Quantum Optics & Quantum Information: Decoherence, Measurement & State Preparation

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Austrian Academy of Sciences

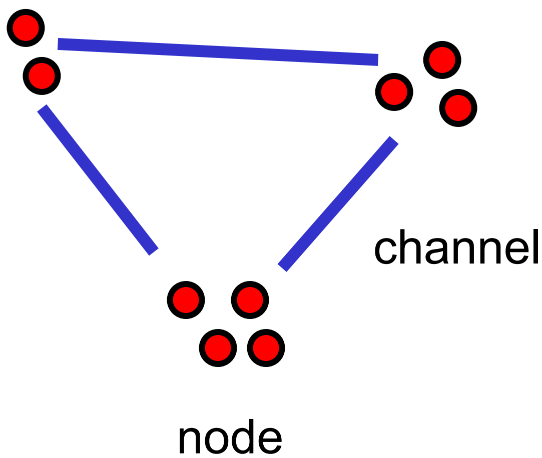
SFB
Coherent Control of Quantum Systems

€U TMR & IP

Institute for Quantum Information

motivation slide in
lectures 1+2

Quantum Network

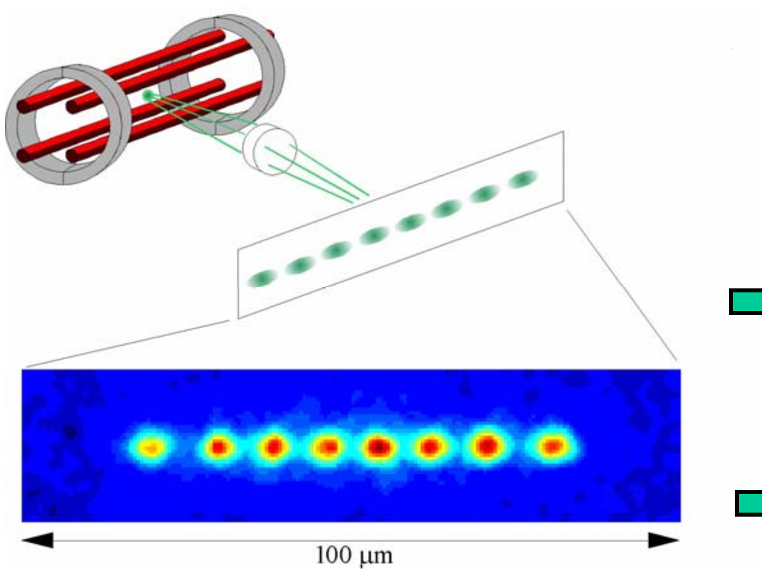


- Nodes: local quantum computing
  - store quantum information
  - local quantum processing
- Channels: quantum communication
  - transmit quantum information

- Lecture 1: Quantum computing with trapped ions
  – entanglement engineering: atoms as qmemory, gates etc.
- Lecture 2: Quantum communication
  – quantum repeater: nested purification protocol
  – implementation: deterministic / probabilistic; photons & atoms
- Lecture 3: Quantum optical systems as *open* quantum systems
  – Measurement, state preparation & decoherence

Lecture 1: Quantum computing with trapped ions

- trapped ions



QC model:

✓ qubits: longlived atomic states

✓ single qubit gates: laser

✓ two qubit gates: via phonon bus

✓ read out: quantum jumps

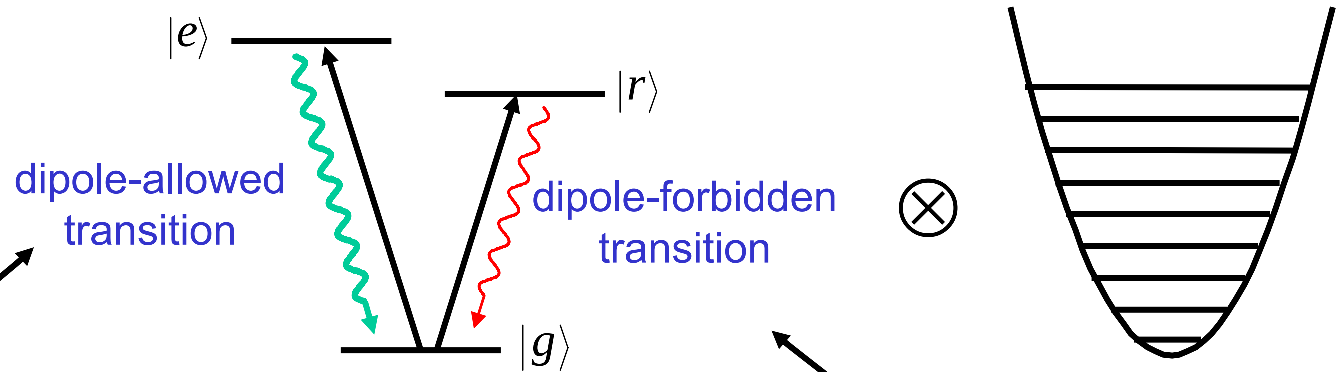
requirements:

✓ state preparation: phonon cooling

✓ [small decoherence]

Trapped ion: the system

- system = internal + external degrees of freedom

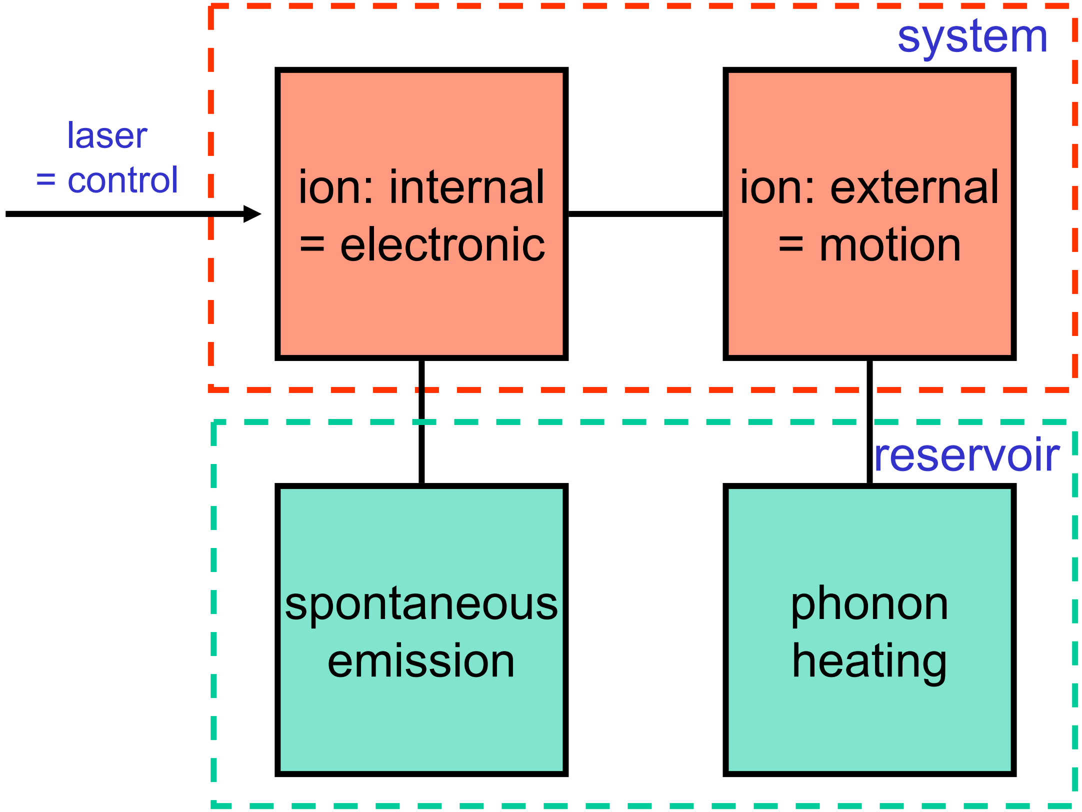


internal: electronic levels

external: motion

- strong dissipation
  - ✓ laser cooling / state preparation
  - ✓ qubit / state measurement
- small dissipation
  - ✓ Hamiltonian: quantum state engineering

System + Reservoir

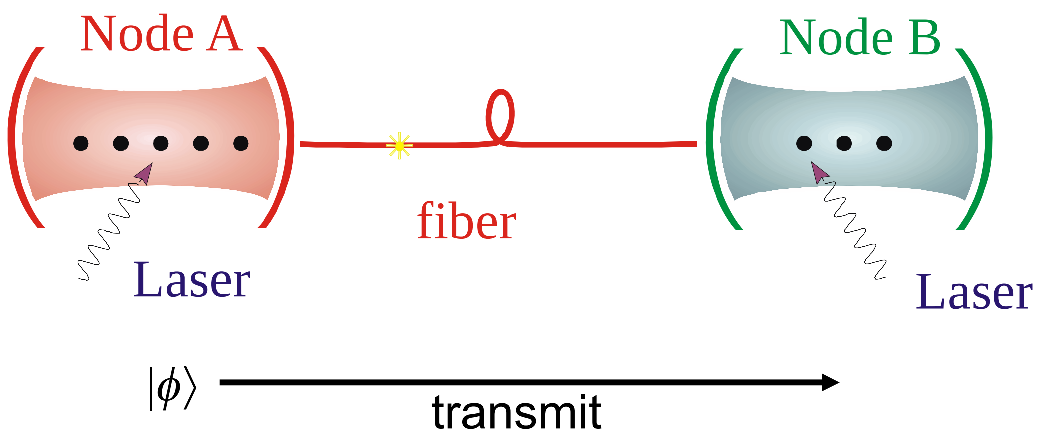


Development of the theory:

- system: Hamiltonian (control)
- reservoir: master equation + continuous measurement theory

Lecture 2: Atom –light interfaces & transmission of qubits

- deterministic transmission of qubits



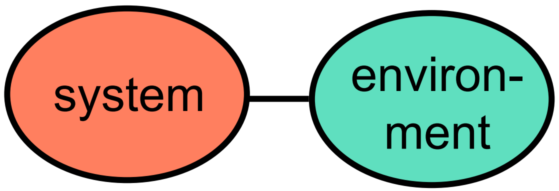
- memory: atoms → - databus: photons → - memory: atoms

- system:
  - ✓ single atoms
  - ✓ high-Q cavities
  - ✓ transmission line / fiber (continuum of modes)

Our approach ...

Quantum Optics ⟺ Quantum Information

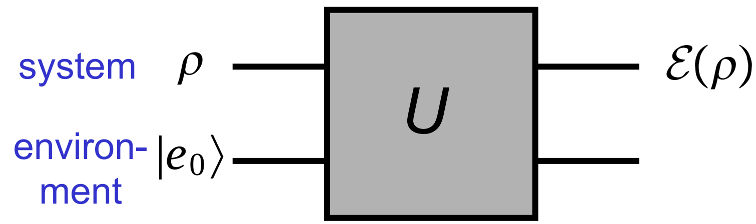
- Open quantum system



harmonic oscillators

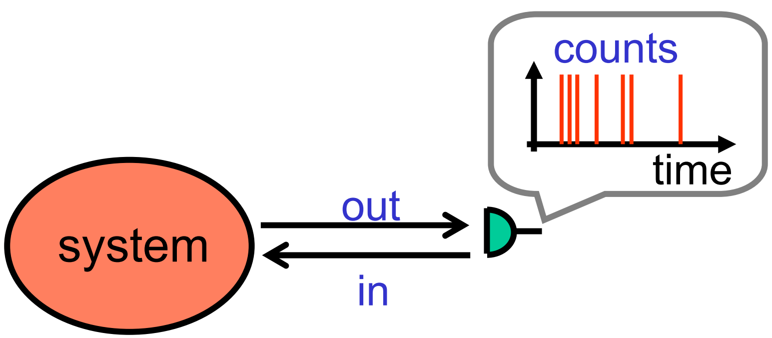
✓ master equation

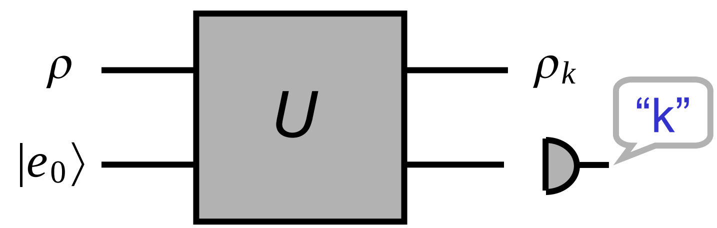
- Quantum operations



$$\rho \rightarrow \mathcal{E}(\rho) = \sum_k E_k \rho E_k^\dagger$$

- Continuous observation





✓ Stochastic Schrödinger Equation

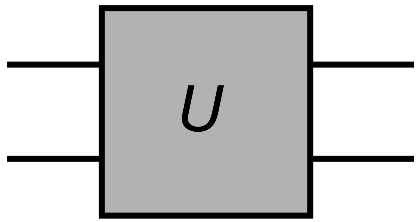
“Quantum Markov processes“

1. Quantum Operations

Ref.: Nielsen & Chuang, Quantum Information and Quantum Computation

Quantum operations

Evolution of a quantum system coupled to an environment:
open quantum system



$$\rho \longrightarrow \mathcal{E}(\rho) = \mathrm{tr}_{\mathrm{env}}[U(\rho \otimes \rho_B)U^\dagger]$$

quantum operation

Operator sum representation:

