A COURSE IN FIELD THEORY

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0 Introduction

Field theory is most successful in describing the process of scattering of particles in the context of the Standard Model, and in particular in the Electromagnetic and Weak Interactions. The Large Electron Positron (LEP) collider operated 1989 till 2000. In a ring of 27 km in diameter, electrons and positrons were accelerated in opposite directions to energies of approximately 45 GeV. This energy is equivalent to half the mass (expressed as energy through $E = mc^2$) of the neutral Z^o vector boson mass, which mediates part of the weak interactions. The Z^o particle can thus be created in electron-positron annihilation, at the regions where the electron and positron beams intersect. As a Z^o can be formed out of an electron and its antiparticle, the positron, it can also decay into these particles. Likewise it can decay in a muon-antimuon pair and other combinations (like hadrons). The cross-section for the formation of Z^o particles shows a resonance peak around the energy where the Z^o particle can be formed. The width of this peak is a measure for the probability of the decay of this particle. By the time you have worked yourself through this course, you should be able to understand how to calculate this cross-section, which in a good approximation is given by

$$\sigma = \frac{4\pi\alpha_e^2 E^2/27}{(E^2 - M_{Z^o}^2)^2 + M_{Z^o}^2\Gamma_{Z^o}^2} \quad , \tag{0.1}$$

expressed in units where $\hbar = c = 1$. $\alpha_e = \frac{e^2}{4\pi} \sim 1/137.037$, is the fine-structure constant, E is twice the beam energy, M_{Z^o} the mass and Γ_{Z^o} the decay rate (or width) of the Z^o vector boson. The latter gets a contribution from all particles in which the Z^o can decay, in particular from the decay in a neutrino and antineutrino of the three known types (electron, muon and tau neutrinos). Any other unknown neutrino type (assuming their mass to be smaller than half the Z^o mass) would contribute likewise. Neutrinos are very hard to detect directly, as they have no charge and only interact through the weak interactions (and gravity) with other matter. With the data obtained from the LEP collider (the figure is from the ALEPH collaboration) one has been able to establish that there are *no* unknown types of light neutrinos, i.e. $N_\nu = 3$, which has important consequences (also for cosmology).

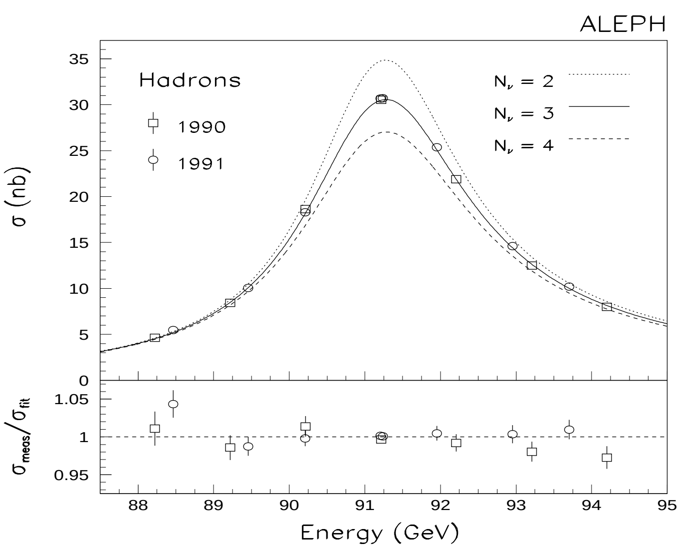

fig. 1

The main aim of this field theory course is to give the student a working knowledge *and understanding* of the theory of particles and fields, with a description of the Standard Model towards the end. We feel that an essential ingredient of any field theory course has to be to teach the student how Feynman rules are derived from first principles. With the path integral approach this is feasible. Nevertheless, it is equally essential that the student learns how to use these rules. This is why the problems form an integral part of this course. As Julius Wess put it during his course as a Lorentz professor at our Institute “you won’t become a good pianist by listening to good concerts”.

These lecture notes reflect the field theory courses I taught in the fall of 1992 at Utrecht, and 1993, 1994, 1996, 1998 and 2000 at Leiden. I owe much to my teachers in this field, Martinus Veltman and Gerard ’t Hooft. As I taught in Utrecht from ’t Hooft’s lecture notes “Inleiding in de gequantiseerde veldentheorie” (Utrecht, 1990) it is inevitable that there is some overlap. In Leiden I spent roughly 25% longer in front of the classroom (3 lectures of 45 minutes each for 14 weeks), which allowed me to spend more time and detail on certain aspects. The set of problems, 40 in total, were initially compiled by Karel-Jan Schoutens with some additions by myself. In their present form, they were edited by Jeroen Snippe.

Of the many books on field theory that exist by now, I recommend the student to consider using “Quantum Field Theory” by C. Itzykson and J.-B. Zuber (McGraw-Hill, New York, 1980) in addition to these lecture notes, because it offers material substantially beyond the content of these notes. I will follow to a large extent their conventions. I also recommend “Diagrammatica: The path to Feynman diagrams”, by M. Veltman (Cambridge University Press, 1994), for its unique style. The discussion on unitarity is very informative and it has an appendix comparing different conventions. For more emphasis on the phenomenological aspects of field theory, which are as important as the theoretical aspects (a point Veltman often emphasised forcefully) I can recommend “Field Theory in Particle Physics” by B. de Wit and J. Smith (North-Holland, Amsterdam, 1986). For path integrals, which form a crucial ingredient of these lectures, the book “Quantum mechanics and path integrals” by R.P. Feynman and A.R. Hibbs (McGraw-Hill, New York, 1978) is a must. Finally, for an introduction to the Standard Model, useful towards the end of this course, the book “Gauge theories of weak interactions”, by J.C. Taylor (Cambridge Univ.Press, 1976), is very valuable.

1 Motivation

Field theory is the ultimate consequence of the attempts to reconcile the principles of relativistic invariance with those of quantum mechanics. It is not too difficult, with a lot of hindsight, to understand why a field needs to be introduced. Although this is not an attempt to do justice to history - and perhaps one should spare the student the long struggle to arrive at a consistent formulation, which most likely has not completely crystalised yet either - but the traditional approach of introducing the concept is not very inspiring and most often lacks physical motivation. In the following discussion I was inspired by “Relativistic Quantum Theory” from V.B. Berestetskii, E.M. Lifshitz and L.P. Pitaevskii (Pergamon Press, Oxford, 1971). The argument goes back to L.D. Landau and R.E. Peierls (1930).

An important consequence of relativistic invariance is that no information should propagate at a speed greater than that of light. Information can only propagate inside the future light cone. Consider the Schrödinger equation

$$i\hbar\frac{\partial\Psi(\vec{x},t)}{\partial t}=H\Psi(\vec{x},t)\quad. \tag{1.1}$$

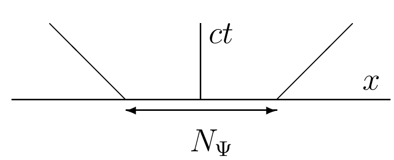

fig. 2

Relativistic invariance should require that $\Psi(\vec{x},t)=0$ for all $(\vec{x},t)$ outside the light cone of the support $N_\Psi=\{\vec{x}|\Psi(\vec{x},0)\neq 0\}$ of the wave function at $t=0$.

Index to Problems