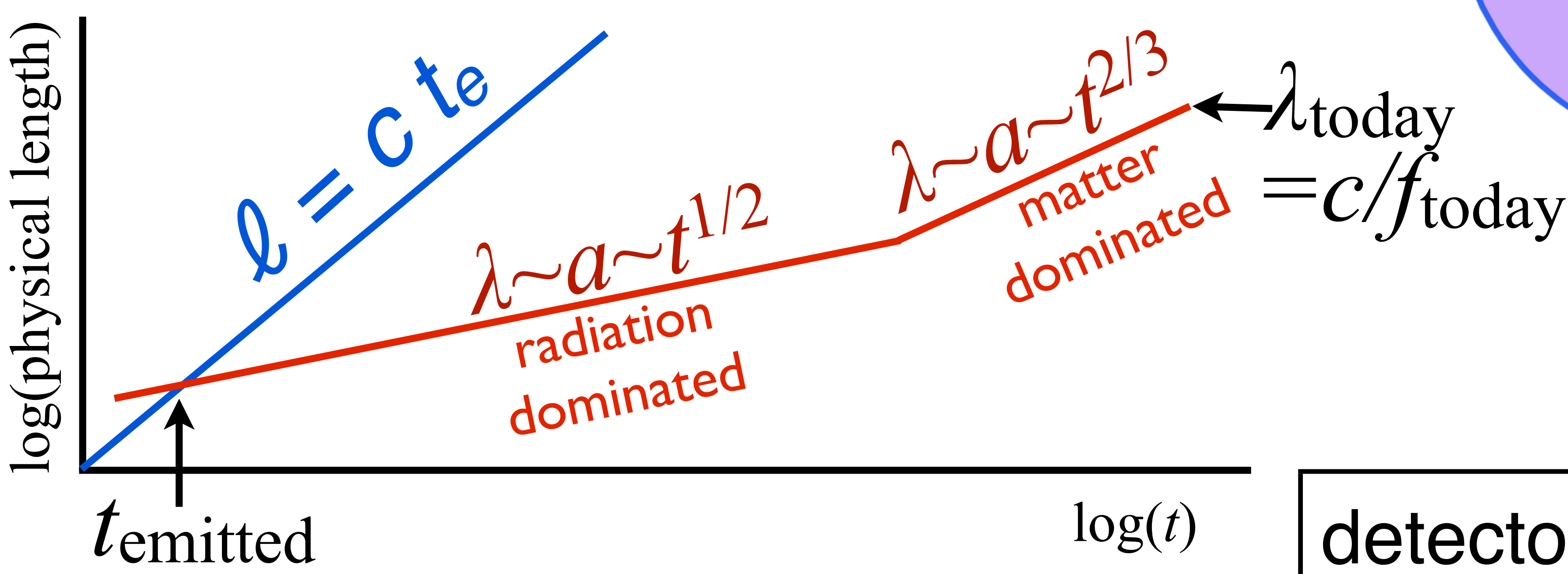


$$\frac{N^{\rm reentry}_{\rm gravitons}}{N^{\rm leave}_{\rm gravitons}} = \frac{(ah)^2_{\rm reentry}}{(ah)^2_{\rm leave}} = \left(\frac{a_R}{a_L}\right)^2 = \exp\left[\frac{2(t_R - t_L)}{\tau}\right]$$

Early-Universe GW Sources

- **Violent physical processes after inflation ends**
  - e.g.: Electroweak phase transition
- **Occur most strongly on scale of horizon**
  - so emitted GW wavelength is $\lambda_e = c\, t_e$, where t_e is the age of universe at emission