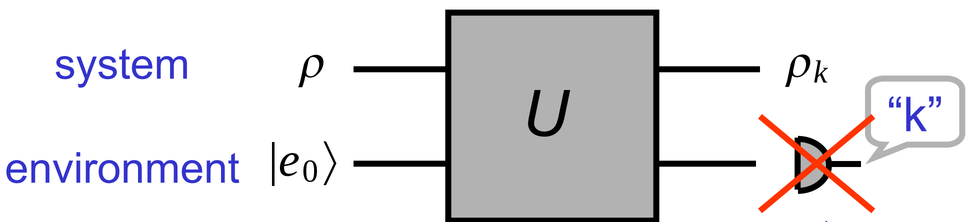
$$\begin{aligned}\rho \longrightarrow \mathcal{E}(\rho) &= \mathrm{tr}_{\mathrm{env}}[U(\rho \otimes |e_0\rangle\langle e_0|)U^\dagger] \\ &= \sum_k \langle e_k|U(\rho \otimes |e_0\rangle\langle e_0|)U^\dagger|e_k\rangle \\ &= \sum_k E_k \rho E_k^\dagger \quad \text{with } E_k = \langle e_k|U|e_0\rangle\end{aligned}$$

operation elements

Properties: $\sum_k E_k^\dagger E_k = 1$

Quantum operations

Measurement of the environment: $P_k \equiv |e_k\rangle\langle e_k|$



state

$$\rho_k \sim \mathrm{tr}_{\mathrm{env}}(|e_k\rangle\langle e_k| U(\rho \otimes |e_0\rangle\langle e_0|)U^\dagger)$$

$$= E_k \rho E_k^\dagger$$

$$\rho_k = E_k \rho E_k^\dagger \,/\, \mathrm{tr}_{\mathrm{sys}}(E_k \rho E_k^\dagger) \qquad \text{(normalized)}$$

probability

$$p_k = \mathrm{tr}_{\mathrm{sys+env}}(|e_k\rangle\langle e_k| U(\rho \otimes |e_0\rangle\langle e_0|)U^\dagger)$$

$$= \mathrm{tr}_{\mathrm{sys}}(E_k \rho E_k^\dagger)$$

Remark: if we do not read out the measurement

$$\rho \longrightarrow \mathcal{E}(\rho) = \sum p_k \rho_k$$

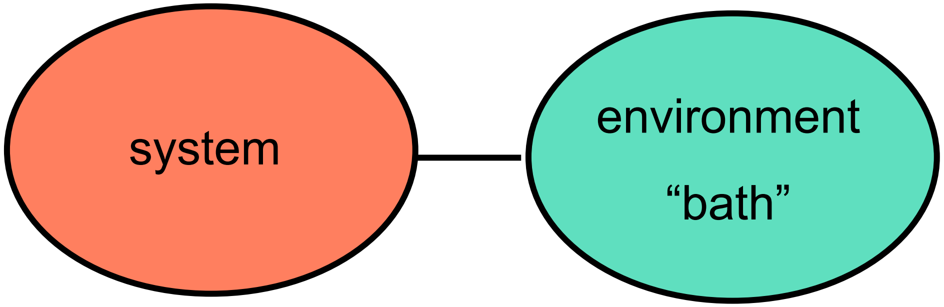
$$= \sum_k E_k \rho E_k^\dagger$$

2. Quantum Noise Models in Quantum Optics

- system + environment model
- formulation
  - operator / c-number stochastic Schrödinger equation
  - [(operator) Langevin equation]

System + environment model

Hamiltonian:



$$H_{\mathrm{tot}} = H_{\mathrm{sys}} + H_B + H_{\mathrm{int}}$$

H_{sys} unspecified

$$H_B = \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega\, \omega\, b^\dagger(\omega) b(\omega) \quad \text{with } [b(\omega), b^\dagger(\omega')] = \delta(\omega-\omega')$$

harmonic oscillators

$$H_{\mathrm{int}} = i\int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega\, \kappa(\omega)[c b^\dagger(\omega) - c^\dagger b(\omega)]$$

system operator

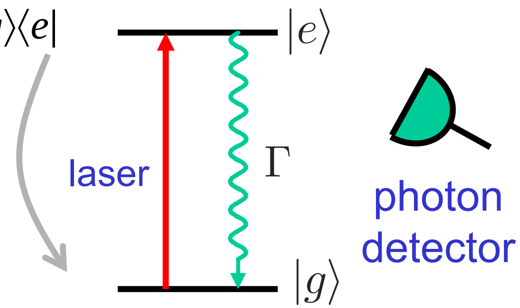
Assumptions:

- rotating wave approximation

Example: spontaneous emission

- driven two-level system undergoing spontaneous emission

$$c \rightarrow \sigma_- = |g\rangle\langle e|$$

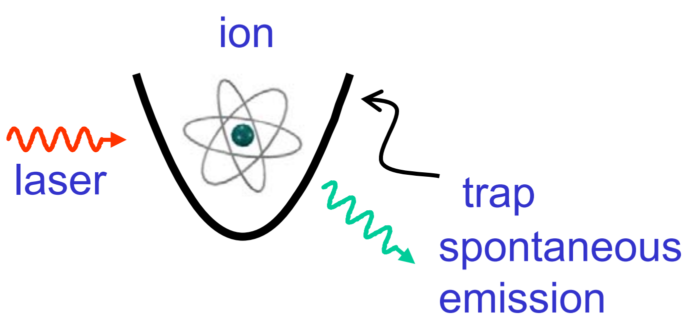


$$H_{\text{sys}} = \omega_{eg}|e\rangle\langle e| - \left(\tfrac{1}{2}\Omega e^{-i\omega_L t}\sigma_+ + \text{h.c.}\right)$$

$$H_{\text{int}} = -\vec{\mu}_{eg} \cdot \vec{E}^{(+)}(0)\sigma_+ + \text{h.c.}$$

$$\rightarrow i\int_{\omega_{eg}-\vartheta}^{\omega_{eg}+\vartheta} d\omega\,\kappa(\omega) b^\dagger(\omega)\sigma_+ + \text{h.c.}$$

- ... including the recoil from spontaneous emission



$$H_{\text{int}} = -\vec{\mu}_{eg} \cdot \vec{E}^{(+)}(\vec{x})\sigma_+ + \text{h.c.}$$

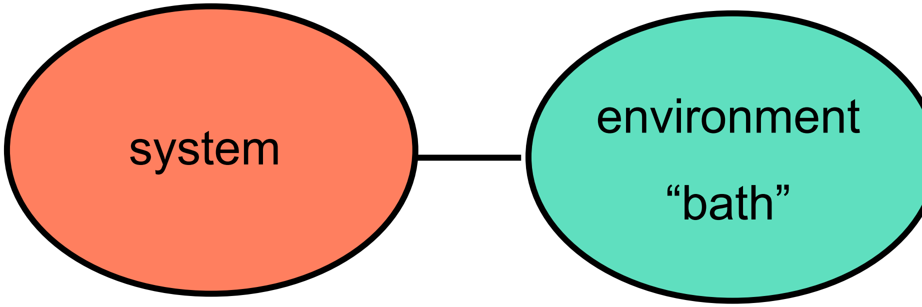
$$\rightarrow \sum_\lambda \int d^3k \ldots b_{\lambda\vec{k}} e^{i\vec{k}\cdot\vec{x}}\sigma_+ + \text{h.c.}$$

recoil

System + environment model

Hamiltonian:

$$H_{\text{tot}} = H_{\text{sys}} + H_B + H_{\text{int}}$$



H_{sys} unspecified

$$H_B = \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega\, \omega\, b^\dagger(\omega) b(\omega) \quad \text{with } [b(\omega), b^\dagger(\omega')] = \delta(\omega-\omega')$$

harmonic oscillators

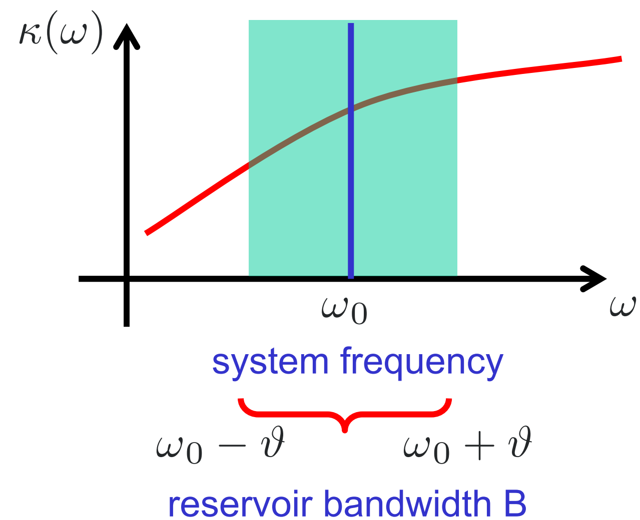
$$H_{\text{int}} = i\int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega\, \kappa(\omega) [c b^\dagger(\omega) - c^\dagger b(\omega)]$$

system operator

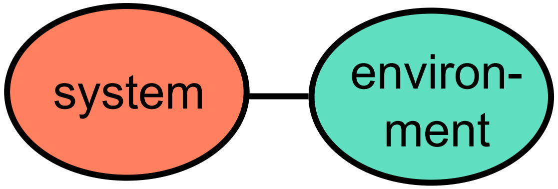
Assumptions:

- rotating wave approximation
- flat spectrum: $\kappa(\omega) \to \sqrt{\gamma/2\pi}$

flat over bandwidth



Schrödinger Equation



- Schrödinger equation

$$\frac{d}{dt}|\Psi_t\rangle = -i\left[H_{\text{sys}} + H_B + H_{\text{int}}\right]|\Psi_t\rangle$$

$|\psi\rangle \otimes |\text{vac}\rangle$

initial condition

- convenient to transform …

$\Psi_t\rangle \longrightarrow e^{-iH_B t}|\Psi_t\rangle$

interaction picture
with respect to bath

$b(\omega) \longrightarrow b(\omega)e^{-i\omega t}$

$H_{\text{sys}} \longrightarrow \tilde{H}_{\text{sys}}$

"rotating frame“
(transform optical
frequencies away)

$c \longrightarrow ce^{-i\omega_0 t}$

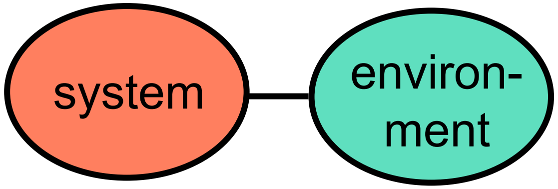
$$\frac{d}{dt}|\tilde{\Psi}_t\rangle = \left[-i\tilde{H}_{\text{sys}} + \left(\underline{\int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega}\, \kappa(\omega)\, \underline{b(\omega)^\dagger e^{\imath(\omega-\omega_0)t}}\right)c - \text{h.c.}\right]|\tilde{\Psi}_t\rangle$$

$\kappa(\omega) \rightarrow \sqrt{\gamma/2\pi}$

flat over bandwidth

$b(t) := \frac{1}{\sqrt{2\pi}}\int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega\, b(\omega)e^{-\imath(\omega-\omega_0)t}$

“noise operators”



- **Schrödinger Equation**

$$\frac{d}{dt}|\Psi_t\rangle = \left[-iH_{\mathrm{sys}} + \sqrt{\gamma}\, b(t)^\dagger c - \sqrt{\gamma}\, c^\dagger b(t)\right]|\Psi_t\rangle$$

$$b(t) := \frac{1}{\sqrt{2\pi}} \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega\, b(\omega) e^{-\imath(\omega-\omega_0)t}$$

"noise operators"

White noise limit $\vartheta \to \infty$

$$[b(t), b^\dagger(s)] = \delta(t-s)$$

$$\langle b(t) b^\dagger(s)\rangle = \delta(t-s)$$

vacuum

$$\tau_{\mathrm{sys}} \gg 1/\vartheta \gg \cancel{\tau_{\mathrm{opt}}}$$

white noise limit $\vartheta \to \infty$

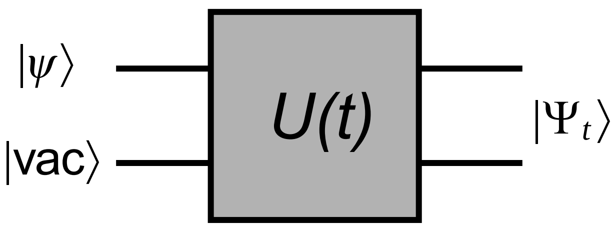
transformed away after RWA

Remarks:

- [We can give precise meaning as a "Quantum Stochastic Schrödinger Equation" within a stochastic Stratonovich calculus]
- We can integrate this equation exactly
  - counting statistics
  - master equation

$\Longleftrightarrow$ quantum operations

3. Integrating the “Quantum Stochastic Schrödinger Equation“

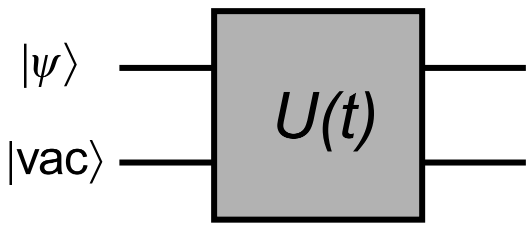


$$|\Psi_0\rangle \longrightarrow |\Psi_t\rangle = e^{-iH_{\mathrm{tot}}t}|\Psi_0\rangle$$

Schrödinger equation:
system + environment

What we want to calculate ...

- **We do not observe the environment:** reduced density operator



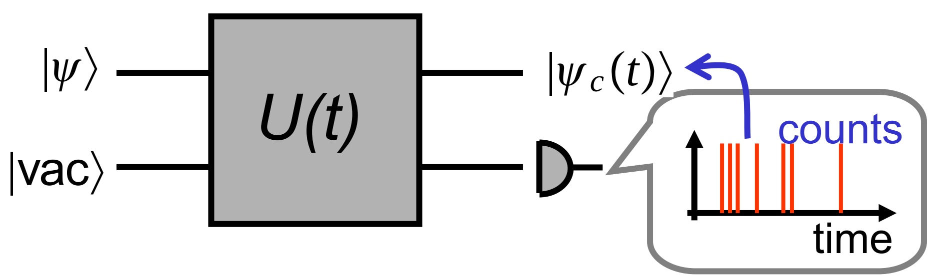
$$\rho_t = \mathrm{tr}_B |\Psi_t\rangle\langle\Psi_t|$$

master equation:

✓ decoherence

✓ preparation of the system (e.g. laser cooling to ground state)

- **We measure the environment:** continuous measurement



conditional wave function:

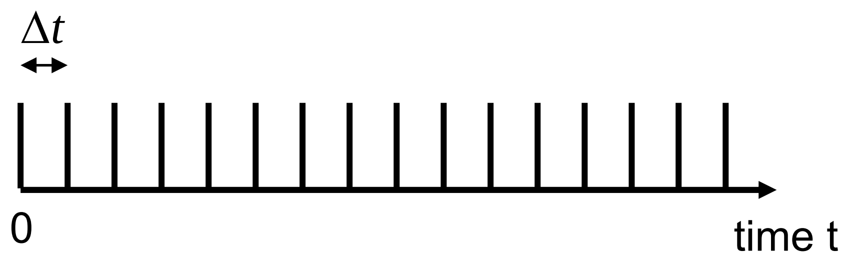
✓ counting statistics

✓ effect of observation on system evolution (e.g. preparation of the (single quantum) system)

Remark: simple man‘s version of conversion from Stratonovich to Ito

Integration in small timesteps

- We integrate the Schrödinger equation in small time steps



$$|\Psi(t = t_f)\rangle \;=\; U(\Delta t_f)\ldots U(\Delta t_1)U(\Delta t_0)|\Psi(0)\rangle$$

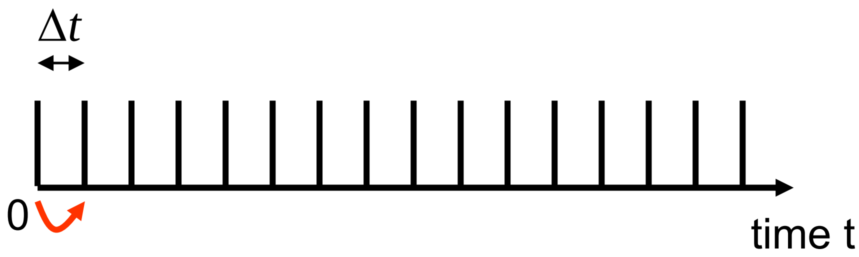
- Remark: choice of time step

$$\tau_{\rm sys} \gg \Delta t \gtrsim 1/\vartheta \gg \cancel{\tau_{\rm opt}}$$

allows us to use perturbation theory in Δt

white noise limit $\vartheta \to \infty$

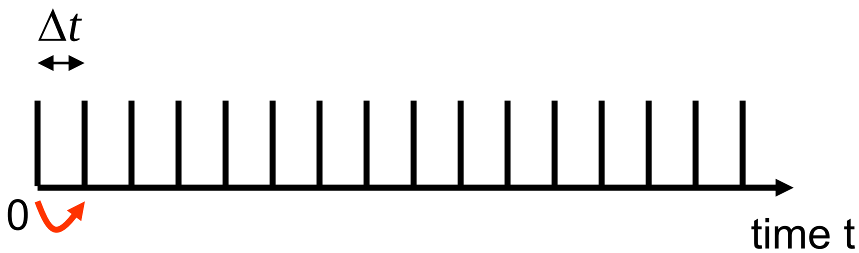
transformed away after RWA



- **First time step:** first order in *Δt*

$$U(\Delta t)|\Psi(0)\rangle = \Big\{\hat{1} - iH_{\text{sys}}\Delta t + \sqrt{\gamma}\, c\int_0^{\Delta t} b^\dagger(t)\,dt - \sqrt{\gamma}\, c^\dagger \int_0^{\Delta t} \cancel{b(t)}\,dt \ldots \Big\}|\Psi(0)\rangle$$

$|\psi\rangle \otimes |\text{vac}\rangle$



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$$+(-i)^2 \gamma c^\dagger c \int_0^{\Delta t} dt \int_0^{t_2} dt'\, b(t) b^\dagger(t') + \ldots \Big\}|\Psi(0)\rangle$$

$|\psi\rangle \otimes |\text{vac}\rangle$

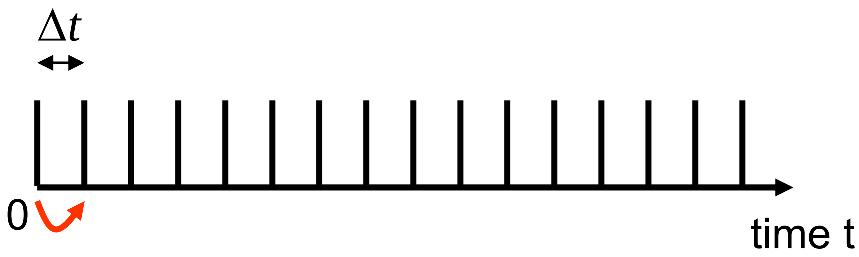
$$\int_0^t dt_2 \int_0^{t_2} dt_1\, b(t_2)\, b^\dagger(t_1)|\text{vac}\rangle = \int_0^t dt_2 \int_0^{t_2} dt_1 \left[b(t_2), b^\dagger(t_1)\right]|\text{vac}\rangle$$

$$= \int_0^t dt_2 \int_0^{t_2} dt_1\, \delta(t_2 - t_1)|\text{vac}\rangle$$

$$= \frac{1}{2}\Delta t|\text{vac}\rangle$$

first order in Δt

1st time step



- **First time step:** to first order in *Δt*

$$
\begin{aligned}
|\Psi(\Delta t)\rangle &= \hat{U}(\Delta t)|\Psi(0)\rangle \\
&= \left\{\hat{1} - iH_{\text{eff}}\,\Delta t + \sqrt{\gamma}\; c\,\Delta B(0)^{\dagger}\right\}|\Psi(0)\rangle
\end{aligned}
$$

We define:

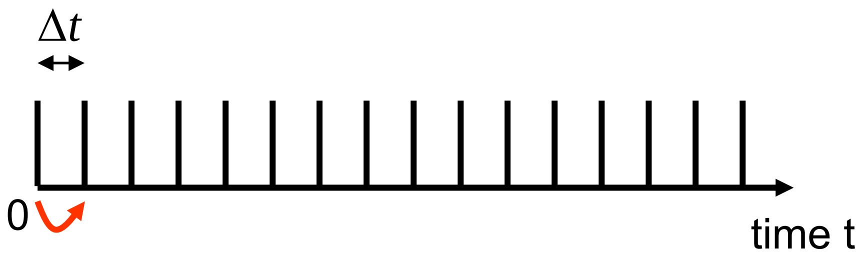
- effective (non-hermitian) system Hamiltonian

$$H_{\text{eff}} := H_{\text{sys}} - \frac{i}{2}\gamma c^{\dagger} c$$

- annihilation / creation operator for a photon in the time slot Δt :

$$\Delta B(t) := \int_t^{t+\Delta t} b(s)\, ds$$

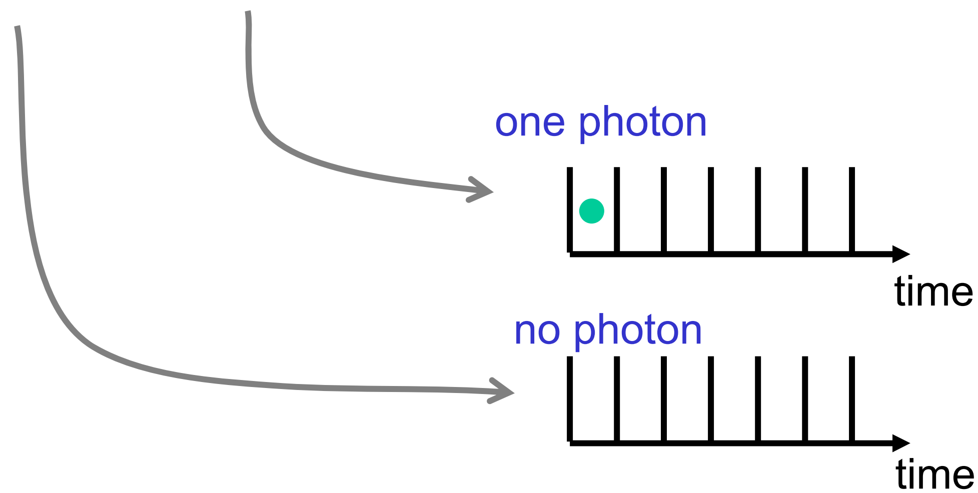
1st time step



- **First time step:** to first order in *Δt*

$$
\begin{aligned}
|\Psi(\Delta t)\rangle &= \hat{U}(\Delta t)|\Psi(0)\rangle \\
&= \left\{\hat{1} - iH_{\text{eff}}\,\Delta t + \sqrt{\gamma}\,c\Delta B(0)^{\dagger}\right\}|\Psi(0)\rangle
\end{aligned}
$$

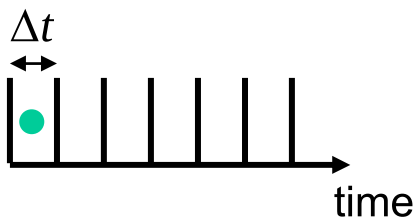
interpretation: superposition of vacuum and one-photon state



Discussion:

annihilation / creation operator for a photon in the time slot Δt :

$$\Delta B(t) := \int_t^{t+\Delta t} b(s)\, ds$$



Remarks and properties:

- commutation relations:

$$[\Delta B(t), \Delta B^\dagger(t')] = \begin{cases} \Delta t & t = t' \quad \text{overlapping intervals} \\ 0 & t \neq t' \quad \text{nonoverlapping intervals} \end{cases}$$

- one-photon wave packet in time slot Δt

$$\frac{\Delta B^\dagger(t)}{\sqrt{\Delta t}} |\text{vac}\rangle \equiv |1\rangle_t \quad \text{(normalized)}$$

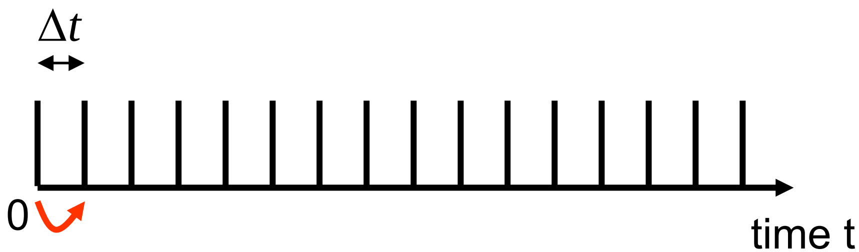
- number operator of photon in time slot t:

$$N(t) = \frac{\Delta B^\dagger(t)}{\sqrt{\Delta t}} \frac{\Delta B(t)}{\sqrt{\Delta t}}$$

- $N(t)$ as set up commuting operators, $[N(t), N(t')] = 0$, which can be measured "simultaneously"

1st time step:

quantum operations



- **Summary of first time step:** to first order in Δt

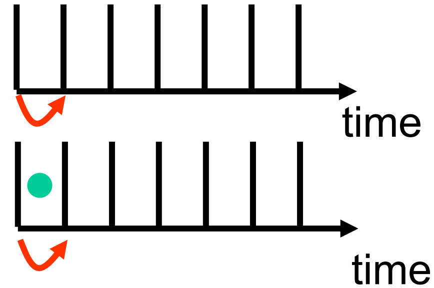
$$
\begin{aligned}
|\Psi(\Delta t)\rangle &= \left[1 - iH_{\text{eff}}\Delta t + \sqrt{\gamma}\, c\, \Delta B^{\dagger}(0)\right]|\Psi(0)\rangle \\
&= |\text{vac}\rangle \otimes \left(1 - iH_{\text{eff}}\Delta t\right)|\psi(0)\rangle + |1\rangle_t \otimes \left(\sqrt{\gamma\Delta t}\, c|\psi(0)\rangle\right) \\
&\equiv |\text{vac}\rangle \otimes E_0|\psi(0)\rangle + |1\rangle_t \otimes E_1|\psi(0)\rangle
\end{aligned}
$$

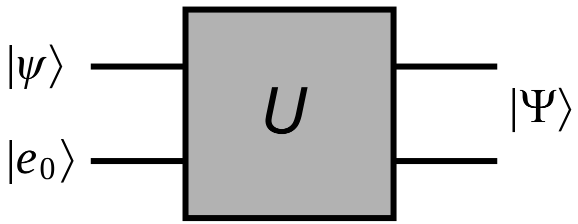
operation elements

where we read off the operation elements

$$E_0 = 1 - iH_{\text{eff}}\Delta t \quad \text{(no photon)}$$

$$E_1 = \sqrt{\gamma\Delta t}\, c \quad \text{(1 photon)}$$

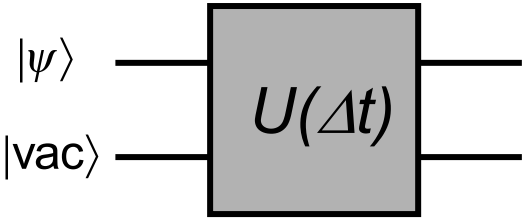




$$
\begin{aligned}
|\psi\rangle|e_0\rangle \to |\Psi\rangle &= U|\psi\rangle|e_0\rangle \\
&= \sum_k |e_k\rangle\langle e_k|U|e_0\rangle|\psi\rangle \equiv \sum |e_k\rangle\, E_k|\psi\rangle
\end{aligned}
$$

Discussion 1:

- **We do not read the detector:** reduced density operator

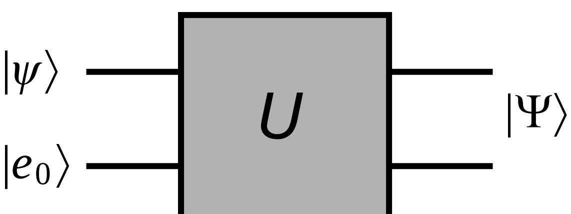


$$\begin{aligned}\rho(\Delta t) &= \mathrm{tr}_B|\Psi(\Delta t)\rangle\langle\Psi(\Delta t)| \\ &= E_0\rho(0)E_0^\dagger + E_1\rho(0)E_1^\dagger \\ &= \big(1 - iH_{\text{eff}}\Delta t\big)\rho(0)\big(1 - iH_{\text{eff}}\Delta t\big)^\dagger + \gamma c\rho(0)c^\dagger\Delta t\end{aligned}$$

no photon one photon

master equation:

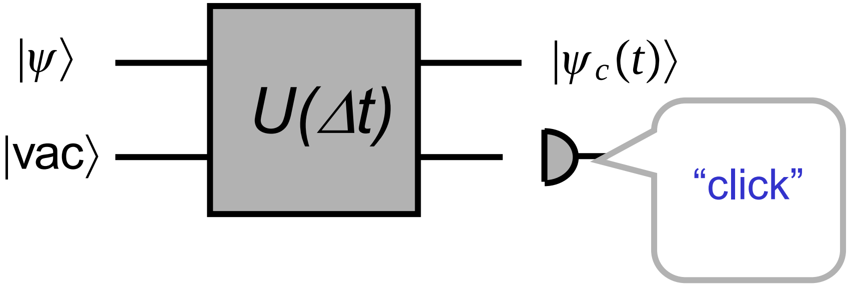
$$\begin{aligned}\rho(\Delta t) - \rho(0) &= -i\big(H_{\text{eff}}\rho(0) - \rho(0)H_{\text{eff}}^\dagger\big)\Delta t + \gamma c\rho(0)c^\dagger\Delta t \\ &\equiv -i\big[H_{\text{sys}}, \rho(0)\big]\Delta t + \frac{1}{2}\gamma(2c\rho(0)c^\dagger - c^\dagger c\rho(0) - \rho(0)c^\dagger c)\Delta t\end{aligned}$$



$$\begin{aligned}\rho \rightarrow \mathcal{E}(\rho) &= \mathrm{tr}_{\text{env}}[U(\rho\otimes|e_0\rangle\langle e_0|)U^\dagger] \\ &= \textstyle\sum_k E_k\rho E_k^\dagger\end{aligned}$$

Discussion 2:

- **We read the detector:**



- **Click:** resulting state

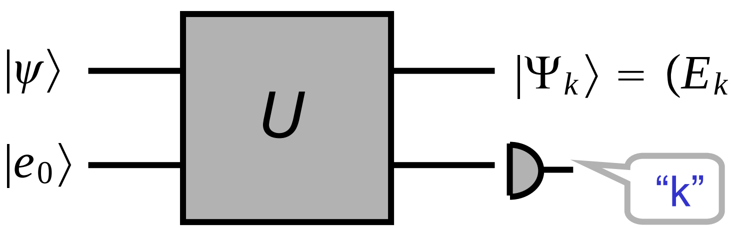
quantum jump operator

$$E_1|\psi(0)\rangle \equiv |\psi^{\text{click}}(\Delta t)\rangle = \sqrt{\gamma \Delta t}\, c|\psi(0)\rangle \quad \text{(quantum jump)}$$

with probability

$$p^{\text{click}} = \text{tr}_{\text{sys}}(E_1 \rho(0) E_1) = \gamma \Delta t \, \|c\psi(0)\|^2$$