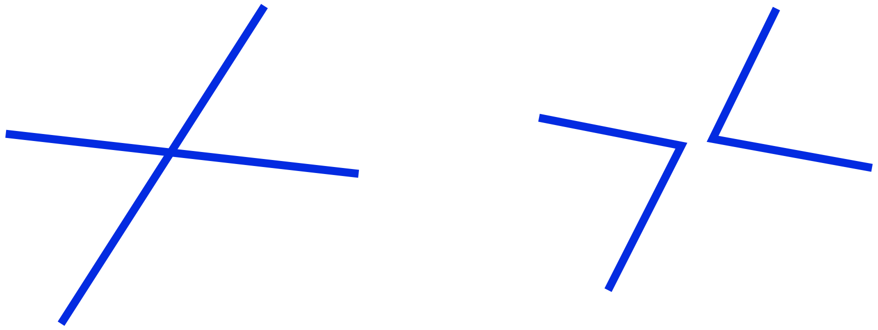




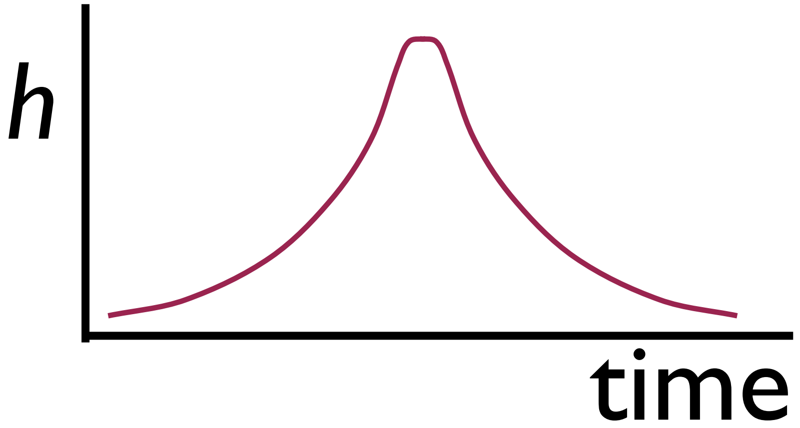
detector	f_{today}	$t_{emitted}$
LIGO	100 Hz	10^{-22} s
LISA	10^{-3} Hz	10^{-12} s
PTA	10^{-8} Hz	10^{-2} s

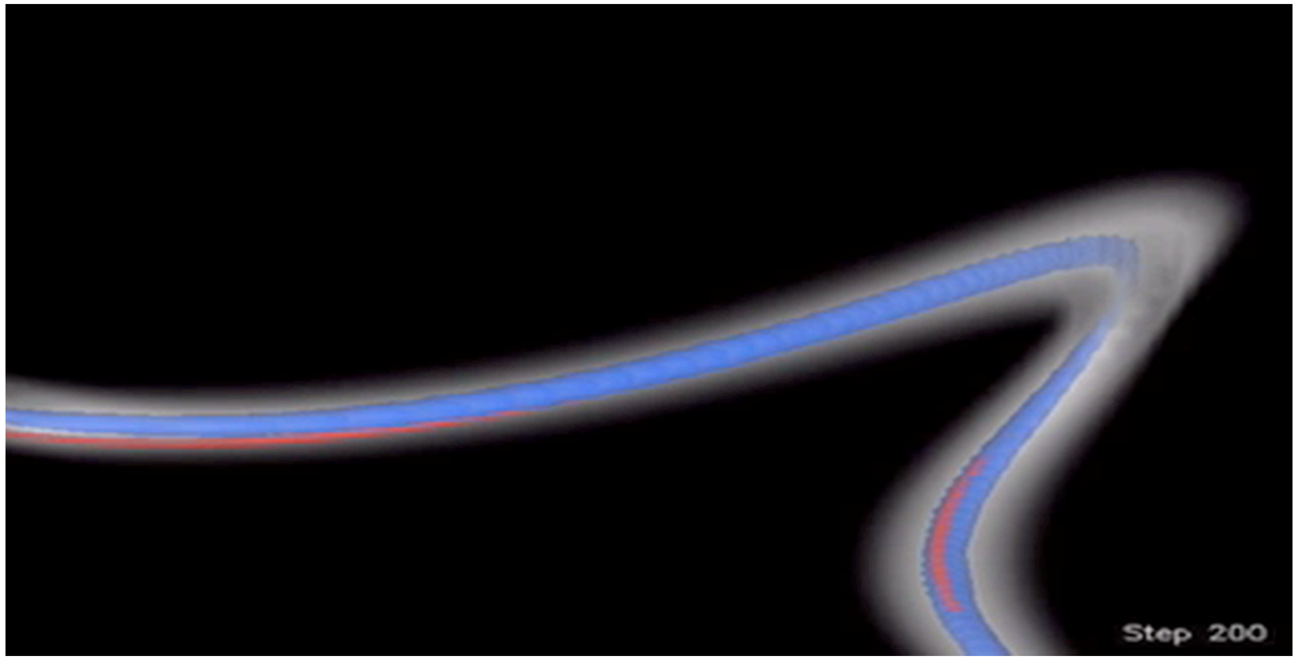
Early-Universe GW Sources

- **Cosmic Strings**
  - may have been formed by inflation of fundamental strings
  - when cross: high probability to reconnect

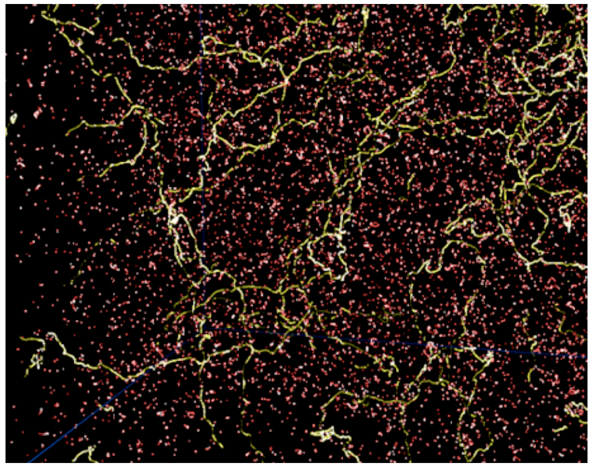


  - kink travels down string at speed of light, radiating GWs strongly in forward direction
  - characteristic waveform





  - network of strings produces stochastic GWs



Conclusions

- **There are many potential sources of gravitational waves**
- **And, as with other new windows, there are likely to be unexpected sources**
- **Next Friday, Oct 2:**
  - GW Detectors
  - GW data analysis: finding signals and extracting their information
  - Put these sources in their astrophysical contexts, and in the contexts of detectors