Rem.: density matrix $\rho_1(0) = E_1 \rho(0) E_1 / \text{tr}(\dots)$

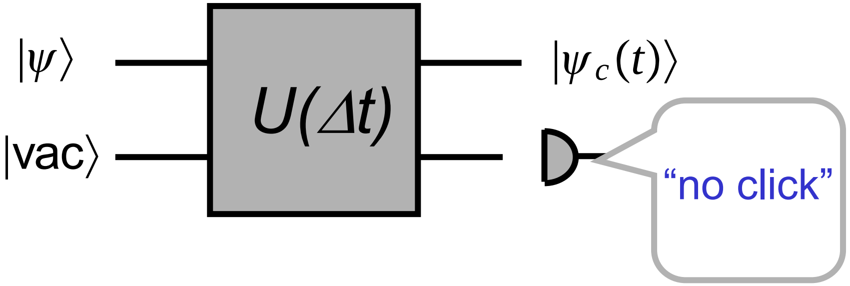


$$|\Psi_k\rangle = (E_k|\psi\rangle)|e_k\rangle / \|\dots\|$$

$$p_k = \|E_k \psi\|^2$$

Discussion 2:

- **We read the detector:**



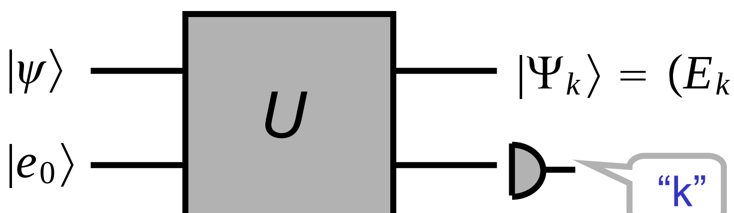
- **No click:** resulting state

decaying norm

$$E_0|\psi(0)\rangle \equiv |\psi^{\text{no click}}(\Delta t)\rangle = \left(1 - iH_{\text{eff}}\Delta t\right)|\psi(0)\rangle \approx e^{-iH_{\text{eff}}\Delta t}|\psi(0)\rangle$$

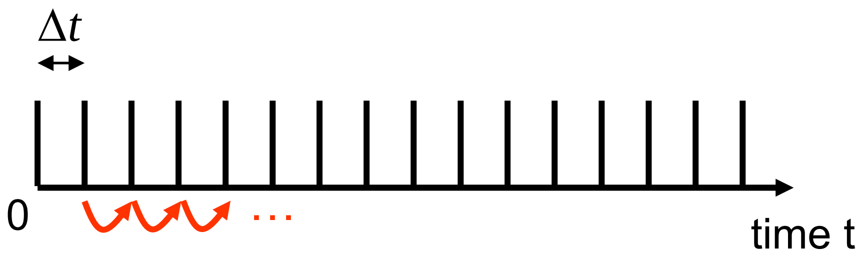
with probability

$$p^{\text{no click}} = \text{tr}_{\text{sys}}(E_0\rho(0)E_0) = \left\| e^{-iH_{\text{eff}}\Delta t}\psi(0) \right\|^2$$



$$|\Psi_k\rangle = (E_k|\psi\rangle)|e_k\rangle/\|\ldots\|$$

$$p_k = \|E_k\psi\|^2$$



- **Second and more time steps:**

$$|\Psi(n\Delta t)\rangle = \left[1 - iH_{\text{eff}}\Delta t + \sqrt{\gamma}\, c\, \Delta B^{\dagger}((n-1)\Delta t)\right]|\Psi((n-1)\Delta t)\rangle$$

stroboscopic integration

$$\equiv \left[1 - iH_{\text{eff}}\Delta t + \sqrt{\gamma}\, c\, \Delta B^{\dagger}((n-1)\Delta t)\right] \times$$

$$\ldots \times \left[1 - iH_{\text{eff}}\Delta t + \sqrt{\gamma}\, c\, \Delta B^{\dagger}(0)\right]|\Psi(0)\rangle$$

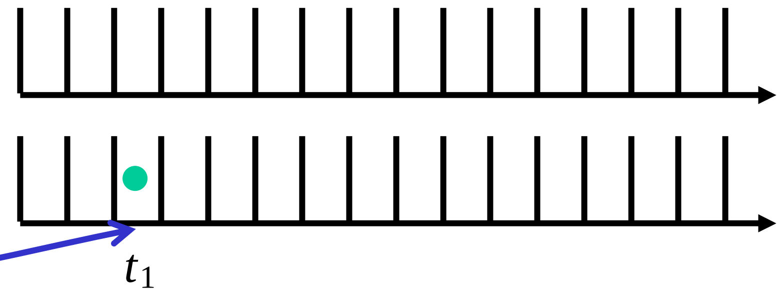
✓ Note: remember … commute in different time slots

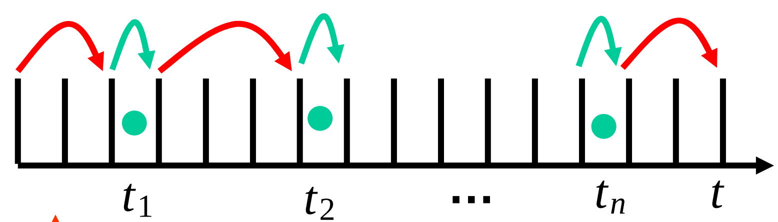
$$[\Delta B(t), \Delta B^{\dagger}(t')] = \begin{cases} \Delta t & t = t' \quad \text{overlapping intervals} \\ 0 & t \neq t' \quad \text{nonoverlapping intervals} \end{cases}$$

Final result for solution of SSE

- **Wave function of the system + environment: entangled state**

$$
\begin{aligned}
|\Psi(t)\rangle \; = \; & |\text{vac}\rangle \otimes e^{-iH_{\text{eff}}t}|\psi(0)\rangle \\
& + (\gamma\Delta t)^{1/2} \sum_{t_1} |1_{t_1}\rangle \otimes e^{-iH_{\text{eff}}(t-t_1)} c\, e^{-iH_{\text{eff}}t_1}|\psi(0)\rangle \\
& + \ldots \\
& + (\gamma\Delta t)^{n/2} \sum_{t_n > \ldots > t_1} |1_{t_1} 1_{t_2} \ldots 1_{t_n}\rangle \otimes e^{-iH_{\text{eff}}(t-t_n)} c \ldots c\, e^{-iH_{\text{eff}}t_1}|\psi(0)\rangle \\
& \ldots
\end{aligned}
$$





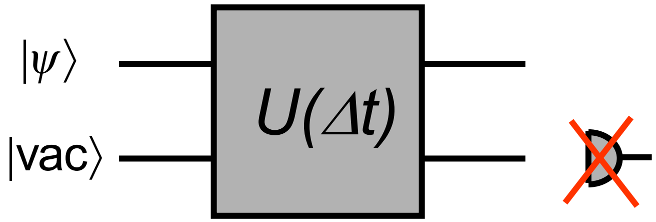
1. system time evolution $|\psi(t|t_1 t_2 \ldots t_n)\rangle$ for a specfic count sequence
2. photon count statistics: probability densities

$$p_{(0,t]}(t_1, t_2, \ldots, t_n) = \|\psi(t|t_1 t_2 \ldots t_n)\|^2$$

click: $|\psi_t\rangle \rightarrow \sqrt{\gamma}\, c|\psi_{t+\Delta t}\rangle$

no click: $|\psi_0\rangle \rightarrow |\psi_t\rangle = e^{-iH_{\text{eff}}t}|\psi_0\rangle$

- **Tracing over the environment we obtain the master equation**



$$\frac{d}{dt}\rho(t) = -i\left[H_{\mathrm{sys}}, \rho(t)\right] + \frac{1}{2}\gamma\left(2c\rho(t)c^{\dagger} - c^{\dagger}c\rho(t) - \rho(t)c^{\dagger}c\right)$$

master equation

✓ Lindblad form

✓ coarse grained time derivative

Ito-Quantum Stochastic Schrödinger Equation

- taking the limit …

$$\begin{aligned} \Delta t &\longrightarrow dt \\ \Delta B(t) &\longrightarrow dB(t) \\ \Delta B^{\dagger}(t) &\longrightarrow dB(t)^{\dagger} \end{aligned}$$

Ito operator noise increments

- Quantum Stochastic Schrödinger Equation

$$d|\Psi(t)\rangle = \left[-\frac{i}{\hbar} H_{\mathrm{sys}}\, dt + \sqrt{\gamma}\, c\, dB^{\dagger}(t) - \sqrt{\gamma}\, c^{\dagger} dB(t)\right]|\Psi(t)\rangle$$

- Properties of Ito increments:
  - point to the future:

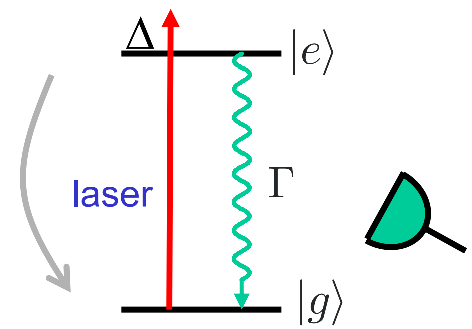
$$dB(t)|\Psi(t)\rangle = 0$$

  - Ito rules:

$$\begin{aligned} [dB(t)]^2 &= [dB^{\dagger}(t)]^2 = 0, \\ dB(t)\, dB^{\dagger}(t) &= dt, \\ dB^{\dagger}(t)\, dB(t) &= 0. \end{aligned}$$

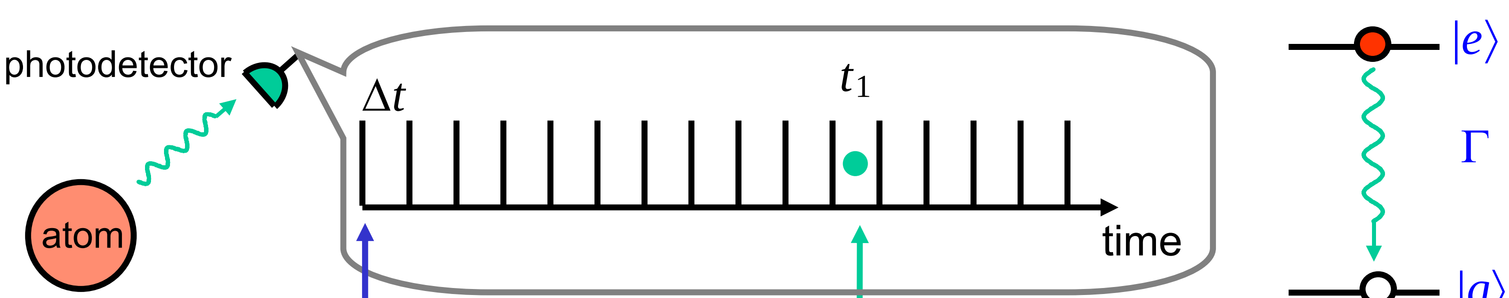
Examples:

- Two-level atom undergoing spontaneous emission
- Driven two-level atom: Optical Bloch Equations



- laser cooling and reservoir engineering of single trapped ion
  – ground state cooling
  – squeezed state generation by reservoir engineering

Example 1: two-level atom undergoing spontaneous decay



initial state $|\psi_c(0)\rangle = c_g|g\rangle + c_e|e\rangle$

while no photon is detected

$$|\psi_c(t)\rangle = \frac{e^{-iH_{\mathrm{eff}}t/\hbar}|\psi_c(0)\rangle}{\|\ldots\|} = \frac{c_g|g\rangle + c_e e^{-\Gamma t/2}|e\rangle}{\|\ldots\|}$$

our knowledge increases that the atom is not in the excited state

a photon is detected $|\psi_c(t+\Delta t)\rangle = \frac{\sqrt{\Gamma}\sigma_-|\tilde{\psi}_c(t)\rangle}{\|\ldots\|} = |g\rangle$

probability that a photon is detected in (t,t+Δt] $\mathcal{P}_1^{(t,t+\Delta t]} = \Gamma|c_e|^2 e^{-\Gamma t}\Delta t$

Example 2: driven two-level atom + spontaneous emission

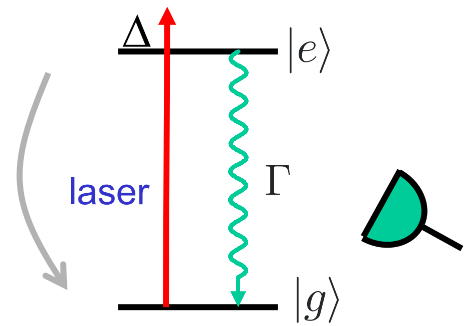
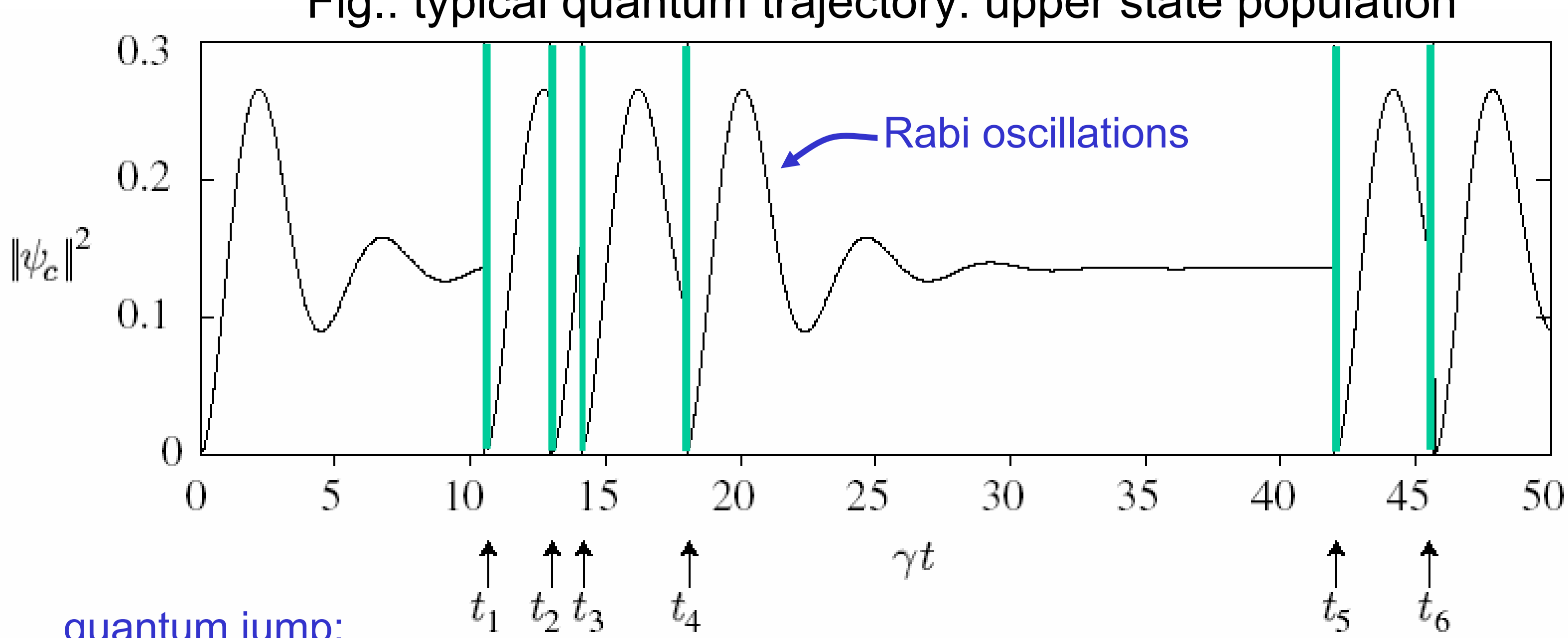


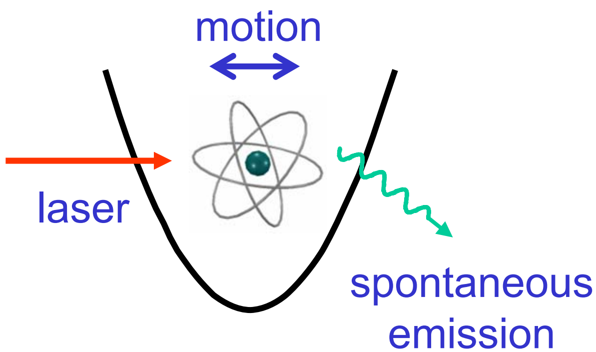
Fig.: typical quantum trajectory: upper state population

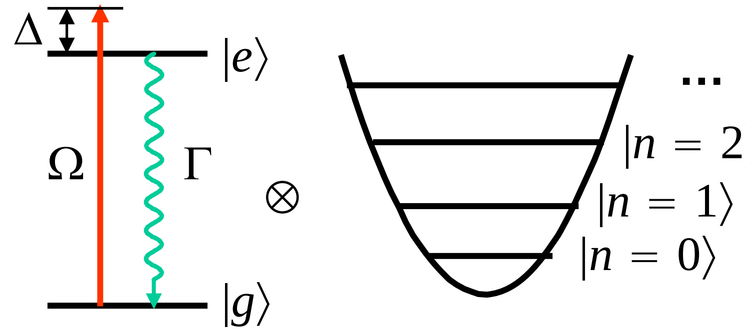


quantum jump:
electron returns to the ground state

(prepares the system)

Example 3: laser cooling of a trapped ion



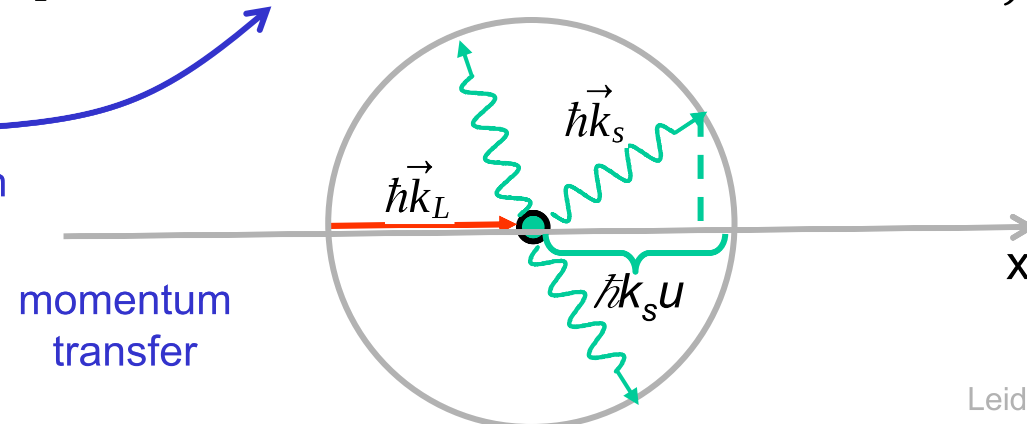


$$H_{\text{sys}} = \left(\frac{\hat{P}^2}{2m} + \frac{1}{2}m\nu^2\hat{X}^2\right) - \Delta|e\rangle\langle e| - \left(\frac{1}{2}\Omega e^{ik\hat{X}}\sigma_- + \text{h.c.}\right)$$

- Master equation (1D):

$$\frac{d}{dt}\rho = -i\left[H_{\text{sys}},\rho\right] + \frac{1}{2}\Gamma\left(2\int_{-1}^{+1} du\, N(u)\left(e^{ik\hat{X}u}\sigma_-\right)\rho\left(\sigma_+ e^{ik\hat{X}u}\right) - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-\right)$$

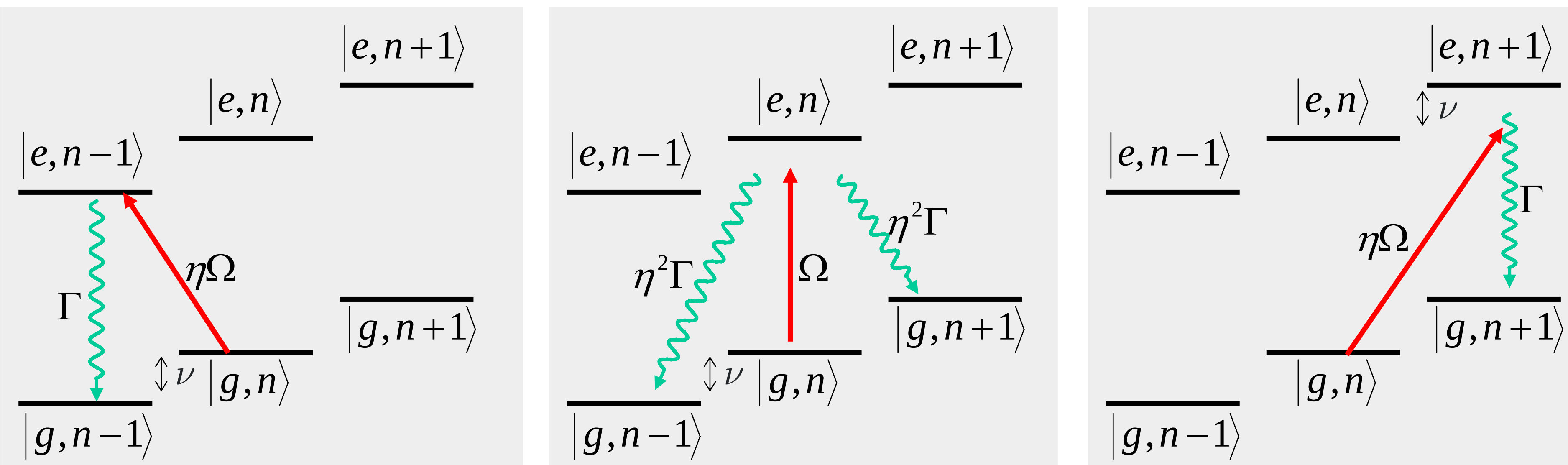
quantum jump operator:
recoil from spontaneous emission



- Lamb-Dicke limit: adiabatic elimination of internal dynamics

$$\dot{\rho} = A_+\left(a\rho a^\dagger - \frac{1}{2}a^\dagger a\rho - \rho\frac{1}{2}a^\dagger a\right) \quad \text{cooling term}$$
$$+ A_-\left(a^\dagger\rho a - \frac{1}{2}aa^\dagger\rho - \rho\frac{1}{2}aa^\dagger\right) \quad \text{heating term}$$

- processes contributing at low intensity

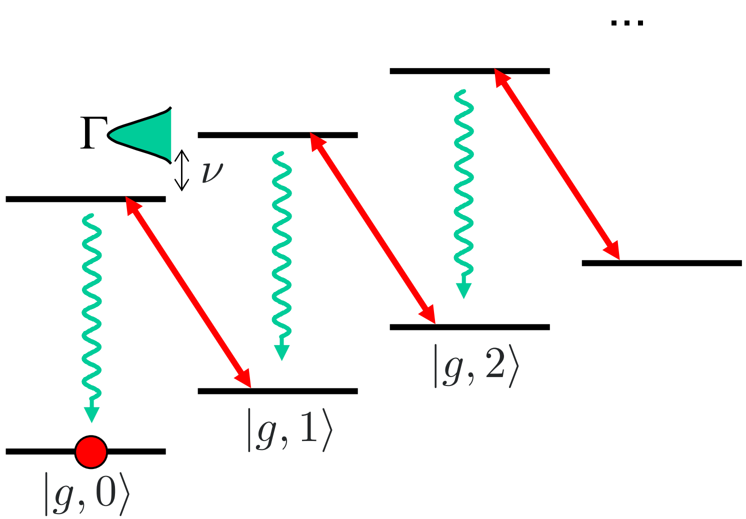


cooling $2\,\mathrm{Re}\,S(-\nu)$ diffusion D heating $2\,\mathrm{Re}\,S(+\nu)$

$$A_\pm = 2\mathrm{Re}[S(\mp\nu) + D]$$

sideband cooling

- ... as optical pumping to the ground state



- master equation

$$\dot{\rho} = A_+\left(a\rho a^\dagger - \tfrac{1}{2}a^\dagger a\rho - \rho\tfrac{1}{2}a^\dagger a\right) \quad (A_+ \gg A_-)$$

- final state

$$\rho_{\text{osc}} \rightarrow |0\rangle\langle 0| \qquad (\Gamma \ll \nu,\ \text{sideband cooling})$$

"dark state" of the jump operator *a*:

$$a|0\rangle = 0$$

Example 4: reservoir engineering / trapped ion

- Consider the master equation (in interaction picture)

$$\dot{\rho} = \frac{1}{2}\gamma(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c)$$

- stationary state = dark state of the jump operator

$$c|\psi_D\rangle = 0 \rightarrow \rho = |\psi_D\rangle\langle\psi_D|$$

assume 1-dim
subspace

prepare a "pure state"

- reservoir engineering

$$c|\psi_D\rangle = 0$$

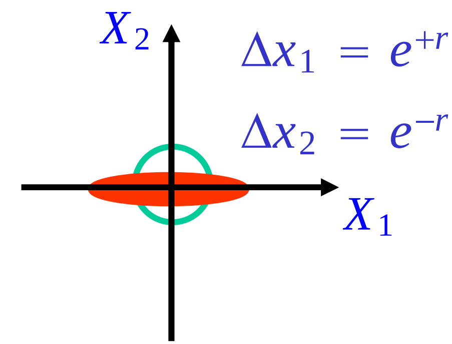
engineer
jump operator

prepare interesting
quantum state

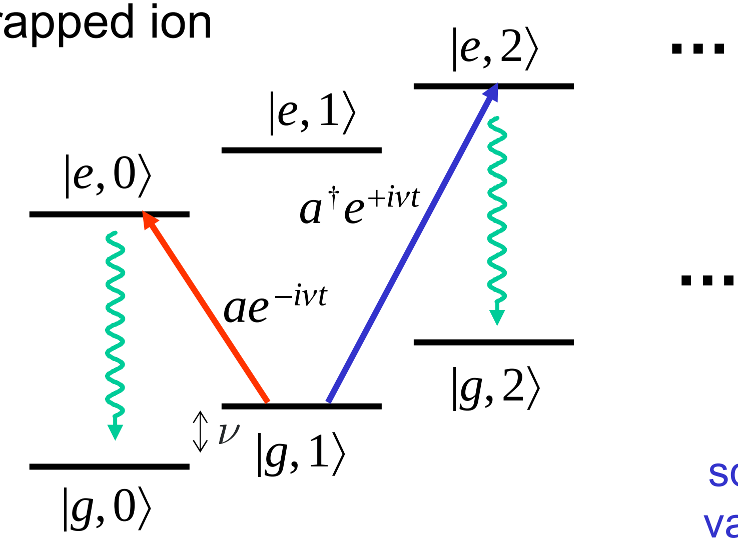
- example: squeezed state of the harmonic oscillator

$$c \equiv \cosh r\, e^{i\epsilon/2} a + \sinh r\, e^{-i\epsilon/2} a^\dagger$$

$|\psi_D\rangle = |r,\epsilon\rangle$ squeezed vacuum

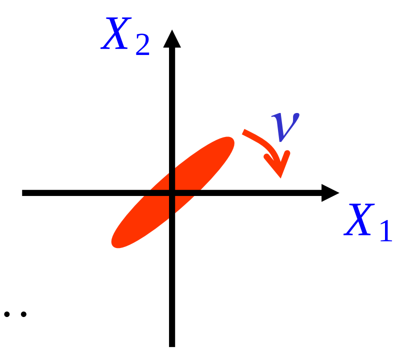


- how? … trapped ion



we "cool" to a squeezed state of motion

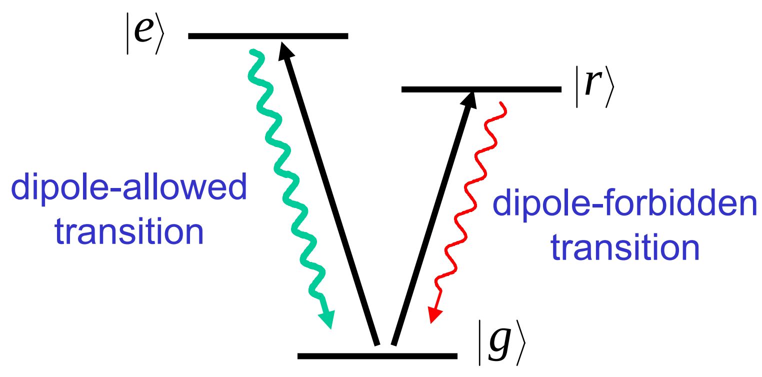
squeezed vacuum in rotating frame



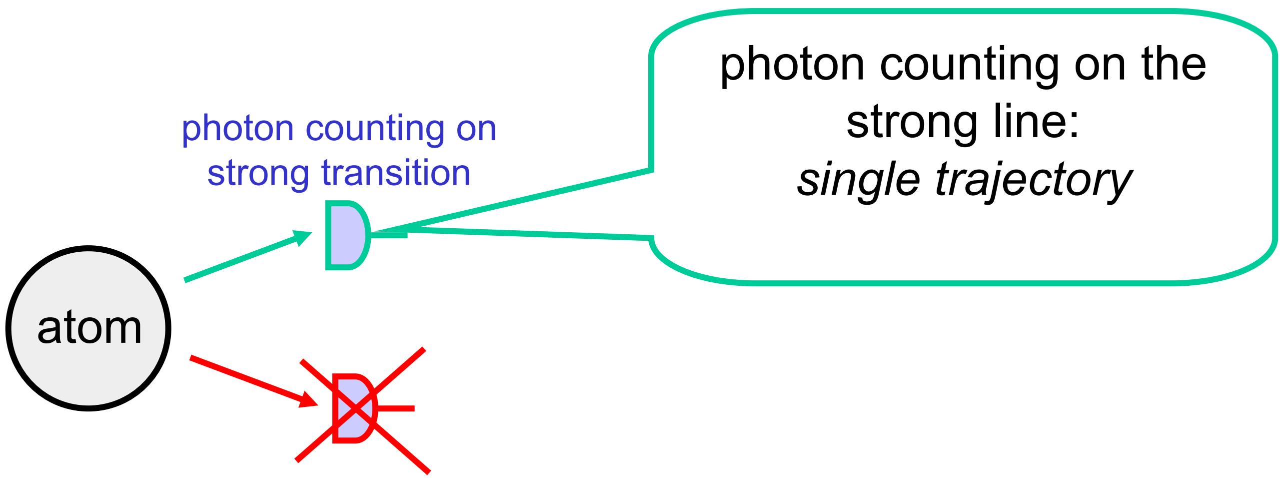
$$\dot\rho = -i[\nu a^\dagger a, \rho] + \gamma \underbrace{(\cosh r\, e^{i\epsilon/2} a e^{-i\nu t} + \sinh r\, e^{-i\epsilon/2} a^\dagger e^{+i\nu t})}_{\text{jump operator}} \rho (\ldots)^\dagger - \ldots$$

Example 5: State measurement & quantum jumps in 3-level systems

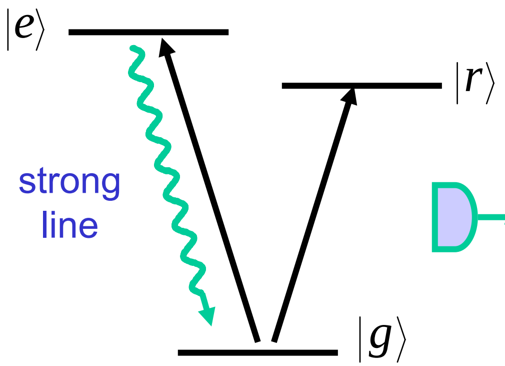
- three level atom

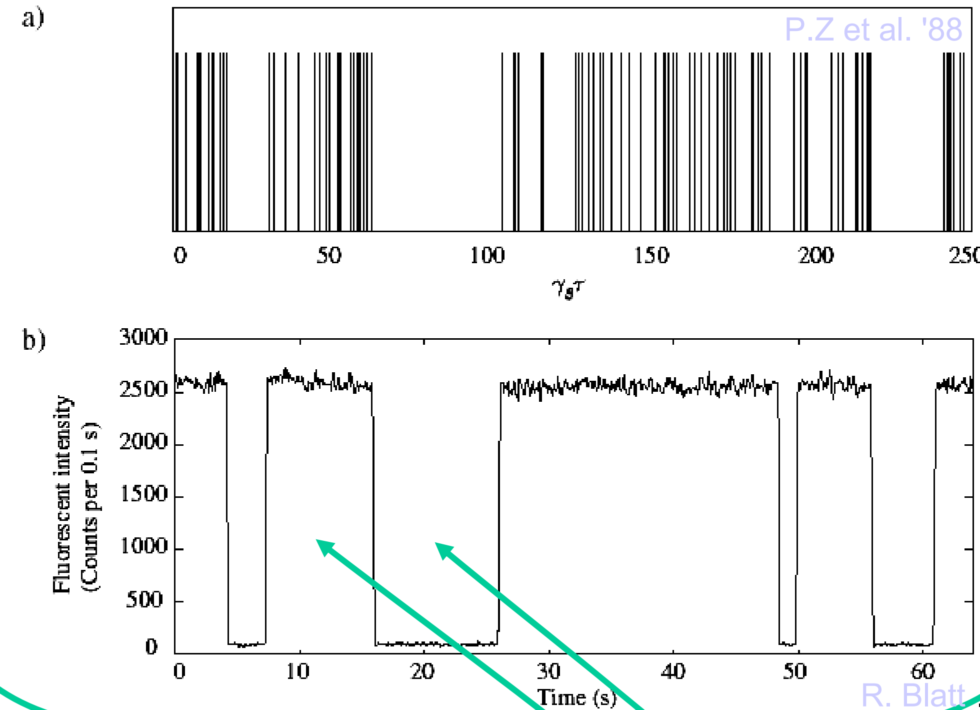


- single atom photon counting



photon counting on strong transition





- ✓ atomic density matrix conditional to observing an emission window

$$\rho_c(t) \longrightarrow |r\rangle\langle r|$$ preparation in metastable state

- ✓ state measurement with 100% efficiency

here: with a weak driving field g - r

$$\psi = \alpha|g\rangle + \beta|r\rangle \qquad \begin{array}{ll} |\alpha|^2 & \ldots \text{ probability } \textbf{NO} \text{ window} \\ |\beta|^2 & \ldots \text{ probability window} \end{array}$$

4. Cascaded Quantum Systems

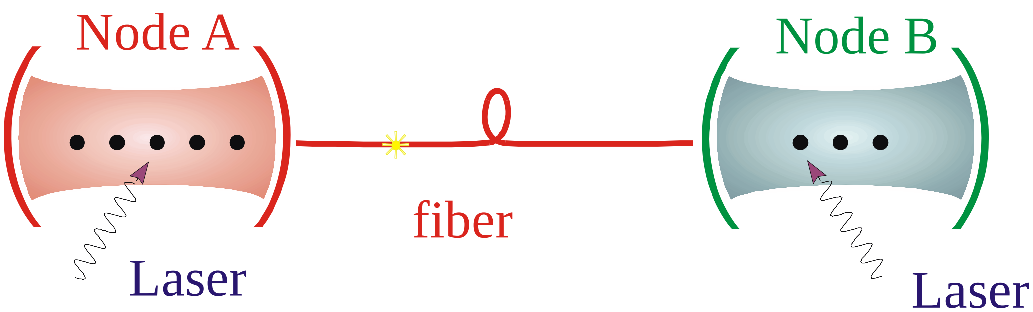
- formal theory
- example
  - optical interconnects

Motivation: Theory of Optical Interconnects

J.I. Cirac, P.Z. H.J. Kimble and H. Mabuchi PRL '97

- A cavity QED implementation

Optical cavities connected by a quantum channel

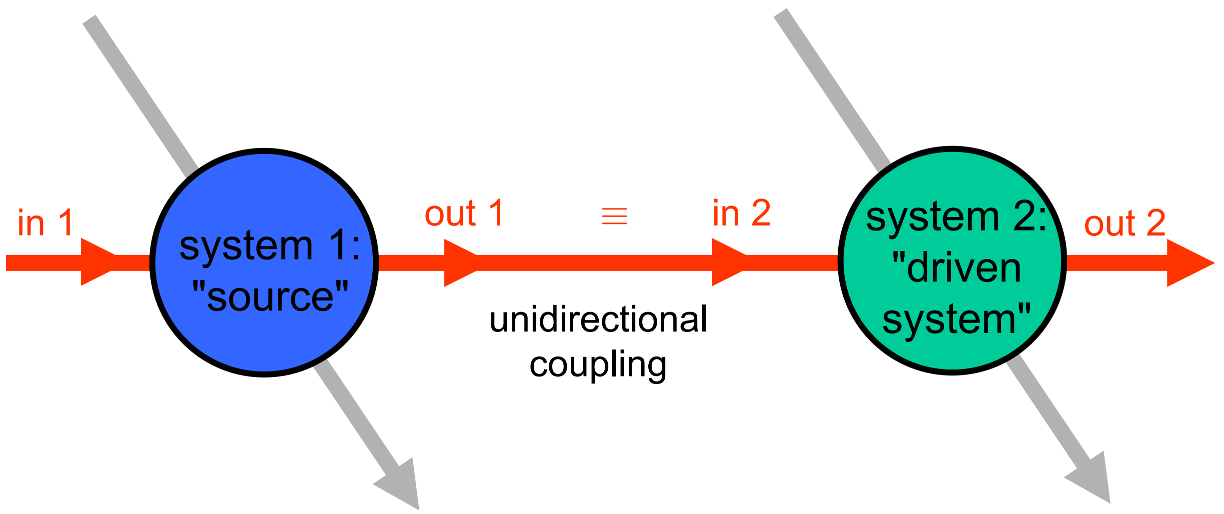


- memory: atoms → - databus: photons → - memory: atoms

- We call this protocol *photonic channel*

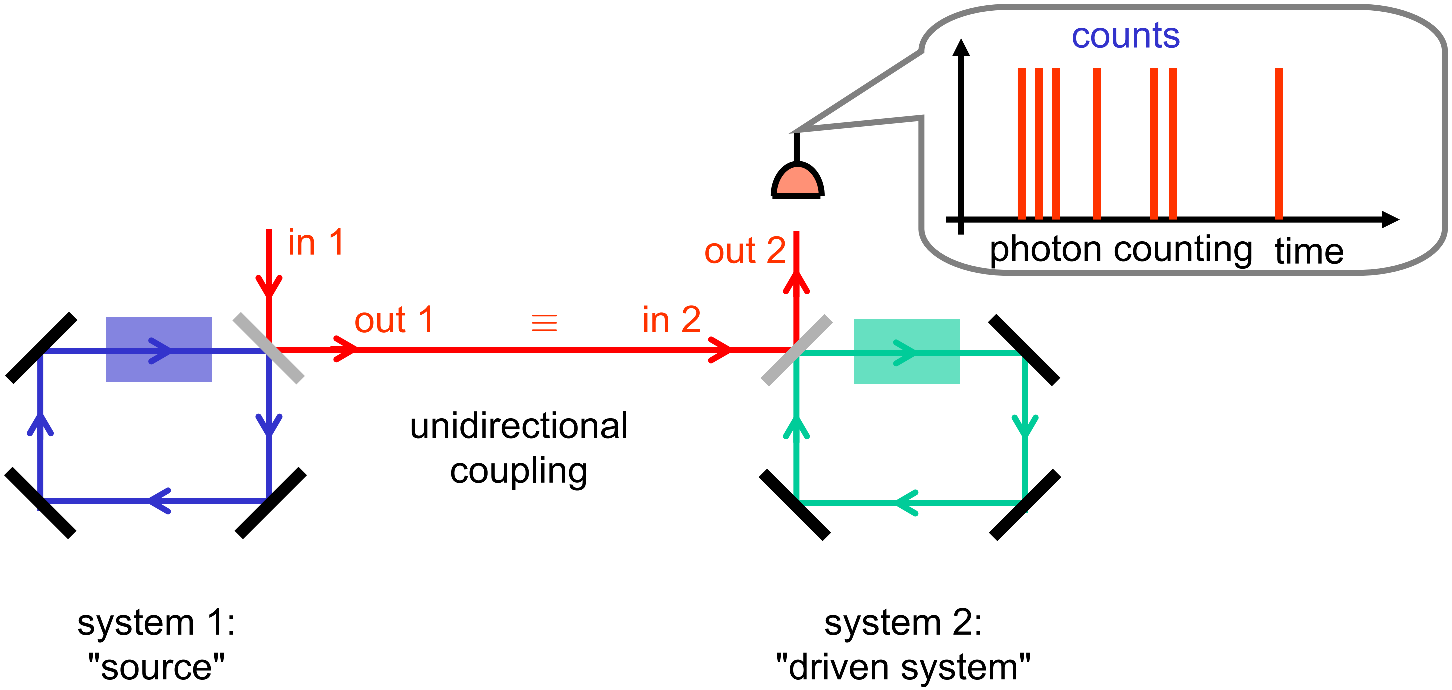
Cascaded Quantum Systems

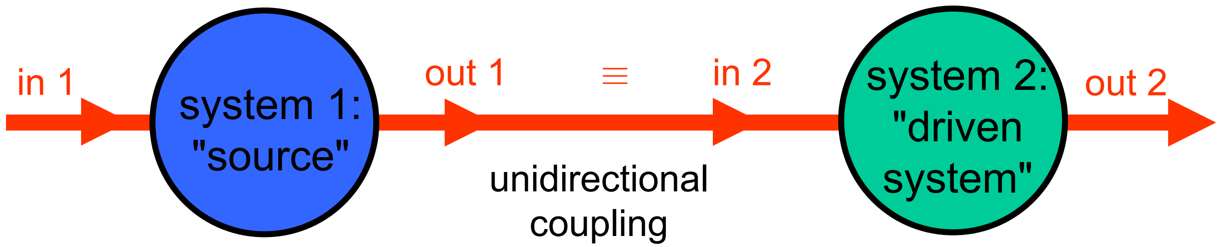
- cascaded quantum system = first quantum system drives a second quantum system: *unidirectional* coupling



Cascaded Quantum Systems

- example of a cascaded quantum system





Hamiltonian

$$H = H_{\text{sys}}(1) + H_{\text{sys}}(2) + H_{\text{B}} + H_{\text{int}}$$

$$H_B = \int_{\omega_0-\vartheta}^{\omega_0+\vartheta} d\omega\, \hbar\omega\, b^\dagger(\omega) b(\omega)$$

with $b(\omega)$ the annihilation operator

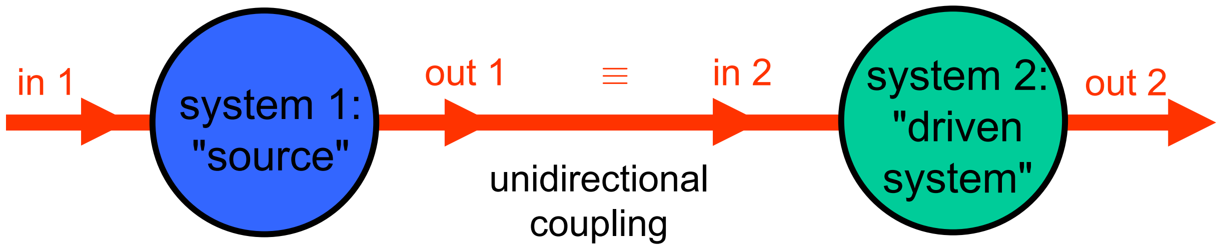
$$[b(\omega), b^\dagger(\omega')] = \delta(\omega - \omega')$$

interaction part

position of
first system

position of
second system

$$\begin{aligned} H_{\text{int}}^{(\vartheta)}(t) = \quad & i\hbar \int d\omega \kappa_1(\omega)[b^\dagger(\omega) e^{-i\omega/c x_1} c_1 - c_1^\dagger b(\omega) e^{+i\omega/c x_1}] \\ & + i\hbar \int d\omega \kappa_2(\omega)[b^\dagger(\omega) e^{-i\omega/c x_2} c_2 - c_2^\dagger b(\omega) e^{+i\omega/c x_2}] \qquad (x_2 > x_1) \end{aligned}$$



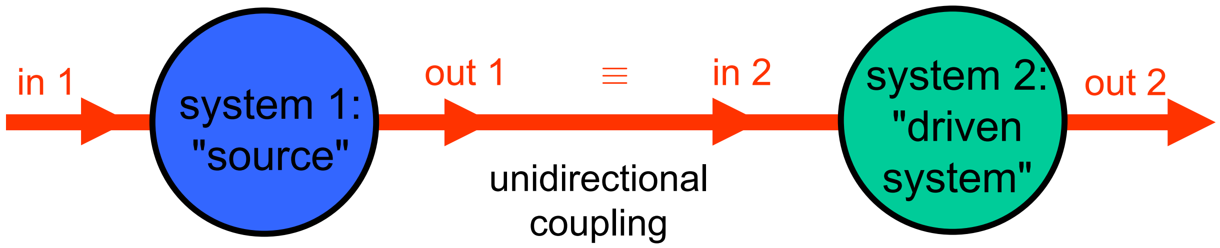
time delay

interaction picture

$$H_{\text{int}}(t) = \imath\hbar\sqrt{\gamma_1}\,[b^\dagger(t)c_1 - b(t)c_1^\dagger] + \imath\hbar\sqrt{\gamma_2}\,[b^\dagger(t^-)c_2 - b(t^-)c_2^\dagger]$$

with $t^- = t - \tau$ where $\tau \to 0^+$

$$b(t) \equiv b_{\text{in}}(t) := \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} d\omega b(\omega) e^{-\imath(\omega-\omega_0)t}$$



Stratonovich SSE

time delay

$$\frac{d}{dt}\Psi(t) = \left\{ -\frac{i}{\hbar}\left(H_{\text{sys}}(1) + H_{\text{sys}}(2)\right) + \right.$$
$$\left. + \sqrt{\gamma_1}\,[b^\dagger(t)c_1 - b(t)c_1^\dagger] + \sqrt{\gamma_2}\,[b^\dagger(t^-)c_2 - b(t^-)c_2^\dagger] \right\} \Psi(t)$$

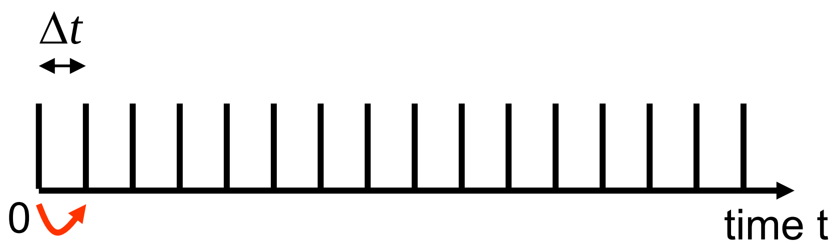
Initial condition:

$$|\Psi\rangle = |\psi\rangle \otimes |\text{vac}\rangle$$

Notation:

$$\sqrt{\gamma_1}\,c_1 \to c_1, \quad \sqrt{\gamma_2}\,c_2 \to c_2, \quad \hbar = 1$$

First time step



$$U(\Delta t)|\Psi(0)\rangle \;=\; \Big\{\hat{1} - i[H(1)+H(2)]\Delta t + (\,c_2 + c_1\,)\int_0^{\Delta t} dt\, b^\dagger(t)$$

$$(-i)^2 \int_0^{\Delta t} dt_1 \int_0^{t_2} dt_2 \underbrace{(-b(t_1)c_1^\dagger - b(t_{\bar{1}})c_2^\dagger)}_{\text{destruction}}\underbrace{(b^\dagger(t_2)c_1 + b^\dagger(t_{\bar{2}})c_2)}_{\text{creation}} + \ldots\Big\}|\Psi(0)\rangle$$

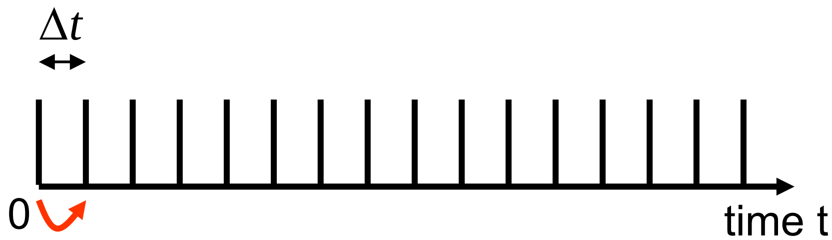
$$= \int_0^{\Delta t} dt_1 \int_0^{t_2} dt_2 (-\delta(t_1-t_2)c_1^\dagger c_1 + \delta(t_1-t_2+\tau)c_1^\dagger c_2$$

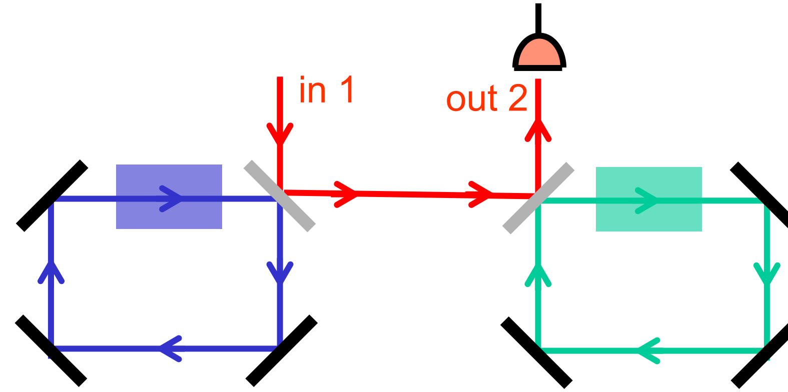
time delay!

$$-\,\delta(t_1-\tau-t_2)c_2^\dagger c_1 - \delta(t_1-t_2)c_2^\dagger c_2)|\mathrm{vac}\rangle$$

$$= \Big(-\frac{1}{2}c_1^\dagger c_1 + 0 \underbrace{- c_2^\dagger c_1}_{\text{reabsorption}} - \frac{1}{2}c_2^\dagger c_2\Big)|\mathrm{vac}\rangle\Delta t$$

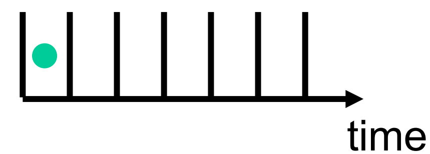
First time step



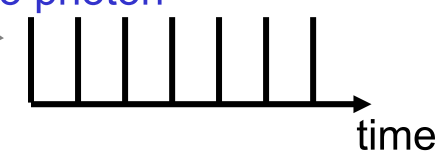


$$U(\Delta t)|\Psi(0)\rangle = \left\{\hat{1} - iH_{\text{eff}}\Delta t + (c_2 + c_1)\Delta B^\dagger(0)\right\}|\Psi(0)\rangle$$

one photon



no photon



- effective Hamiltonian

$$H_{\text{eff}} = H(1) + H(2) - i\frac{1}{2}c_1^\dagger c_1 - i\frac{1}{2}c_2^\dagger c_2 - \underline{ic_2^\dagger c_1} \text{ reabsorption}$$

$$= \underbrace{\left\{H(1) + H(2) + i\frac{1}{2}(c_1^\dagger c_2 - c_2^\dagger c_1)\right\}}_{\text{hermitian}} \underbrace{- i\frac{1}{2}c^\dagger c}_{\text{decay}} \quad (\text{with } c = c_2 + c_1)$$

- we identify $(c_2 + c_1)$ with the "jump operator"

Summary of results:

- Ito-type stochastic Schrödinger equation:

$$\begin{aligned} d|\Psi(t)\rangle &= |\Psi(t+dt)\rangle - |\Psi(t)\rangle \\ &= \Big\{\hat{1} - iH_{\mathrm{eff}}dt + (c_1 + c_2)dB^\dagger(t)\Big\}|\Psi(0)\rangle \end{aligned}$$

$$H_{\mathrm{eff}} = H_{\mathrm{sys}} + i\tfrac{1}{2}(c_1^\dagger c_2 - c_2^\dagger c_1) - i\tfrac{1}{2}c^\dagger c$$

- master equation for source + system:

Version 1:

$$\tfrac{d}{dt}\rho = -i(H_{\mathrm{eff}}\rho - \rho H_{\mathrm{eff}}^\dagger) + \tfrac{1}{2}(2c\rho c^\dagger - c^\dagger c\rho - \rho c^\dagger c)$$

Lindblad form

Version 2:

$$\begin{aligned} \frac{d}{dt}\rho &= -i[H_{\mathrm{sys}},\rho] \\ &+ \frac{1}{2}\{2c_1\rho c_1^\dagger - \rho c_1^\dagger c_1 - c_1^\dagger c_1\rho\} + \frac{1}{2}\{2c_2\rho c_2^\dagger - \rho c_2^\dagger c_2 - c_2^\dagger c_2\rho\} \\ &- \underline{\{[c_2^\dagger, c_1\rho] + [\rho c_1^\dagger, c_2]\}}. \end{aligned}$$

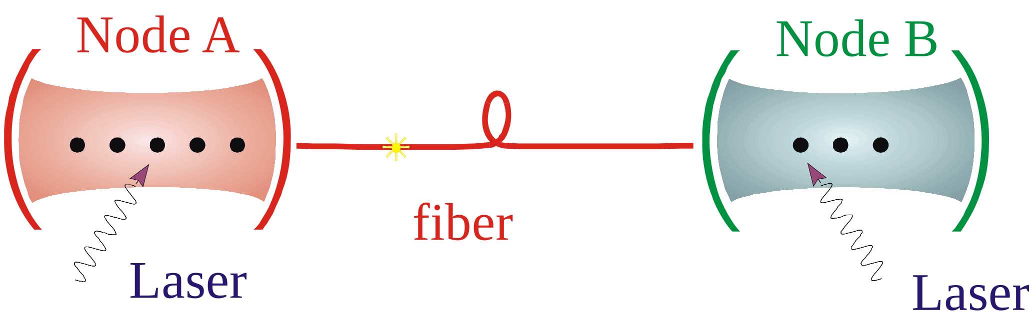
unidirectional coupling of source to system

Example: Optical Interconnects

J.I. Cirac, P.Z. H.J. Kimble and H. Mabuchi PRL '97

- A cavity QED implementation

Optical cavities connected by a quantum channel

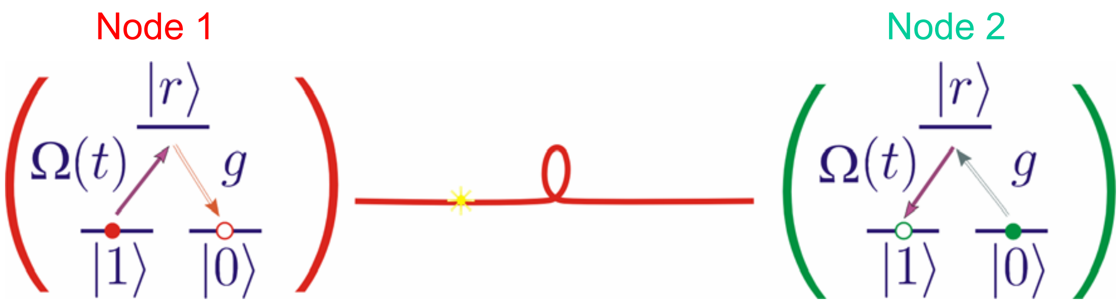


memory: atoms → databus: photons → memory: atoms

- We call this protocol *photonic channel*

System

- System



- Hamiltonian: eliminate the excited state adiabatically

Hamiltonian $$H = H_1 + H_2$$

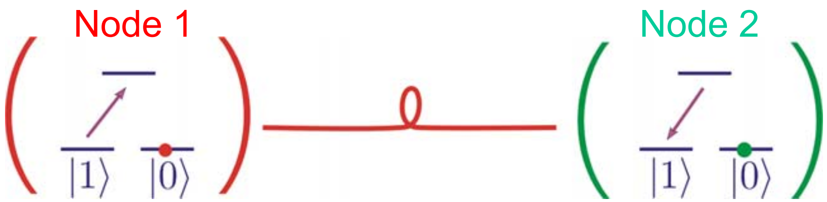
node i $$\hat{H}_i = -\delta \hat{a}_i^\dagger \hat{a}_i - i g_i(t) \left[|1\rangle_i\langle 0| a - \text{h.c.} \right] \quad (i = 1, 2)$$

Raman detuning $$\delta = \omega_L - \omega_c$$

Rabi frequency $$g_i(t) = \frac{g\Omega_i(t)}{2\Delta}$$

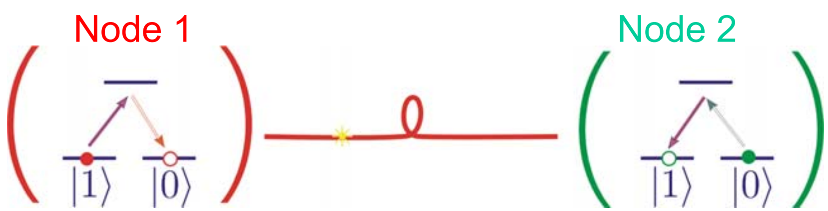
Ideal transmission

- sending the qubit in state 0



$$|0\rangle|0\rangle \quad \rightarrow \quad |0\rangle \ |0\rangle$$

- sending the qubit in state 1



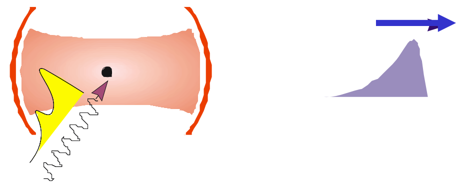
$$|1\rangle|0\rangle \quad \rightarrow \quad |0\rangle \ |1\rangle$$

- superpositions

$$[\alpha \ |0\rangle + \beta|1\rangle] \ |0\rangle \quad \rightarrow \quad |0\rangle \, [\alpha \ |0\rangle + \beta \ |1\rangle]$$

Physical picture as guideline for solution

- Ideal transmission = no reflection from the second cavity
- Physical picture as guideline for solution: "time reversing cavity decay"
  - consider one cavity alone



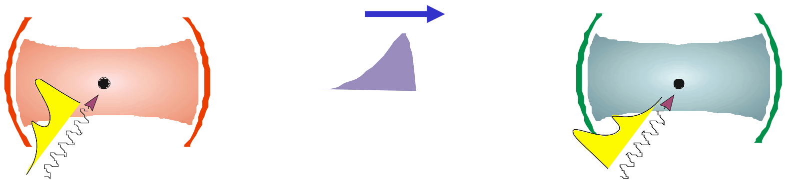
decay

  - run the movie backwards

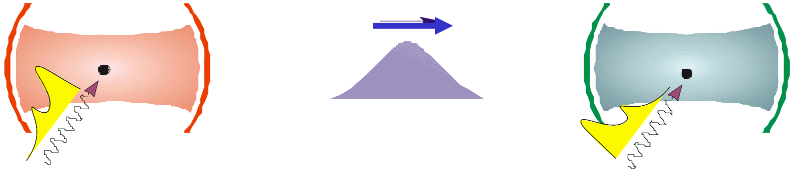
restore

inverse laser pulse

– two cavities



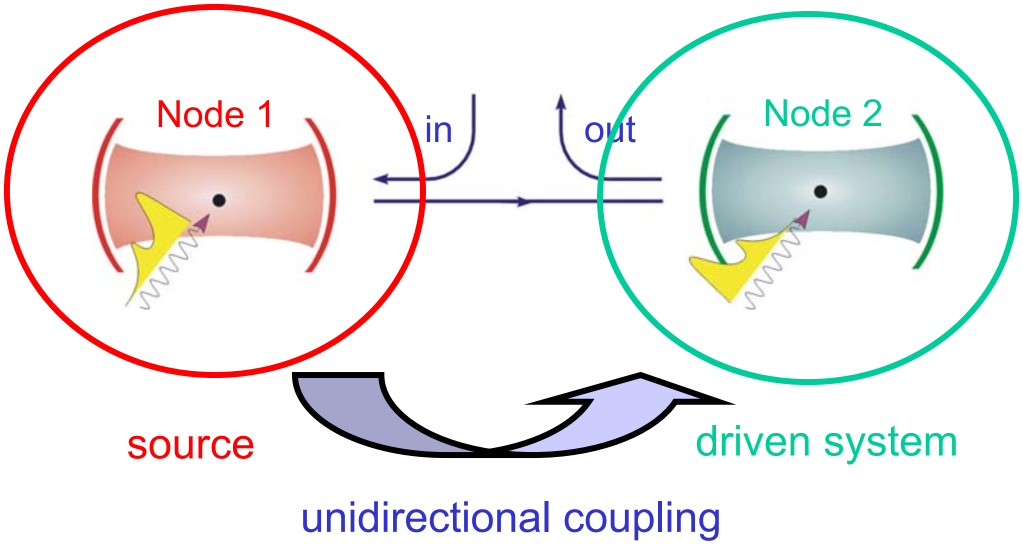
– design laser pulses to make the outgoing wavepacket symmetric



– we try a solution where the laser pulses are the time reverse of each other

Description ... as a cascaded quantum systems

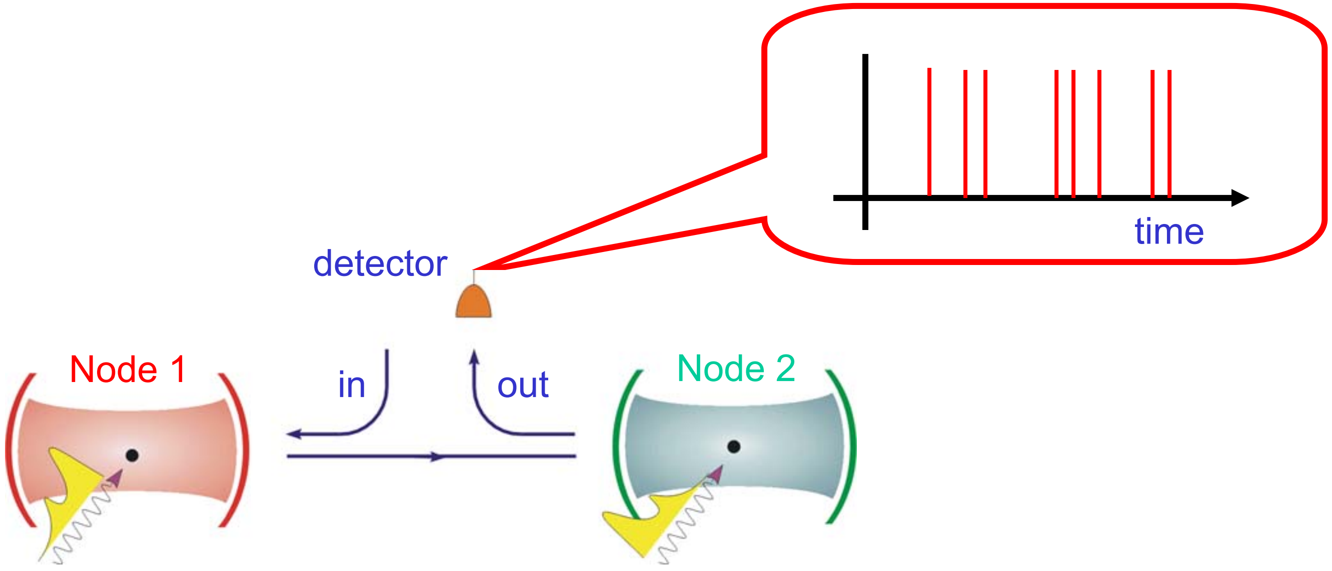
- cascaded quantum system



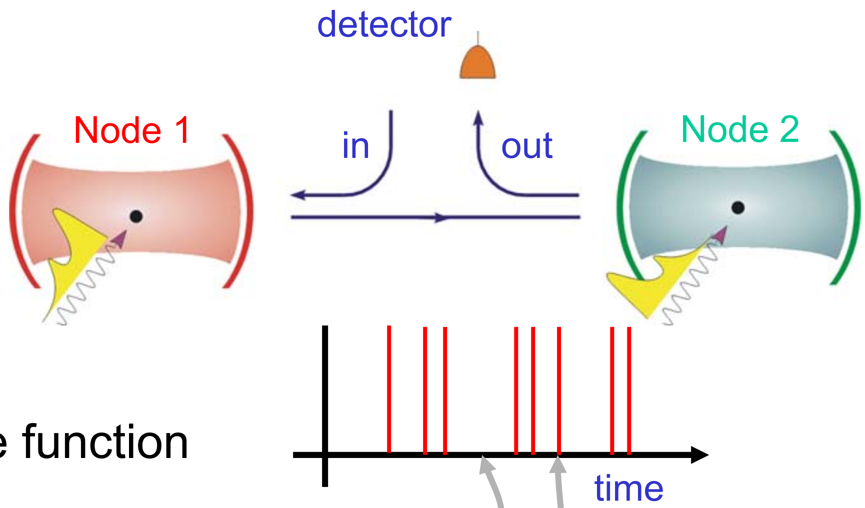
- a theory of cascaded quantum systems H. Carmichael and C. Gardiner, PRL '94

... quantum trajectories

- Quantum trajectory picture: *evolution conditional to detector clicks*



- We want *no reflection*: this is equivalent to requiring that the detector never clicks (= dark state of the cascaded quantum system)



- system wave function $|\Psi_c(t)\rangle$

- between the quantum jumps the wave function evolves with

$$\hat{H}_{\text{eff}}(t) = \hat{H}_1(t) + \hat{H}_2(t) - i\kappa\left(\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2 + 2\,\hat{a}_2^\dagger\,\hat{a}_1\right)$$

- quantum jump

$$|\psi_c(t+dt)\rangle \propto \hat{c}|\psi_c(t)\rangle \qquad (\text{with } \hat{c} = \hat{a}_1 + \hat{a}_2)$$

- probability for a jump $\propto \langle\psi_c(t)|\hat{c}^\dagger\hat{c}|\psi_c(t)\rangle$

- condition that no jump occurs

$$\langle\psi_c(t)|\hat{c}^\dagger\hat{c}|\psi_c(t)\rangle \overset{!}{=} 0 \quad \Longrightarrow \quad \hat{c}|\psi_c(t)\rangle = 0 \quad \forall t$$

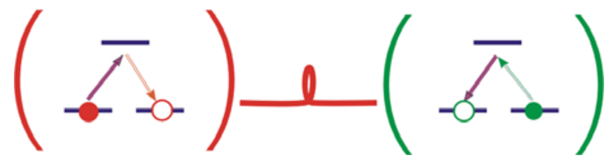
no reflection

Equations

- Wave function for quantum trajectories: ansatz

atoms cavity modes

$$
\begin{aligned}
|\Psi_c(t)\rangle \quad &= \quad |00\rangle|00\rangle \\
&+ \quad \Big[\alpha_1(t)|10\rangle|00\rangle + \alpha_2(t)|01\rangle|00\rangle \\
&\qquad + \beta_1(t)|00\rangle|10\rangle + \beta_2(t)|00\rangle|01\rangle\Big].
\end{aligned}
$$

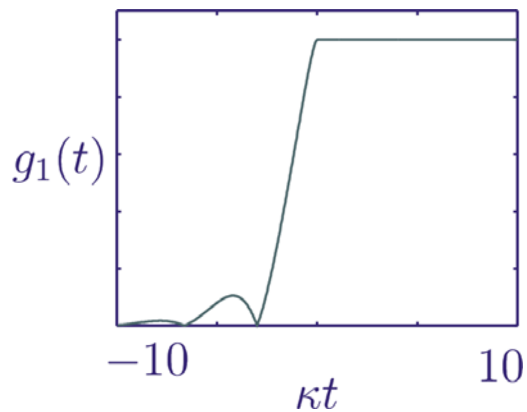


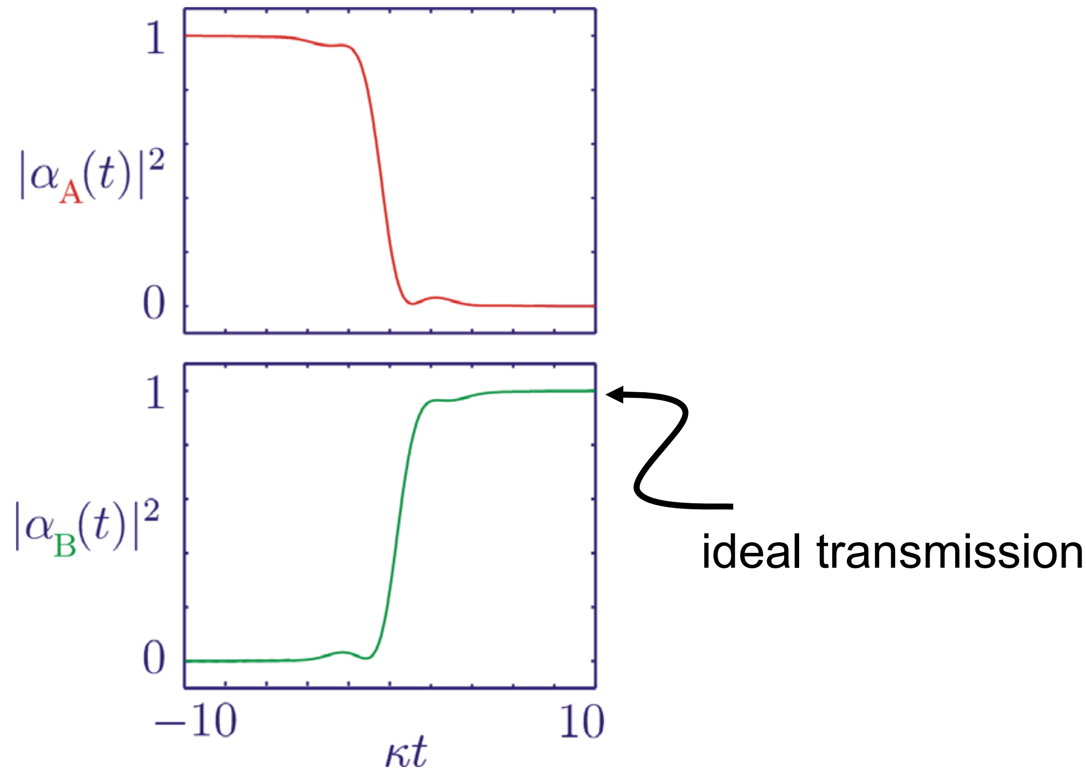
ONE excitation in system

- we derive equations of motion ... and impose the dark state conditions
- we find exact analytical solutions for pulse shapes leading to "no reflection" ...

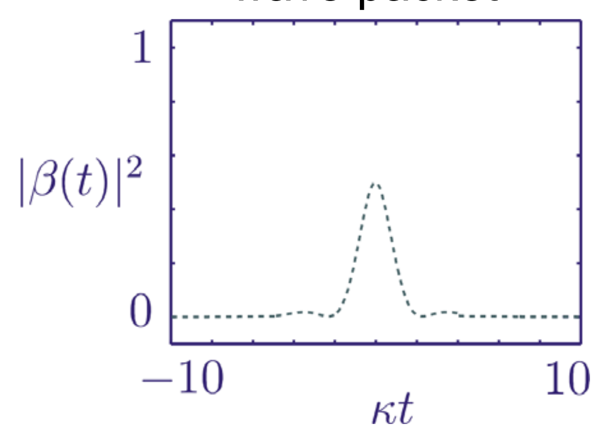
Results

pulse shape





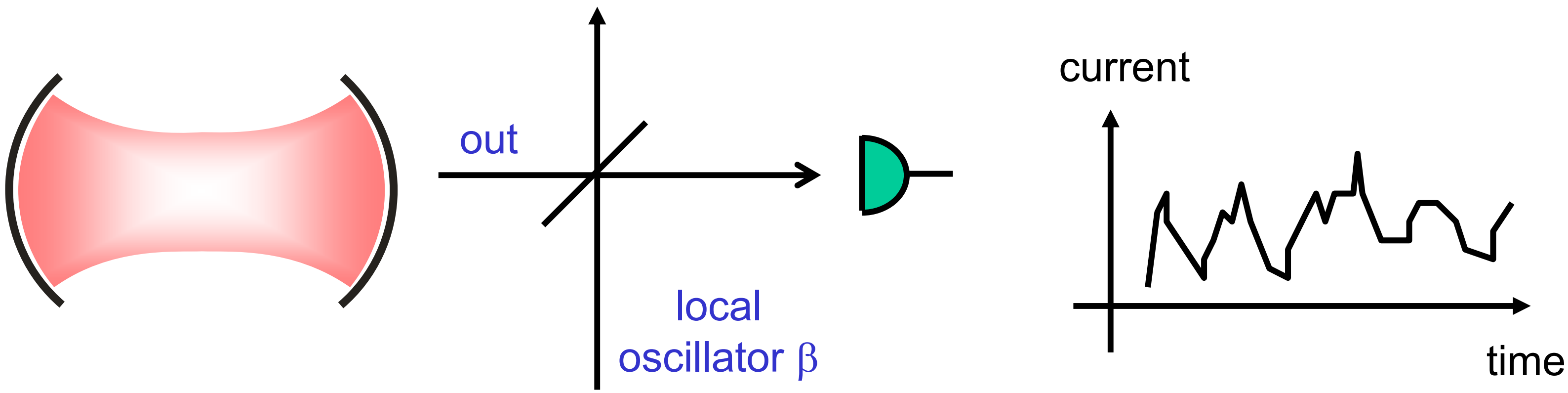
wave packet



similar theory developed for …

Homodyne Detection

- homodyne detection



- conditional system wave function

$$d|\psi_X(t)\rangle \;= \Big[(-iH - \tfrac{1}{2}\gamma c^\dagger c)dt + \sqrt{\gamma}\, c\, dX(t)\Big]|\psi_X(t)\rangle$$

with $dX(t) = \sqrt{\gamma}\,\langle x(t)\rangle_c dt + dW(t)$ and $dW(t)$ a Wiener increment

homodyne current

$c+c^{+}$

shot noise